Dynamics of Systems

CTB 2300

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Contents of Lecture 13

1. **Equation of motion for the water level in a harbour connected with Sea by parallel and sequential canals**
2. **Hydraulic 2DOF system**

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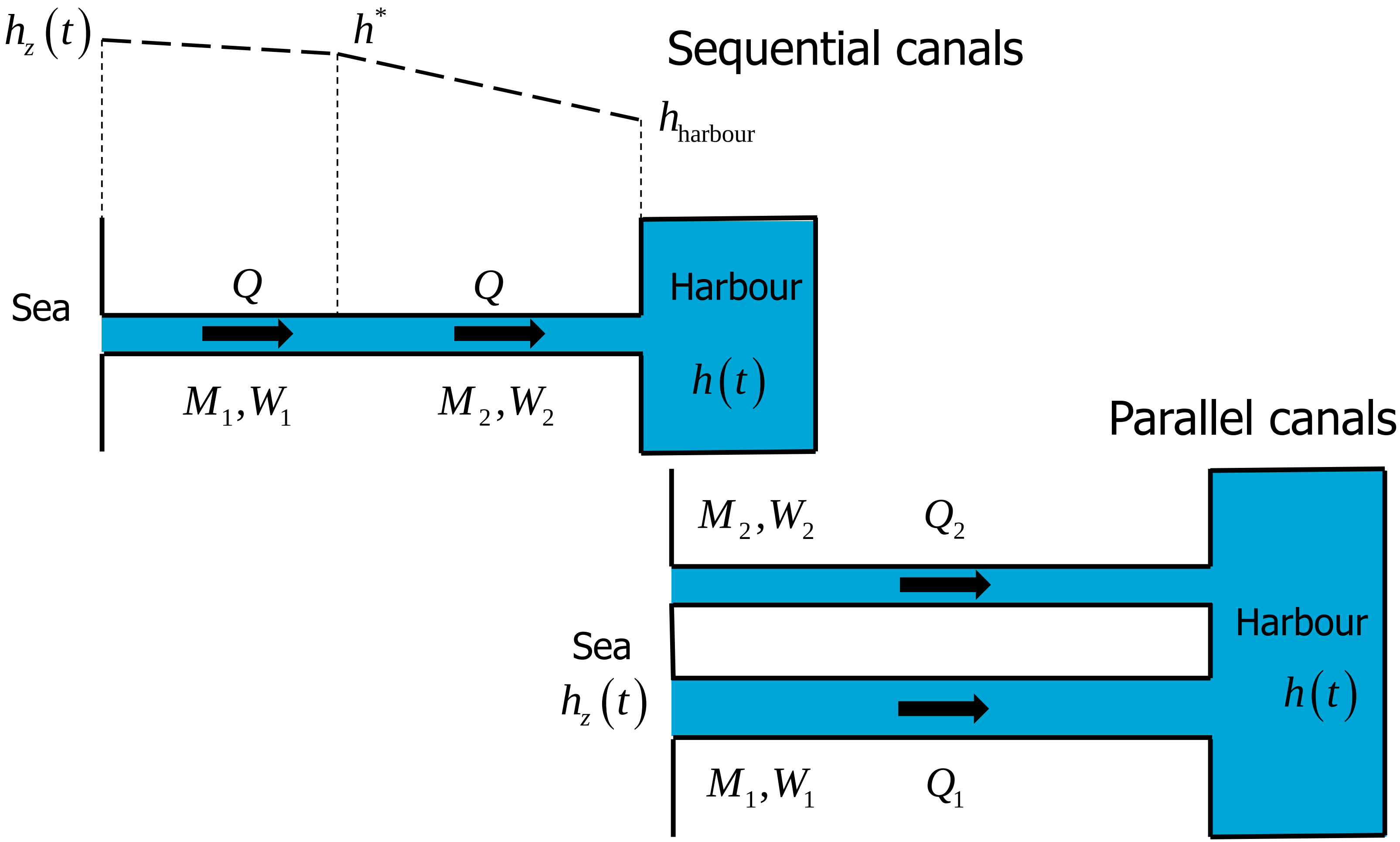
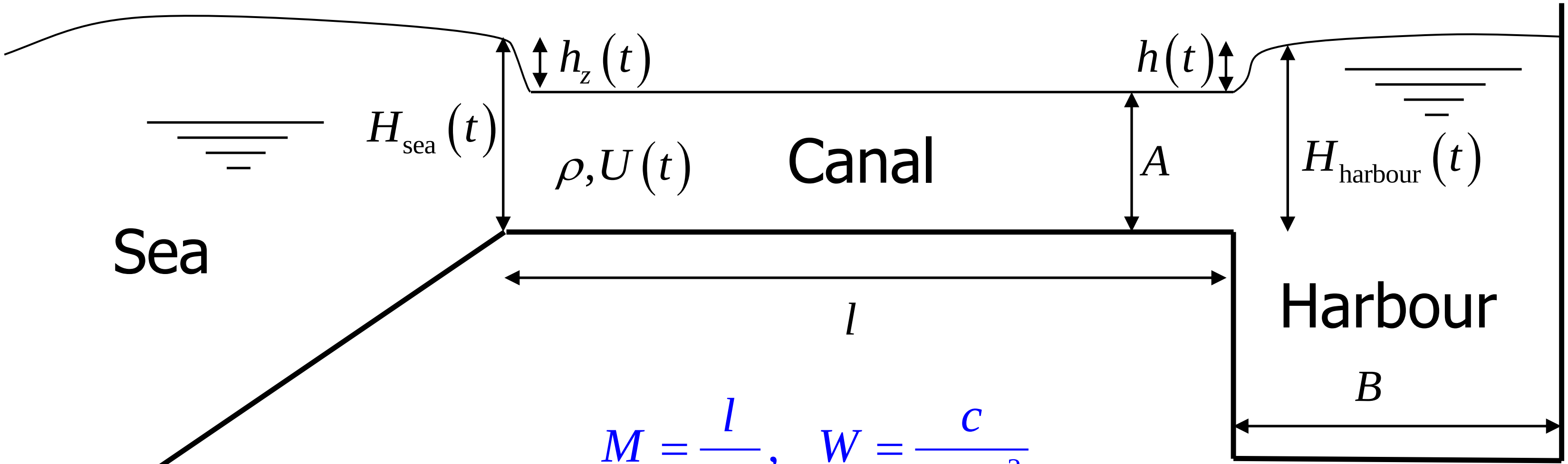
Sequential and parallel canals



Recollection of Lecture 12



$$M = \frac{l}{gA}, \quad W = \frac{c}{\rho g A^2}$$

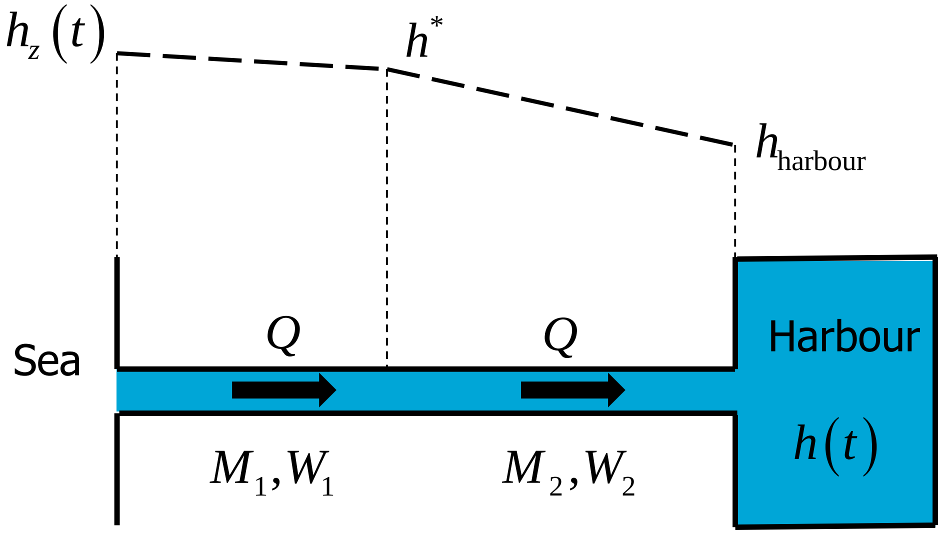
$$\begin{cases} \dfrac{d(m_{canal}U)}{dt} = A(p_{sea} - p_{harbour}) - cU \\ m_{canal} = \rho A l \end{cases} \Rightarrow M\dot{Q} + WQ = h_z - h \qquad (Q = AU)$$

$$\begin{cases} \dfrac{d(m_{harbour})}{dt} = \rho A U \\ m_{harbour} = \rho B H_{harbour} \end{cases} \Rightarrow B\dot{h} = Q$$

Discharge (debiet): Q

Inertia (traagheid): M

2 canals in sequence



$$\begin{cases} M_i \dot{Q}_i + W_i Q_i = h_{\text{from}} - h_{\text{to}} \\ B\dot{h} = Q_{\text{in}} - Q_{\text{out}} \end{cases}$$

$$\begin{cases} M_1 \dot{Q} + W_1 Q = h_z - h^* \\ M_2 \dot{Q} + W_2 Q = h^* - h \\ B\dot{h} = Q \end{cases} \Rightarrow \begin{cases} (M_1 + M_2)\dot{Q} + (W_1 + W_2) Q = h_z - h \\ B\dot{h} = Q \end{cases}$$

$$(M_1 + M_2) B\ddot{h} + (W_1 + W_2) B\dot{h} + h = h_z$$

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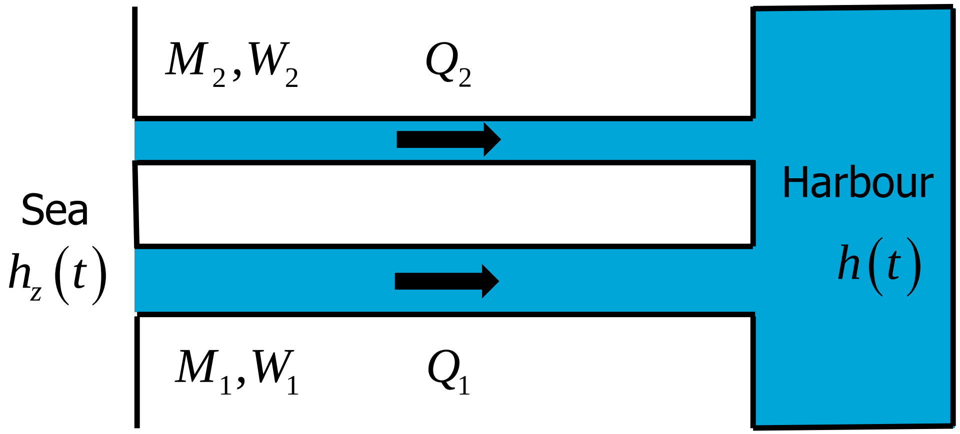
2 different canals in parallel

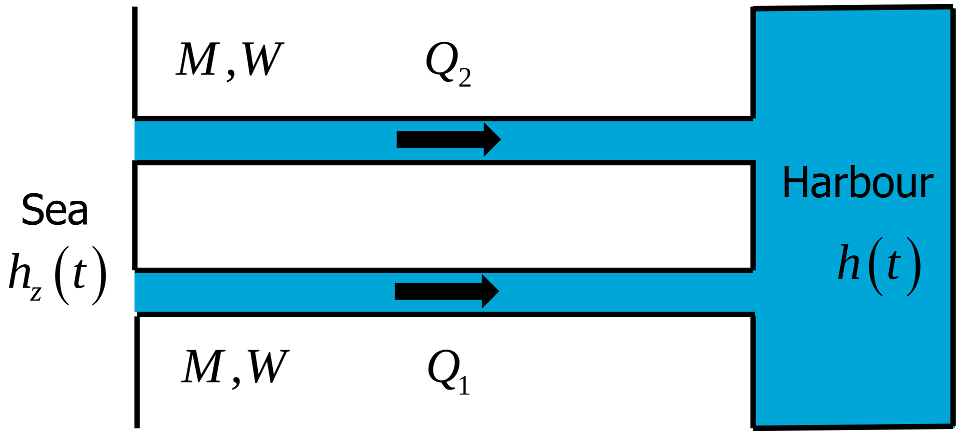


$$\begin{cases} M_i\dot{Q}_i + W_i Q_i = h_{\text{from}} - h_{\text{to}} \\ B\dot{h} = Q_{\text{in}} - Q_{\text{out}} \end{cases}$$

$$\begin{cases} M_1\dot{Q}_1 + W_1 Q_1 = h_z - h \\ M_2\dot{Q}_2 + W_2 Q_2 = h_z - h \\ B\dot{h} = Q_1 + Q_2 \end{cases}$$

1.5 degrees of freedom

2 identical canals in parallel



$$\begin{cases} M_i\dot{Q}_i + W_i Q_i = h_{\text{from}} - h_{\text{to}} \\ B\dot{h} = Q_{\text{in}} - Q_{\text{out}} \end{cases}$$

$$\begin{cases} M\dot{Q}_1 + WQ_1 = h_z - h \\ M\dot{Q}_2 + WQ_2 = h_z - h \\ B\dot{h} = Q_1 + Q_2 \end{cases} \Rightarrow \begin{cases} M\left(\dot{Q}_1 + \dot{Q}_2\right) + W\left(Q_1 + Q_2\right) = 2\left(h_z - h\right) \\ B\dot{h} = Q_1 + Q_2 \end{cases}$$

$$\boxed{MB\ddot{h} + WB\dot{h} + 2h = 2h_z}$$

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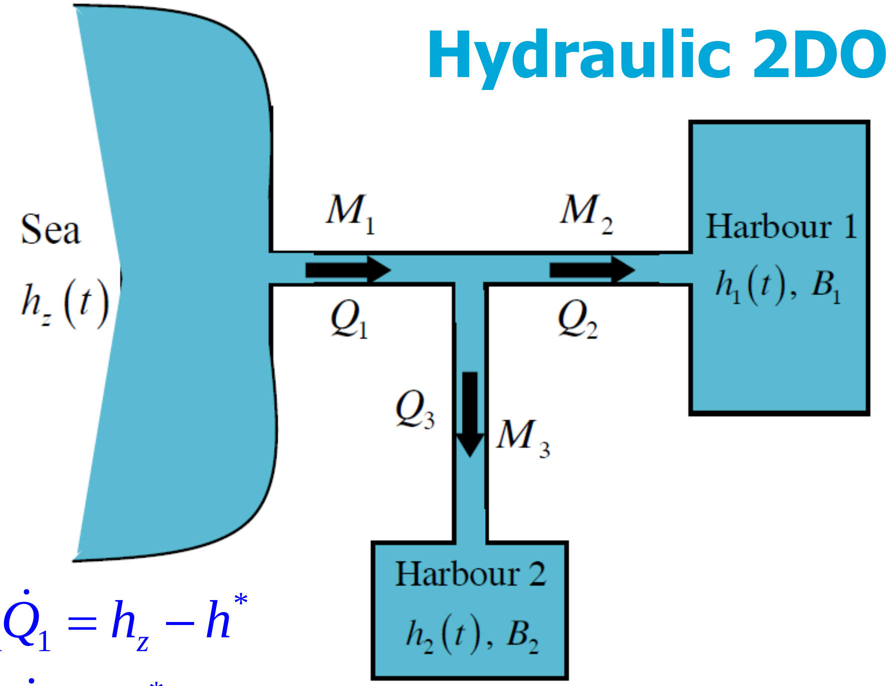
Hydraulic 2DOF system



$$\begin{cases} M_i\dot{Q}_i + W_i Q_i = h_{\text{from}} - h_{\text{to}} \\ B\dot{h} = Q_{\text{in}} - Q_{\text{out}} \end{cases}$$

$$\begin{cases} M_1\dot{Q}_1 = h_z - h^* \\ M_2\dot{Q}_2 = h^* - h_1 \\ M_3\dot{Q}_3 = h^* - h_2 \\ B_1\dot{h}_1 = Q_2 \\ B_2\dot{h}_2 = Q_3 \\ Q_1 = Q_2 + Q_3 \end{cases}$$

$$\Rightarrow \begin{bmatrix} B_1(M_1+M_2) & B_2 M_1 \\ B_1 M_1 & B_2(M_1+M_3) \end{bmatrix} \begin{bmatrix} \ddot{h}_1 \\ \ddot{h}_2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} h_z \\ h_z \end{bmatrix}$$

$$\boxed{\begin{bmatrix} \frac{B_1^2}{B_2}(M_1+M_2) & B_1 M_1 \\ B_1 M_1 & B_2(M_1+M_3) \end{bmatrix} \begin{bmatrix} \ddot{h}_1 \\ \ddot{h}_2 \end{bmatrix} + \begin{bmatrix} \frac{B_1}{B_2} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} \frac{B_1}{B_2} h_z \\ h_z \end{bmatrix}}$$

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