CIE 3150 / CTB 3335 – Concrete Structures 2

Examples from previous exams

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General information

Material properties:

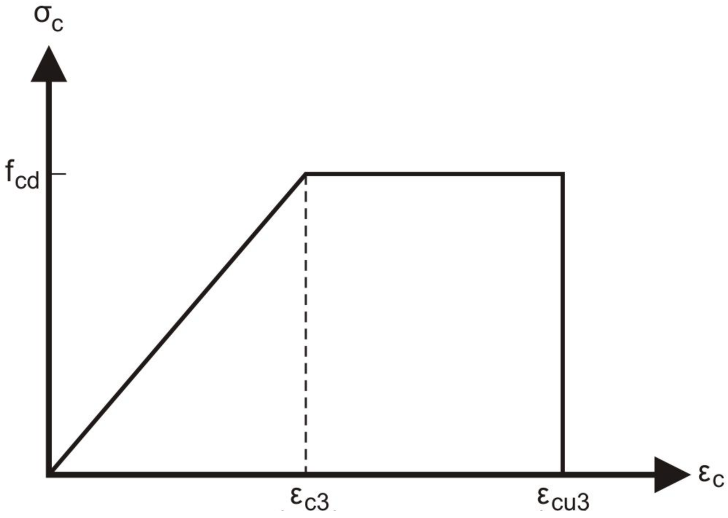
The design compressive strength of concrete: $f_{cd} = \dfrac{\alpha_{cc} f_{ck}}{\gamma_c}$ with $\alpha_{cc} = 1{,}0$ and $\gamma_c = 1{,}5$

The design tensile strength of concrete: $f_{ctd} = \dfrac{\alpha_{ct} f_{ctk,0,05}}{\gamma_c}$ with $\alpha_{ct} = 1{,}0$ and $\gamma_c = 1{,}5$

Concrete strength class C40/50:
compression:
$f_{ck} = 40$ N/mm^2
tension:
$f_{ctk,0,05} = 2{,}5$ N/mm^2

Concrete strength class C45/55:
compression:
$f_{ck} = 45$ N/mm^2
tension:
$f_{ctk,0,05} = 2{,}7$ N/mm^2

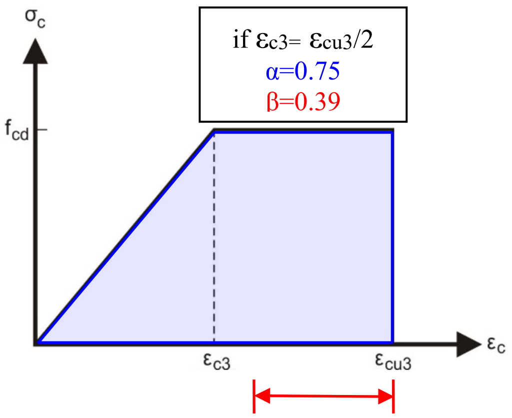
Stress-strain diagram for concrete in compression



Additional information

$$\alpha = \frac{(\varepsilon_{cu3} - \varepsilon_{c3}) + \dfrac{\varepsilon_{c3}}{2}}{\varepsilon_{cu3}}$$

$$\beta = \frac{\dfrac{(\varepsilon_{cu3} - \varepsilon_{c3})^2}{2} + ((\varepsilon_{cu3} - \varepsilon_{c3}) + \dfrac{1}{3} \cdot \varepsilon_{c3}) \cdot \dfrac{\varepsilon_{c3}}{2}}{\alpha \cdot \varepsilon_{cu3}{}^2}$$



if $\varepsilon_{c3} = \varepsilon_{cu3}/2$
α=0.75
β=0.39

Concrete strength class up to C50/60:

$\varepsilon_{cu3} = 3{,}5$ ‰
$\varepsilon_{c3} = 1{,}75$ ‰

Normal strength concrete compression zone characteristics of a rectangular cross-section ($\leq$ C50/60)

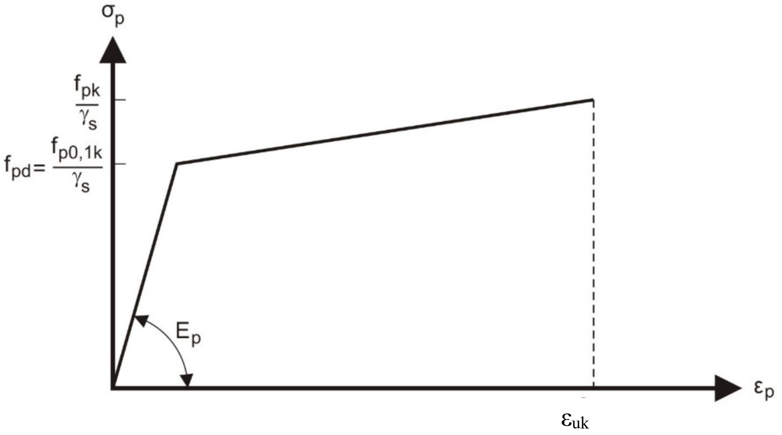
sectional area factor $\alpha = 0{,}75$ ($A = \alpha\, b\, x_u$)
distance factor $\beta = 0{,}39$ ($y = \beta\, x_u$)

Prestressing steel:

Mechanical properties prestressing steel

Strength class	Type	Tensile strength		Failure strain	0,1% fractile	Permissible tensile stress			Kink in σ-ε diagram (ULS)	Modulus of elasticity
						During stressing	During stressing with accurate jacking	Initial stress		
		f_{pk}	f_{pk}/γ_s	ε_{pu}	$f_{p0,1k}$	$\sigma_{p,max}$	$\sigma_{p,max}$	σ_{pm0}	f_{pd}	E_p
		MPa	MPa	‰	MPa	MPa	MPa	MPa	MPa	GPa
Y1030H	bar	1030	936	35	927	773[1]	773[b]	773	843	205 or 170
Y1670C	wire	1670	1518	35	1503	1336	1428	1253	1366	205
Y1770C	wire	1770	1609	35	1593	1416	1513	1328	1448	205
Y1860S7	strand	1860	1691	35	1674	1488	1590	1395	1522	195

Stress-strain diagram of prestressing steel



Y1860S7: $f_{pk} / \gamma_s = 1860 / 1,1 = 1691$ N/mm^2

$f_{pd} = f_{p0,1k} / \gamma_s = 1674 / 1,1 = 1522$ N/mm^2

$\varepsilon_{uk} = 35$ ‰

max. initial stress $\sigma_{pi} = \sigma_{pm0} = 1395$ N/mm^2 ; $\sigma_{p,max} = 1488$ N/mm^2 (at jacking)

$E_p = 195 \cdot 10^3$ N/mm^2

Note:
The strain allowed in ULS (ε_{ud}) is related to ε_{uk}.
The ratio is $\varepsilon_{ud} / \varepsilon_{uk} = 1,0$ in these examples. A National Annex to EN 1992-1-1 might prescribe a different ratio, e.g. 0,9. Apart from a cut-off at ε_{ud}, it has no further impact on the stress-strain diagram.

Prestressing force including frictional loss:

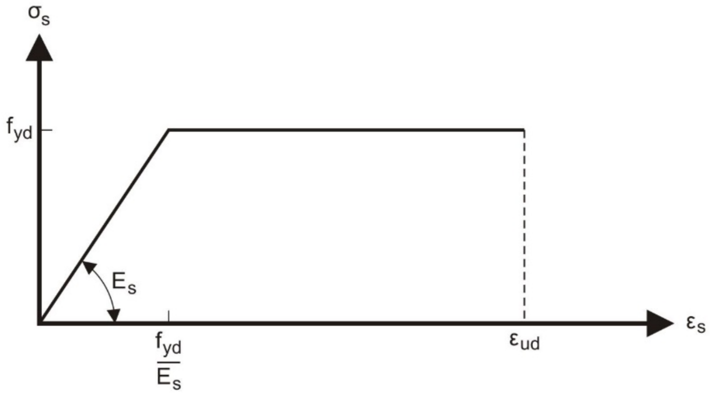
$$P_{m0}(x) = P_{m0}(x = 0) \cdot e^{-\mu(\theta + k\,x)}$$

friction coefficient μ
Wobble-factor k

Reinforcing steel: B500:

f_{yd} = 500 / 1,15 = 435 N/mm^2
bond factor: ξ_s = 1,0
E_s = 200 · 10^3 N/mm^2

Stress-strain diagram of reinforcing steel



ε_{ud} = 45 ‰

Load specifications (general):

Partial load factors in ultimate limit state (ULS) design:
permanent load : γ_G = 1,2
variable load : γ_Q = 1,5
prestressing load : γ_P = 1,0

Partial load factors in serviceability limit state (SLS) design:
All loads : γ = 1,0

Reinforced/prestressed concrete: volumetric mass (density) = 25 kN/m^3

Crack width:

$$w_{\max} = \frac{1}{2} \cdot \frac{f_{ctm}}{\tau_{bm}} \cdot \frac{Ø}{\rho_{p,eff}} \cdot \frac{1}{E_s} \cdot (\sigma_s - \alpha \cdot \sigma_{sr} + \beta \cdot \varepsilon_{cs} \cdot E_s)$$

where

σ_s steel stress in a crack under external tensile load

σ_{sr} maximum steel stress in the crack in the crack formation stage
tension: follows from the cracking axial force (based on f_{ctm})
bending: follows from the cracking bending moment (based on f_{ctm})

ε_{cs} shrinkage of the concrete (> 0)

$\rho_{p,eff}$ reinforcement ratio $A_s/A_{c,eff}$ based on the cross-sectional area of the "hidden tensile member"

$h_{c,eff}$ height of the effective concrete tensile zone with regard to cracking**:**

f_{ctm} mean concrete tensile strength

τ_{bm} mean bond stress between concrete and steel

Values for τ_{bm}, α and β for various conditions.

	crack formation stage	stabilised cracking stage
short term loading	$\alpha = 0,5$ $\beta = 0$ $\tau_{bm} = 2,0 f_{ctm}$	$\alpha = 0,5$ $\beta = 0$ $\tau_{bm} = 2,0 f_{ctm}$
long term or dynamic loading	$\alpha = 0,5$ $\beta = 0$ $\tau_{bm} = 1,6 f_{ctm}$	$\alpha = 0,3$ $\beta = 1$ $\tau_{bm} = 2,0 f_{ctm}$

The height of the effective area is:

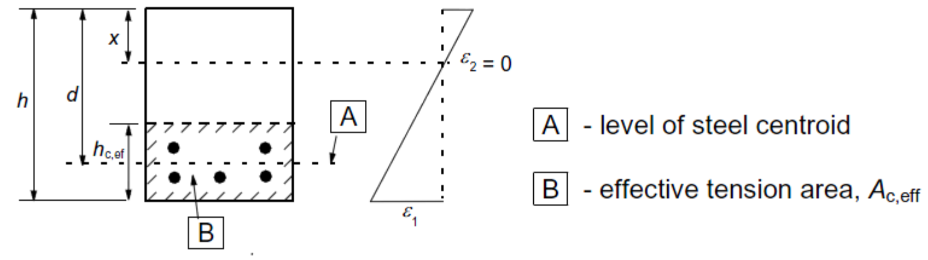
tension:

$$h_{c,eff} = 2,5\,(h - d)$$
$$h_{c,eff} \leq h/2$$

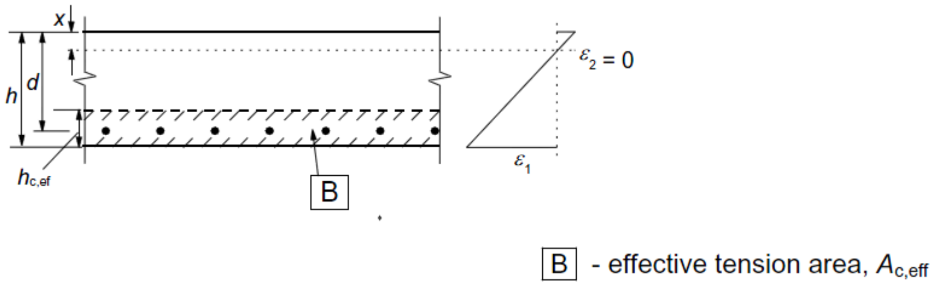
bending:

$$h_{c,eff} = 2,5\,(h - d)$$
$$h_{c,eff} \leq \left(h - x\right)/3$$

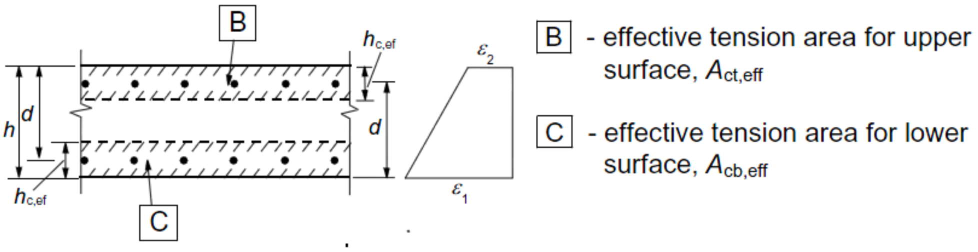
Effective concrete area:



a) Beam



b) Slab



c) Member in tension

Example 1 – Slab, crack width and punching shear resistance

A building uses the outrigger system. According to the requirements of the architect, the exterior columns are reinforced concrete columns and the ground floor is left open. The structural system is given in Figure 1.1. The thickness of the concrete floor is 200 mm. The cross-sections of the square columns are 175 mm × 175 mm.

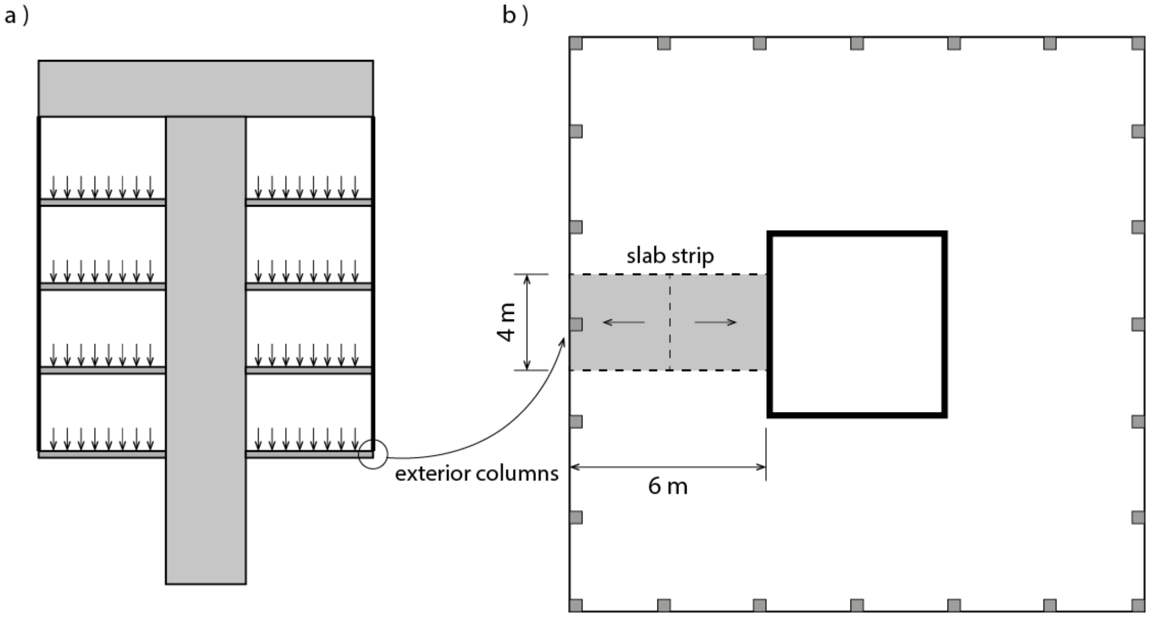


Figure 1.1 Design of a building.

Parameters:

Concrete strength class : C25/30
weight density concrete : $\gamma_c = 25$ kN/m^3
mean concrete tensile strength : $f_{ctm} = 2.6$ N/mm^2
bond strength : $\tau_{bm} = 2 f_{ctm}$
modulus of elasticity : $E_{cm} = 31000$ N/mm^2

Reinforcement class B500B : $f_{yd} = 435$ N/mm^2
modulus of elasticity : $E_s = 200000$ N/mm^2

Concrete Column : 175 mm × 175 mm
Concrete slab
thickness : 200 mm
floor finishing : 0.4 kN/m^2
Variable load : $q_{Qk} = 4.0$ kN/m^2
Quasi-permanent combination (SLS) : $\psi_2 = 0.5$ (for q_Q)
Partial load factors (ULS) : $\gamma_G = 1.2$
: $\gamma_Q = 1.5$

Additional information:

Crack width control:

$$w_{max} = \frac{1}{2}\frac{f_{ctm}}{\tau_{bm}}\frac{\text{Ø}}{\rho_{s,eff}}\frac{1}{E_s}(\sigma_s - \alpha\,\sigma_{sr})$$

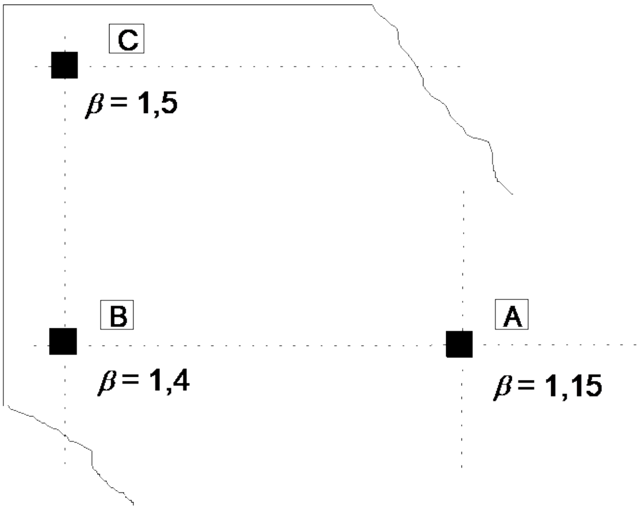
$$\sigma_{sr} = \frac{N_{crack}}{A_s} = \frac{A_c f_{ctm}\left(1+\frac{E_s}{E_c}\rho_s\right)}{A_s}$$

Table 15,III Values for τ_{bm}, α and β from eq. (15.15) for various conditions. The values for α between brackets are the recalibrated values as applied in the Eurocode by means of the coefficient k_t (EN 1992-1-1 eq. (7.9))

	crack formation stage	stabilized cracking stage
Short term loading	α = 0,5 (0,6) β = 0 $\tau_{bm} = 2{,}0\,f_{ctm}$	α = 0,5(0,6) β = 0 $\tau_{bm} = 2{,}0\,f_{ctm}$
long term or dynamic loading	α = 0,5 (0,6) β = 0 $\tau_{bm} = 1{,}6\,f_{ctm}$	α = 0,3 (0,4) β = 1 $\tau_{bm} = 2{,}0\,f_{ctm}$

Punching shear; axial force and bending moment:

Approximated safe values for β (load eccentricity factor) according to the Eurocode:



Question 1.1

Calculate the (design) load of the exterior column at ULS and SLS load combinations.

Hint: Consider the shaded part of the slab in Figure 1.1(b) as a simply supported one-way slab, which implies that half of the distributed load on this slab strip is transferred to the exterior column.

Answer 1.1

self weight:	$h_{\text{slab}} \times \gamma_c = 0.2 \times 25$	$= 5.0$	kN/m^2
floor finishing:		$= 0.4$	kN/m^2
dead weight:	q_{Gk}	$= 5.4$	kN/m^2
variable load:	q_{Qk}	$= 4.0$	kN/m^2

ULS:

$$q_{Ed} = \gamma_G \times q_{Gk} + \gamma_Q \times q_{Qk} = 1.2 \times 5.4 + 1.5 \times 4.0 = 12.48\text{ kN/m}^2$$

$$P_{Ed} = q_{Ed} \times a \times b = 12.48 \times (6.0/2) \times 4.0 = 149.76\text{ kN}$$

SLS:

$$q = q_{Gk} + \psi_2 \times q_{Qk} = 5.4 + 0.5 \times 4.0 = 7.4\text{ kN/m}^2$$

$$P = q \times a \times b = 7.4 \times (6.0/2) \times 4.0 = 88.8\text{ kN}$$

Question 1.2

The tensile reinforcement in the columns at the <u>lowest floor level</u> is 4 bars Ø12 mm. Check the capacity of the column (ULS).

Answer 1.2

$$A_s = 4 \times \frac{1}{4}\pi Ø^2 = 4 \times \frac{1}{4}\pi \times 12^2 = 452.39\text{ mm}^2$$

$$\sigma_s = \frac{P_{\text{Ed}}}{A_s} = \frac{149.76}{452.39} \times 1000 = 331\text{ MPa} < 435\text{ MPa}$$

Result: ULS capacity is **OK**.

Question 1.3

Will the exterior columns crack JUST AFTER the installation of the slab?

Answer 1.3

Column tensile force from self-weight of the slab P_0 and column cracking force P_{cr}:

$$P_0 = 5{,}4 \times 4{,}0 \times (6{,}0/2) = 64{,}8kN$$

$$P_{cr} = f_{ctm}A_c + f_{ctm}\alpha_e A_s$$

$$\alpha_e = \frac{E_s}{E_c} = \frac{200000}{31000} = 6.45$$

Cracking force (concrete and steel contributions):

$P_{\text{cr}} = 2.6 \times 175^2 + 2.6 \times 6.45 \times 452.39 = 87.2\text{ kN}$, $P_{cr} > P_0 \rightarrow$ the column will **not** crack

Question 1.4

Will the exterior column crack after the building is in use (SLS)? If so, calculate the long term crack width $w_{\max}$ and check whether the calculated crack width is smaller than 0.20 mm. If not, calculate at which variable load q_{Qk} the column cracks and calculate w_{max} at that load. (Assume that the full column cross-section is effective).

Answer 1.4

SLS load; see Answer 1.1:

$P = 88.8kN > 87.2kN = P_{cr}$ → the column will crack.

Calculate the steel stress directly after cracking σ_{sr} and the steel stress σ_s in SLS:

$$\sigma_{sr} = \frac{f_{ctm}}{\rho}(1+\alpha_e \rho)$$

$$\text{where } \rho = \frac{A_s}{A_c} = \frac{452.39}{175^2} = 1.48\%$$

$$\sigma_{sr} = \frac{2.6}{0.0148}(1+6.45\times 0.0148) = 192.8MPa$$

$$\sigma_s = \frac{P_{Ed}}{A_s} = \frac{88.8}{452.39}\times 1000 = 196.29MPa$$

$$w_{\max} = \frac{1}{2}\frac{f_{\mathrm{ctm}}}{\tau_{\mathrm{bm}}}\frac{Ø}{\rho}\frac{1}{E_{\mathrm{s}}}(\sigma_{\mathrm{s}} - \alpha\sigma_{\mathrm{sr}} + \beta\varepsilon_{sc}\,\mathrm{E}_s)$$

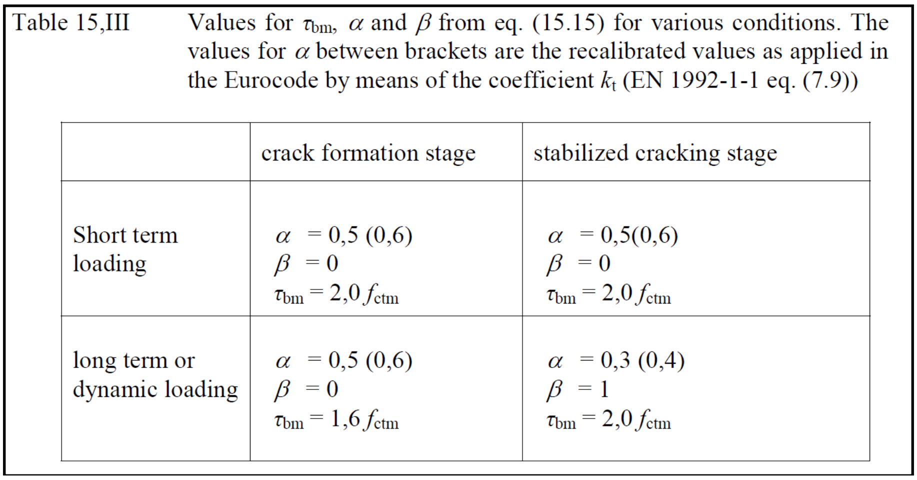


Table 15,III Values for τ_{bm}, α and β from eq. (15.15) for various conditions. The values for α between brackets are the recalibrated values as applied in the Eurocode by means of the coefficient k_t (EN 1992-1-1 eq. (7.9))

	crack formation stage	stabilized cracking stage
Short term loading	α = 0,5 (0,6) β = 0 $\tau_{\mathrm{bm}} = 2{,}0 f_{\mathrm{ctm}}$	α = 0,5(0,6) β = 0 $\tau_{\mathrm{bm}} = 2{,}0 f_{\mathrm{ctm}}$
long term or dynamic loading	α = 0,5 (0,6) β = 0 $\tau_{\mathrm{bm}} = 1{,}6 f_{\mathrm{ctm}}$	α = 0,3 (0,4) β = 1 $\tau_{\mathrm{bm}} = 2{,}0 f_{\mathrm{ctm}}$

$\alpha = 0{,}3$, assuming $\varepsilon_{sc} = 0 \rightarrow$

$$w_{\max} = \frac{1}{2}\times\frac{1}{2}\times\frac{12}{0.0148}\times\frac{1}{200000}\times(196.29 - 0.3\times 192.8) = 0.14\text{ mm} < 0.2\text{ mm}; \textbf{OK}$$

Question 1.5

Is the calculated crack width related to the phase “fully developed crack pattern” or not?

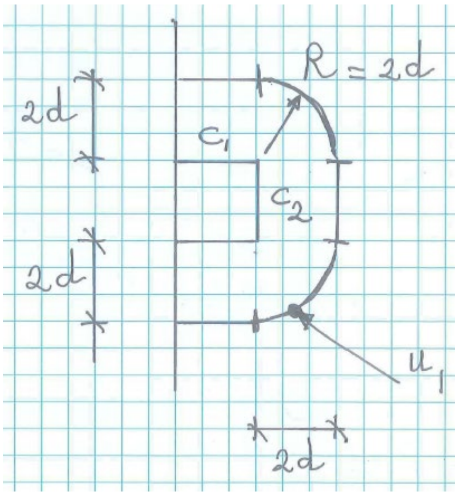
Answer 1.5

The SLS load is greater than the cracking load and, therefore, the tensile member is in the “fully developed crack pattern stage” (stabilized cracking); not in the “not fully developed crack pattern stage” (crack formation stage).
Answer: Yes.

Question 1.6

Draw the control perimeter u_1 of the edge column, and calculate its value.

Answer 1.6.



Calculate the length of the control perimeter u_1 of the edge column:
First calculate *d,* the effective slab height.

Assume that the slab is reinforced with rebars, mesh Ø 12 mm, and has a concrete cover of 20 mm.

$$\left.\begin{aligned} d_x &= 200-20-12/2 = 174\, mm \\ d_y &= 200-20-12-12/2 = 162\, mm \end{aligned}\right\} \rightarrow d = \frac{1}{2}\left(d_x + d_y\right) = 168\, mm$$

$$R = 2d = 2 \times 168 = 336 \text{ mm}$$

$$u_1 = 2c_1 + c_2 + 2 \cdot 4d \cdot \pi \cdot \frac{1}{4} = 2 \cdot 175 + 175 + 2 \cdot 4 \cdot 168 \cdot \pi \cdot \frac{1}{4} = 1581 \text{ mm}$$

Question 1.7

At the column - slab connection, the slab is provided with a reinforcement mesh at the top; Ø12 spaced 200 mm in both *x* and *y* direction. Check the punching shear capacity of the edge column. Assume that $\beta = 1{,}4$.

Answer 1.7

The design value of the punching shear stress of the edge column can be calculated as (use the load transferred to the edge column, P_{Ed}, from **Answer 1.1)**:

$$v_{Ed} = \beta \frac{V_{Ed}}{u_1 d} = \beta \frac{P_{Ed}}{u_1 d} = 1{,}4 \cdot \frac{149{,}76 \cdot 10^3}{1581 \cdot 168} = 0{,}79 \text{ MPa}$$

The punching shear resistance stress of the edge column is:

$$v_{\mathrm{Rd,c}} = 0{,}12k(100\rho f_{\mathrm{ck}})^{\frac{1}{3}} + k_1\sigma_{\mathrm{cp}} \text{ with a minimum of}$$

$$v_{\min} = 0{,}035k^{3/2}\sqrt{f_{\mathrm{ck}}} + k_1\sigma_{\mathrm{cp}}$$

where σ_{cp} is the concrete stress in the cross-section from an axial load and/or prestressing. In this case σ_{cp}=0.

A reinforcement mesh Ø12-200 is applied:

$$a_{sx} = a_{sy} = \frac{d^2\pi}{4} \cdot \frac{1000}{s} = \frac{12^2\pi}{4} \cdot \frac{1000}{200} = 565{,}49 \text{ mm}^2/\text{m}$$

$$\left.\begin{array}{l} \rho_x = \dfrac{a_{sx}}{d_x} = \dfrac{565{,}49}{174 \cdot 1000} = 0{,}32\% \\[2ex] \rho_y = \dfrac{a_{sy}}{d_y} = \dfrac{565{,}49}{162 \cdot 1000} = 0{,}35\% \end{array}\right\} \rightarrow \rho = \sqrt{\rho_x \rho_y} = 0{,}34\%$$

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2{,}0$$

$$k = 1 + \sqrt{\frac{200}{168}} > 2 \rightarrow k = 2{,}0$$

$$\text{C25}/30 \rightarrow f_{\mathrm{ck}} = 25 \text{ MPa}$$

$$\left.\begin{array}{l} v_{\mathrm{Rd,c}} = 0{,}12 \cdot 2 \cdot (100 \cdot 0{,}0034 \cdot 25)^{\frac{1}{3}} = 0{,}49 \text{ MPa} \\ v_{\min} = 0{,}035 \cdot 2^{3/2}\sqrt{25} = 0{,}49 \text{ MPa} \end{array}\right\} \rightarrow v_{\mathrm{Rd,c}} = \max\left\{v_{\mathrm{Rd,c}}, v_{\min}\right\} = 0{,}49 \text{ MPa} < v_{\mathrm{Ed}} \ (= 0{,}79 \text{ MPa})$$

The punching shear capacity of the slab is too small.

Structural measures should be taken. These measures can consist of the application of punching shear reinforcement (hooks, stirrups or dowels) or a drop panel (the local increase of slab thickness). Take into account that the upper limit for punishing shear resistance is $k_{\max} \times v_{\mathrm{Rd,c}} = 1.6 \times 0{,}49 \text{ MPa} = 0.78\text{MPa} < v_{\mathrm{Ed}} \ (= 0{,}79 \text{ MPa})$ and therefore, in this case, strictly speaking, only application of punching shear reinforcement is not enough.

Question 1.8

When the punching shear capacity is too small, a drop panel can be added to increase the thickness of the slab locally. Calculate the thickness of the drop panel to avoid the use of punching shear reinforcement.

Answer 1.8

The thickness of the slab should be such that the design punching shear stress is smaller than the punching shear resistance stress:

$$v_{\mathrm{Ed}} = \beta \frac{V_{\mathrm{Ed}}}{u_1 d} = \beta \frac{P_{\mathrm{Ed}}}{u_1 d} = 1{,}4 \cdot \frac{149{,}76 \cdot 10^3}{(3c + 2d\pi)d} \leq 0{,}49 \text{ MPa}$$

$$\text{where } \; c = 175 \text{ mm} \quad \rightarrow \quad 6{,}28d^2 + 525d - 427886 \geq 0$$

$$d \geq 223 \text{ mm}$$

$$h \geq 223 + 20 + 12 = 255 \text{ mm}$$

Make the drop panel such, that the local height of the slab is $h \geq 270$ mm.

Example 2 – Slab

A concrete slab floor is supported by beams and columns. The floor plan is given in Figure 2.1. The beams can be modelled as rigid line supports. Rotation is allowed on the beams.

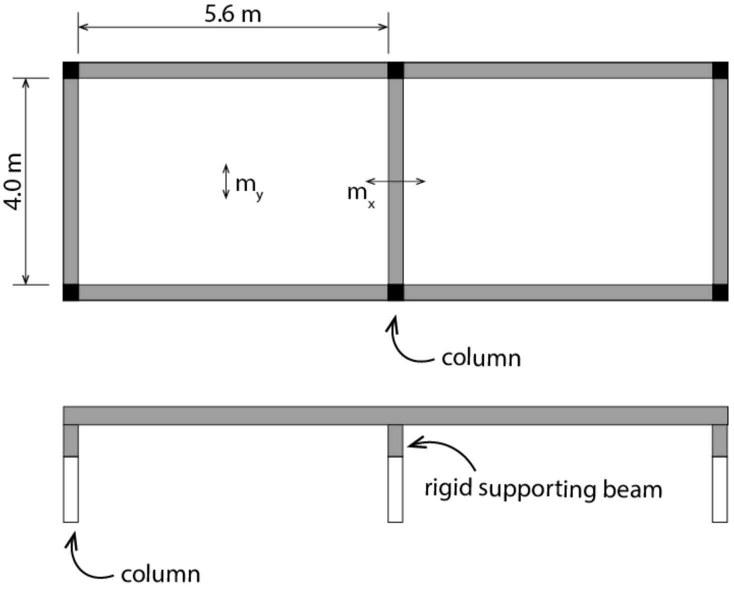


Figure 2.1 Floor plan of a two-way slab

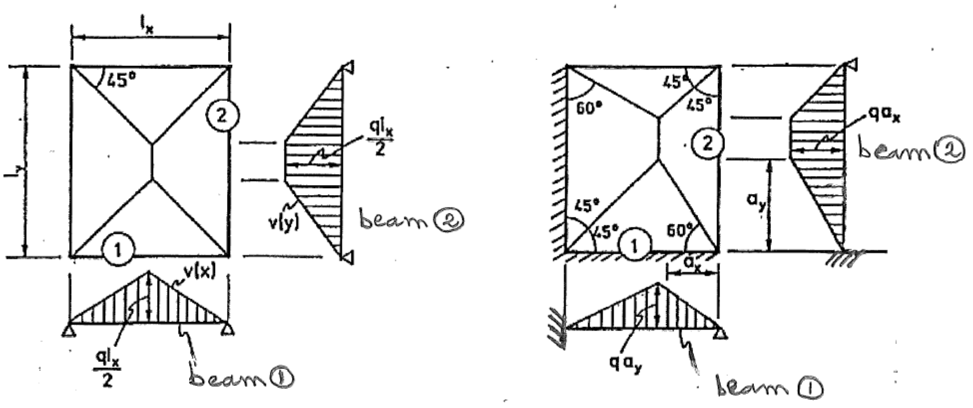
Parameters:

Total slab thickness : 250 mm
Concrete cover : 20 mm
Design load : 15 kN/m^2
Concrete strength class : C25/30
Reinforcement class B500B : f_{yd} = 435 N/mm^2

Additional information:

Concrete slabs

Shear force calculation:



Question 2.1

Determine the maximum design sagging moment m_y in the middle of the slab and the hogging moment m_x at the intermediate beam position using the **strip method**. For the positions of m_x and m_y, see Figure. 2.1.

Hint: Transfer the load in two directions using the table presented below (Figure 2.2).

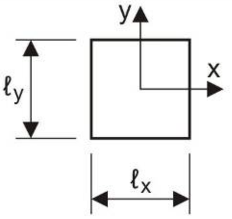
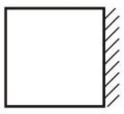
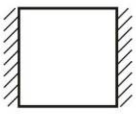
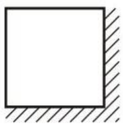
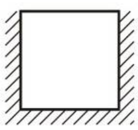
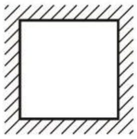
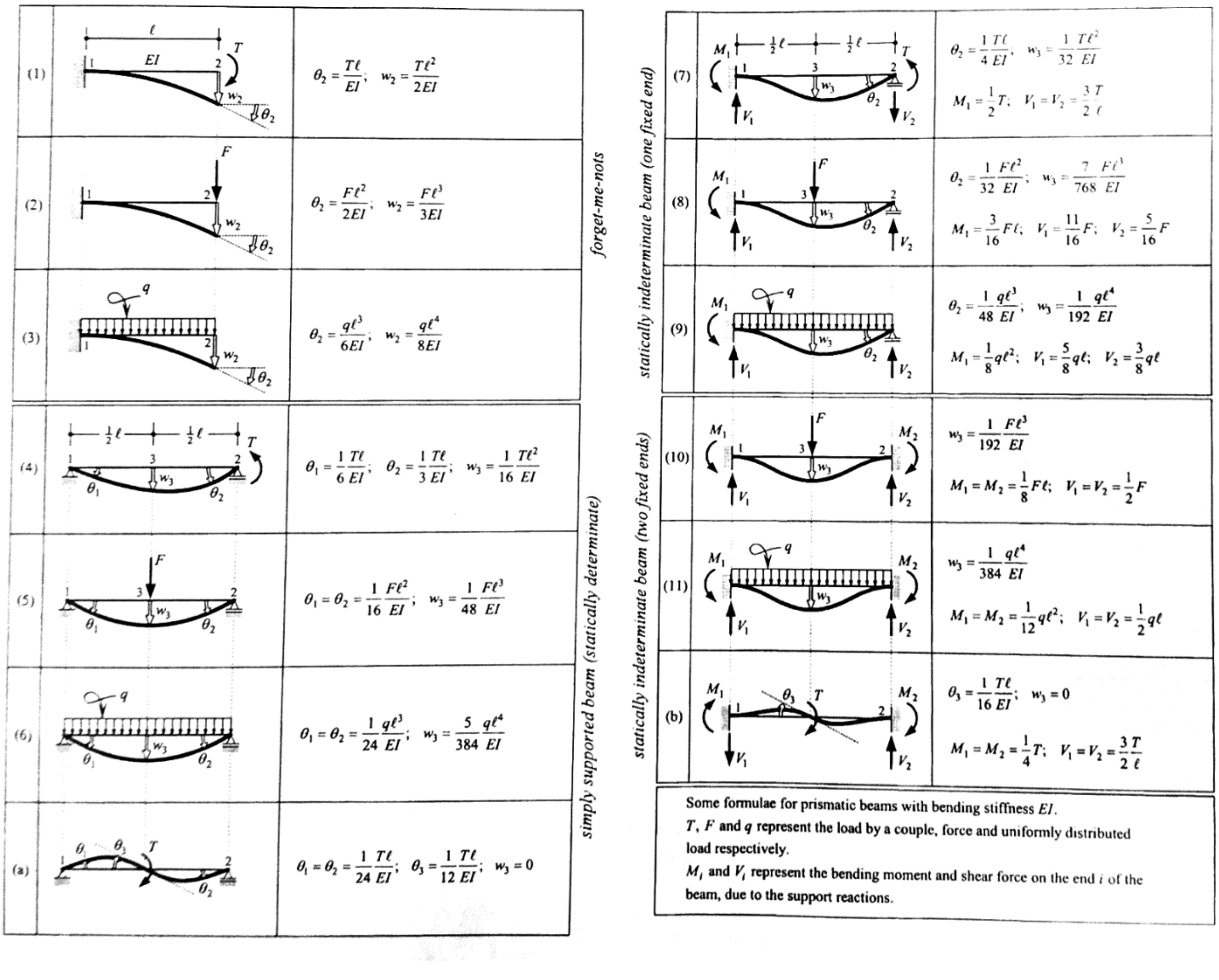
						
$k_x =$	$\frac{\ell_y^4}{\ell_x^4+\ell_y^4}$	$\frac{5\ell_y^4}{2\ell_x^4+5\ell_y^4}$	$\frac{5\ell_y^4}{\ell_x^4+5\ell_y^4}$	$\frac{\ell_y^4}{\ell_x^4+\ell_y^4}$	$\frac{2\ell_y^4}{\ell_x^4+2\ell_y^4}$	$\frac{\ell_y^4}{\ell_x^4+\ell_y^4}$
	$k_y = 1 - k_x$					

Figure 2.2 Strip method load transfer results (uniformly distributed load)

Bending moment table of beams with different boundary conditions:



forget-me-nots	(1)	$\theta_2 = \frac{T\ell}{EI}$; $w_2 = \frac{T\ell^2}{2EI}$
	(2)	$\theta_2 = \frac{F\ell^2}{2EI}$; $w_2 = \frac{F\ell^3}{3EI}$
	(3)	$\theta_2 = \frac{q\ell^3}{6EI}$; $w_2 = \frac{q\ell^4}{8EI}$
simply supported beam (statically determinate)	(4)	$\theta_1 = \frac{1}{6}\frac{T\ell}{EI}$; $\theta_2 = \frac{1}{3}\frac{T\ell}{EI}$; $w_3 = \frac{1}{16}\frac{T\ell^2}{EI}$
	(5)	$\theta_1 = \theta_2 = \frac{1}{16}\frac{F\ell^2}{EI}$; $w_3 = \frac{1}{48}\frac{F\ell^3}{EI}$
	(6)	$\theta_1 = \theta_2 = \frac{1}{24}\frac{q\ell^3}{EI}$; $w_3 = \frac{5}{384}\frac{q\ell^4}{EI}$
	(a)	$\theta_1 = \theta_2 = \frac{1}{24}\frac{T\ell}{EI}$; $\theta_3 = \frac{1}{12}\frac{T\ell}{EI}$; $w_3 = 0$
statically indeterminate beam (one fixed end)	(7)	$\theta_2 = \frac{1}{4}\frac{T\ell}{EI}$; $w_3 = \frac{1}{32}\frac{T\ell^2}{EI}$ $M_1 = \frac{1}{2}T$; $V_1 = V_2 = \frac{3}{2}\frac{T}{\ell}$
	(8)	$\theta_2 = \frac{1}{32}\frac{F\ell^2}{EI}$; $w_3 = \frac{7}{768}\frac{F\ell^3}{EI}$ $M_1 = \frac{3}{16}F\ell$; $V_1 = \frac{11}{16}F$; $V_2 = \frac{5}{16}F$
	(9)	$\theta_2 = \frac{1}{48}\frac{q\ell^3}{EI}$; $w_3 = \frac{1}{192}\frac{q\ell^4}{EI}$ $M_1 = \frac{1}{8}q\ell^2$; $V_1 = \frac{5}{8}q\ell$; $V_2 = \frac{3}{8}q\ell$
statically indeterminate beam (two fixed ends)	(10)	$w_3 = \frac{1}{192}\frac{F\ell^3}{EI}$ $M_1 = M_2 = \frac{1}{8}F\ell$; $V_1 = V_2 = \frac{1}{2}F$
	(11)	$w_3 = \frac{1}{384}\frac{q\ell^4}{EI}$ $M_1 = M_2 = \frac{1}{12}q\ell^2$; $V_1 = V_2 = \frac{1}{2}q\ell$
	(b)	$\theta_3 = \frac{1}{16}\frac{T\ell}{EI}$; $w_3 = 0$ $M_1 = M_2 = \frac{1}{4}T$; $V_1 = V_2 = \frac{3}{2}\frac{T}{\ell}$

Some formulae for prismatic beams with bending stiffness EI.
T, F and q represent the load by a couple, force and uniformly distributed load respectively.
M_i and V_i represent the bending moment and shear force on the end i of the beam, due to the support reactions.

Answer 2.1

Uniformly distributed load: 15 kN/m^2.

The load transfer results from figure 2.2 are based on having the same deflection for *x* and *y* direction strips.

According to Figure 2.2, the second slab type from the left hand side:

$$\rightarrow\ l_x = 5.6m \quad l_y = 4.0m$$

$$k_x = \frac{5l_y{}^4}{2l_x{}^4 + 5l_y{}^4} = \frac{5\times 4.0^4}{2\times 5.6^4 + 5\times 4.0^4} = 0.394$$

$$k_y = 1 - 0.394 = 0.606$$

Result:
39,4% of the uniformly distributed load is transferred in *x*-direction; 60,6% in *y*-direction.

y-direction (model: 1 span; simply supported at both supports):

$$m_y = \frac{1}{8}\times q\times k_y\times l_y{}^2 = \frac{1}{8}\times 15\times 0.606\times 4.0^2 = 18.2 \text{ kNm/m}$$

x-direction (model: 2 spans; simply supported at both end supports; uniform load over both spans):
Bending moment at the intermediate support:

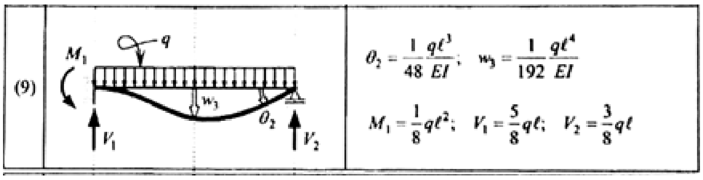
$$m_x = \left(-\frac{1}{8}\right)\times q\times k_x\times l_x{}^2 = \frac{-1}{8}\times 15\times 0.394\times 5.6^2 = -23.2 \text{ kNm/m}$$

Additional information

x-direction:
Bending moment at midspan position:

$$m_x = \frac{1}{2}\left(-\frac{1}{8}\right)qk_xl_x{}^2 + \frac{1}{8}qk_xl_x{}^2 = \frac{-1}{16}\cdot 15\cdot 0.394\cdot 5.6^2 = +11.6 \text{ kNm/m}$$

Note that the maximum positive bending moment in *x*-direction is not at midspan position, but at $^3/_8 l$ from an end support, see figure below ($M_{\text{span,max}}$ at the position where the shear force $V = 0$).



(9)

$\theta_2 = \frac{1}{48}\frac{q\ell^3}{EI}$; $w_3 = \frac{1}{192}\frac{q\ell^4}{EI}$

$M_1 = \frac{1}{8}q\ell^2$; $V_1 = \frac{5}{8}q\ell$; $V_2 = \frac{3}{8}q\ell$

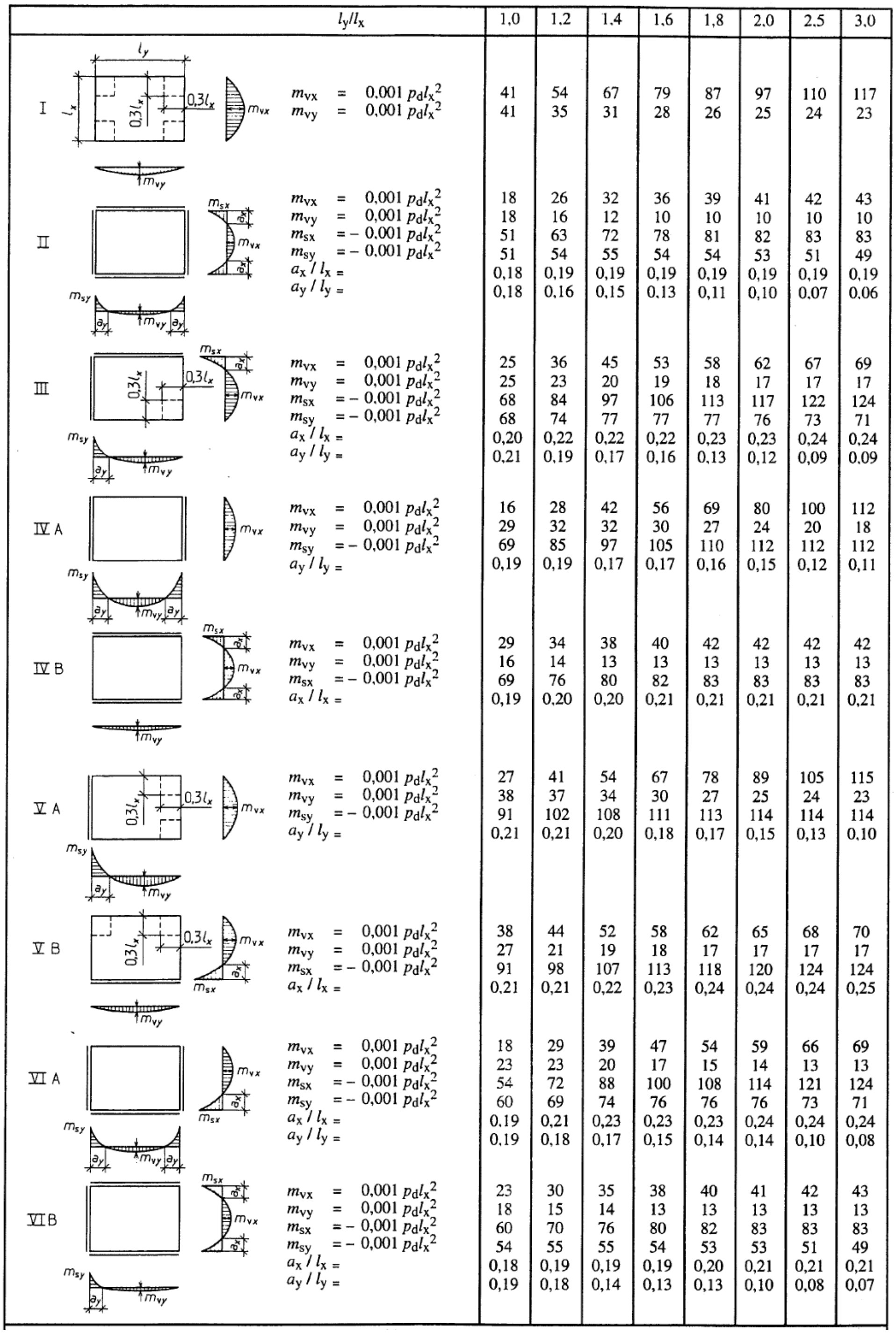
Result:

$$M_{\max} = \frac{3}{8}ql\times\frac{3}{8}l - \frac{1}{2}q(\frac{3}{8}l)^2 = \frac{9}{128}ql^2$$

Question 2.2

Determine the maximum design sagging moment m_y in the middle of the slab and the hogging moment m_x at the intermediate beam position with the **theory of elasticity**. For the position of m_x and m_y, see Figure. 2.1.

Hint: Use the table presented in Figure 2.3 (NEN6720 table 18)



	l_y/l_x	1,0	1,2	1,4	1,6	1,8	2,0	2,5	3,0
I	$m_{vx} = 0{,}001\, p_d l_x^2$	41	54	67	79	87	97	110	117
	$m_{vy} = 0{,}001\, p_d l_x^2$	41	35	31	28	26	25	24	23
II	$m_{vx} = 0{,}001\, p_d l_x^2$	18	26	32	36	39	41	42	43
	$m_{vy} = 0{,}001\, p_d l_x^2$	18	16	12	10	10	10	10	10
	$m_{sx} = -\,0{,}001\, p_d l_x^2$	51	63	72	78	81	82	83	83
	$m_{sy} = -\,0{,}001\, p_d l_x^2$	51	54	55	54	54	53	51	49
	$a_x / l_x =$	0,18	0,19	0,19	0,19	0,19	0,19	0,19	0,19
	$a_y / l_y =$	0,18	0,16	0,15	0,13	0,11	0,10	0,07	0,06
III	$m_{vx} = 0{,}001\, p_d l_x^2$	25	36	45	53	58	62	67	69
	$m_{vy} = 0{,}001\, p_d l_x^2$	25	23	20	19	18	17	17	17
	$m_{sx} = -\,0{,}001\, p_d l_x^2$	68	84	97	106	113	117	122	124
	$m_{sy} = -\,0{,}001\, p_d l_x^2$	68	74	77	77	77	76	73	71
	$a_x / l_x =$	0,20	0,22	0,22	0,22	0,23	0,23	0,24	0,24
	$a_y / l_y =$	0,21	0,19	0,17	0,16	0,13	0,12	0,09	0.09
IV A	$m_{vx} = 0{,}001\, p_d l_x^2$	16	28	42	56	69	80	100	112
	$m_{vy} = 0{,}001\, p_d l_x^2$	29	32	32	30	27	24	20	18
	$m_{sy} = -\,0{,}001\, p_d l_x^2$	69	85	97	105	110	112	112	112
	$a_y / l_y =$	0,19	0,19	0,17	0,17	0,16	0,15	0,12	0,11
IV B	$m_{vx} = 0{,}001\, p_d l_x^2$	29	34	38	40	42	42	42	42
	$m_{vy} = 0{,}001\, p_d l_x^2$	16	14	13	13	13	13	13	13
	$m_{sx} = -\,0{,}001\, p_d l_x^2$	69	76	80	82	83	83	83	83
	$a_x / l_x =$	0,19	0.20	0,20	0,21	0,21	0,21	0,21	0,21
V A	$m_{vx} = 0{,}001\, p_d l_x^2$	27	41	54	67	78	89	105	115
	$m_{vy} = 0{,}001\, p_d l_x^2$	38	37	34	30	27	25	24	23
	$m_{sy} = -\,0{,}001\, p_d l_x^2$	91	102	108	111	113	114	114	114
	$a_y / l_y =$	0.21	0,21	0,20	0,18	0,17	0,15	0,13	0.10
V B	$m_{vx} = 0{,}001\, p_d l_x^2$	38	44	52	58	62	65	68	70
	$m_{vy} = 0{,}001\, p_d l_x^2$	27	21	19	18	17	17	17	17
	$m_{sx} = -\,0{,}001\, p_d l_x^2$	91	98	107	113	118	120	124	124
	$a_x / l_x =$	0.21	0,21	0,22	0,23	0,24	0,24	0,24	0,25
VI A	$m_{vx} = 0{,}001\, p_d l_x^2$	18	29	39	47	54	59	66	69
	$m_{vy} = 0{,}001\, p_d l_x^2$	23	23	20	17	15	14	13	13
	$m_{sx} = -\,0{,}001\, p_d l_x^2$	54	72	88	100	108	114	121	124
	$m_{sy} = -\,0{,}001\, p_d l_x^2$	60	69	74	76	76	76	73	71
	$a_x / l_x =$	0.19	0,21	0,23	0,23	0,23	0,24	0,24	0,24
	$a_y / l_y =$	0.19	0,18	0,17	0,15	0,14	0,14	0,10	0,08
VI B	$m_{vx} = 0{,}001\, p_d l_x^2$	23	30	35	38	40	41	42	43
	$m_{vy} = 0{,}001\, p_d l_x^2$	18	15	14	13	13	13	13	13
	$m_{sx} = -\,0{,}001\, p_d l_x^2$	60	70	76	80	82	83	83	83
	$m_{sy} = -\,0{,}001\, p_d l_x^2$	54	55	55	54	53	53	51	49
	$a_x / l_x =$	0,18	0,19	0,19	0,19	0,20	0,21	0,21	0,21
	$a_y / l_y =$	0,19	0,18	0,14	0,13	0,13	0,10	0,08	0,07

plate support conditions:
- solid line = line support, no rotational fixity
- two solid lines = line support and fully fixed (no rotation)

m_{vx} is the positive moment per unit length in a cross-section along the longer edge (l_y)

m_{vy} is the positive moment per unit length in a cross-section along the shorter edge (l_x)

m_{sx} is the negative moment per unit length in a cross-section along the longer edge (l_y)

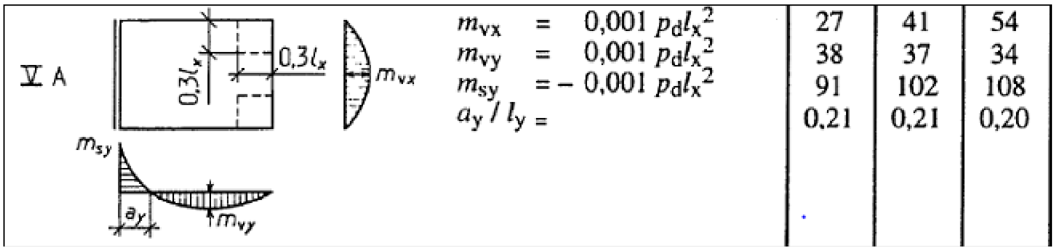
m_{sy} is the negative moment per unit length in a cross-section along the shorter edge (l_x)

p_d is the design value of the uniformly distributed load

Figure 2.3: Bending moments in a slab (uniformly distributed load) according to the theory of elasticity

Answer 2.2

Use the following part of Figure 2.3 (slab type VA; line supports; 3 sides vertically supported only; 1 side completely fixed):



V A	$m_{vx} = 0{,}001\ p_d l_x^2$	27	41	54
	$m_{vy} = 0{,}001\ p_d l_x^2$	38	37	34
	$m_{sy} = -\ 0{,}001\ p_d l_x^2$	91	102	108
	$a_y / l_y =$	0,21	0,21	0,20

According to NEN 6720 table 18 → $l_x = 4.0m \quad l_y = 5.6m \quad \frac{l_y}{l_x} = 1.4$. → *Use the coefficients from the last column.*

$$m_y = m_{vx} = 0.001 \times p_d \times l_x^2 \times 54 = 0.054 \times 15 \times 4.0^2 = 12.96 \text{ kNm/m}$$

$$m_x = m_{sy} = -0.001 \times p_d \times l_x^2 \times 108 = -0.108 \times 15 \times 4.0^2 = -25.92 \text{ kNm/m}$$

Question 2.3

An engineer decides to apply a rebar mesh of Ø8 – 200 mm in both directions in the slab as both bottom and top reinforcement. Is this amount of reinforcement applied sufficient to resist the design bending moments? If not, give your own design. (Use the moment calculated in Question 2.2; *z* can be approximated by 0.9*d*).

Answer 2.3

Calculate the amount of reinforcement, the effective height of the cross-section and the bending moment resistance of the cross-section (assume that the reinforcement in *x*-direction is in the first layer from top and bottom):

$$a_s = \frac{1000}{200}\frac{8^2}{4}\pi = 5 \cdot 50.2 = 251 \text{ mm}^2/\text{m}$$

$$m_{\text{Rd}} = A_s \times f_{yd} \times z = A_s \times f_{yd} \times 0.9d$$

$$d = 250 - 20 - \frac{8}{2} = 226 \text{ mm}$$

$$m_{\text{Rd}} = \frac{251 \times 435 \times 0.9 \times 226}{1000000} = 22 \text{ kNm/m}$$

The maximum bending moment is at the intermediate support position.
Absolute value: 25,92 kNm/m: $m_{\text{Rd}} < m_{\text{Ed}}$ → **not OK**

Additional information:

Apply a mesh Ø8-150mm (PS 335, A = 335 mm^2/m)

$$m_{\mathrm{Rd}} = \frac{50.2 \times \dfrac{1000}{150} \times 435 \times 0.9 \times 226}{1000000} = 29.33\ \mathrm{kNm/m} > m_{\mathrm{Ed,x}}\ ;\ \mathrm{OK}$$

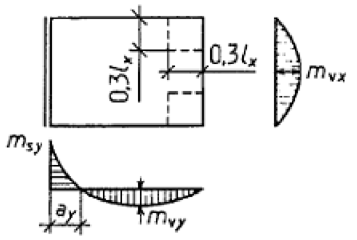
Question 2.4

Indicate with a drawing the part(s) of the slab where torsional reinforcement is needed.

Answer 2.4

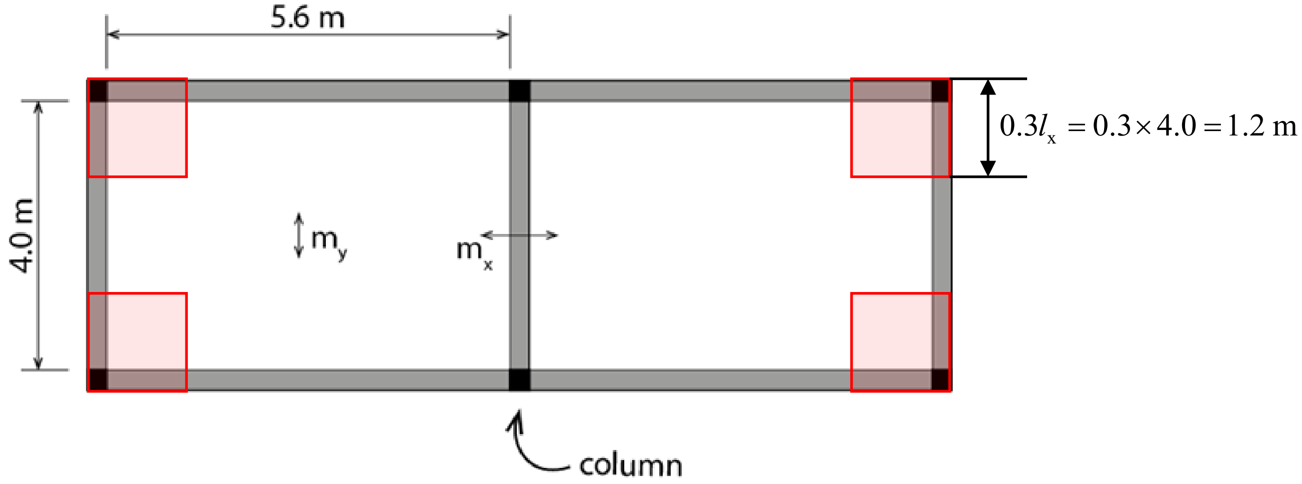
Use the following part of Figure 2.3: Slab type VA; line supports; 3 sides vertically supported only; 1 side completely fixed):

Ⅴ A



$m_{vx} = 0{,}001\ p_d l_x^2$	27	41	54
$m_{vy} = 0{,}001\ p_d l_x^2$	38	37	34
$m_{sy} = -\ 0{,}001\ p_d l_x^2$	91	102	108
$a_y / l_y =$	0,21	0,21	0,20

Torsional reinforcement is required in 2 parts of slab type VA, each 0,3l_x * 0,3 l_x (length * width)

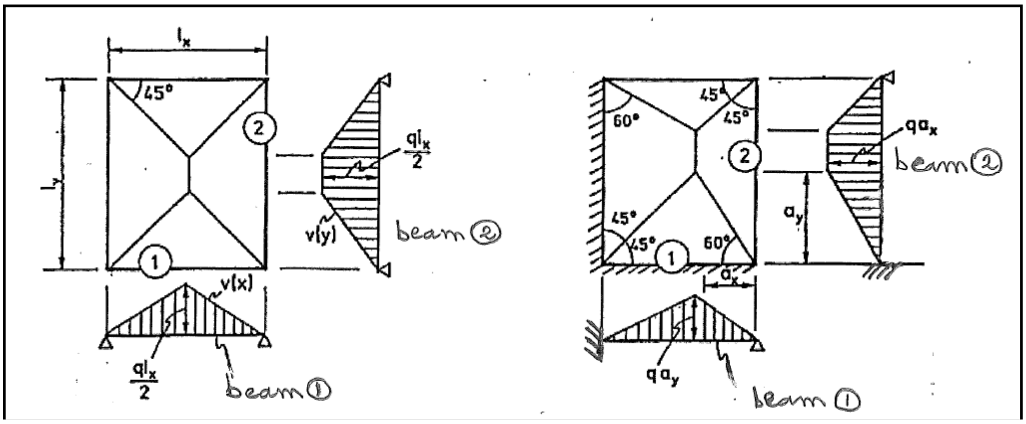


Question 2.5

Indicate with a sketch at which position the maximum shear stress occurs, and determine the design value of the shear stress.

Concrete slabs

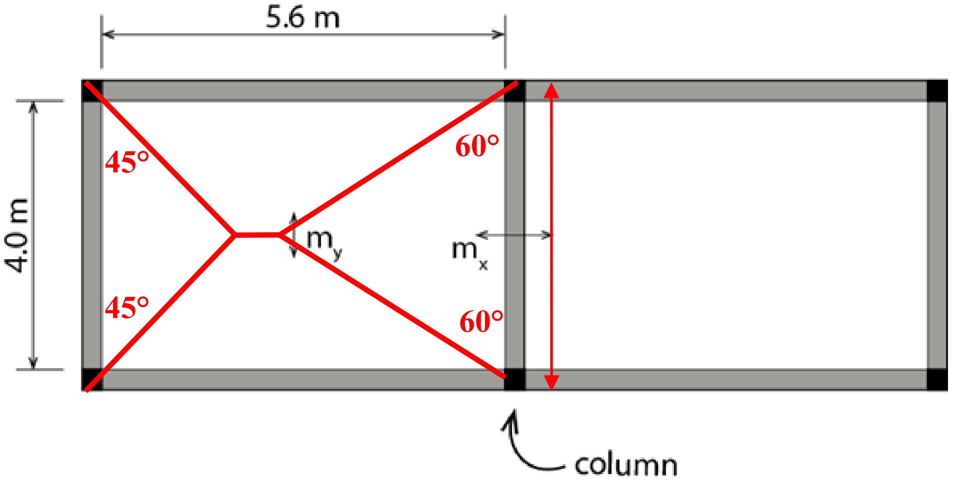
Shear force calculation:



Answer 2.5

The load transfer is modelled using envelopes. The 90º corner angles are split:

- 2 times 45º for a corner where two identical types of supports meet (vertical, rigid line support; no rotational fixity)
- 30º + 60º where a vertical, rigid line support (no rotational fixity) and a completely fixed line support meet.



The maximum shear force per unit length occurs at the intermediate support (load transfer in *x*-direction).

$$v_{\mathrm{Ed,max}} = \frac{V_{\mathrm{Ed}}}{d} = \frac{\left(\frac{l_{\mathrm{y}}}{2}\right)\tan 60° \cdot q_{\mathrm{d}}}{d} = \frac{\left(\frac{4000}{2}\right)\cdot\sqrt{3}\cdot 15\cdot 10^{-3}}{226} = 0{,}23\ \mathrm{MPa}$$

Additional information:

The total load transferred to the line support can be assumed to be uniformly distributed over the length of the support (l_{y}), and then the average value of the shear stress in the slab is:

$$v_{\mathrm{Ed}} = \frac{V_{\mathrm{Ed}}}{l_{\mathrm{y}}d} = \frac{\left(\frac{l_{\mathrm{y}}}{2}\right)\tan 60° \cdot l_{\mathrm{y}} \cdot \frac{1}{2} \cdot q_{\mathrm{d}}}{l_{\mathrm{y}}d} = \frac{\left(\frac{4000}{2}\right)\cdot\sqrt{3}\cdot 4000 \cdot \frac{1}{2} \cdot 15\cdot 10^{-3}}{4000\cdot 226} = 0{,}115\ \mathrm{MPa}$$

Question 2.6

Check the shear capacity of the slab.

Answer 2.6

The design value of the shear resistance of a concrete structure without shear reinforcement has a minimum value that can be always applied (EN 1992-1-1, eq. (6.3N)):

$$v_{\mathrm{min}} = 0{,}035 k^{3/2} \sqrt{f_{\mathrm{ck}}}$$

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2{,}0$$

$$k = 1 + \sqrt{\frac{200}{226}} = 1{,}94$$

$$\mathrm{C25/30} \rightarrow f_{\mathrm{ck}} = 25\ \mathrm{MPa}$$

$v_{\mathrm{min}} = 0{,}035 \cdot 1{,}94^{3/2} \sqrt{25} = 0{,}47\ \mathrm{MPa} > v_{\mathrm{Ed}} \rightarrow$ The shear capacity of the slab is greater than the design value of the shear stress; **OK**.

Example 3 – Slab and crack width

A reinforced concrete balcony slab is supported at two sides by beams, see fig. 3.1. It is not connected to the main building at the longer edge. The clear spacing between the two supporting beams is 3,35 m. The slab is subjected to its self-weight, a permanent load and a live load. The reinforcement in the length direction (*x* direction) is Ø10 mm – 200 mm (5 bars per meter). The concrete cover is 15 mm. The slab thickness is 150 mm. The width of the beams is 300 mm.

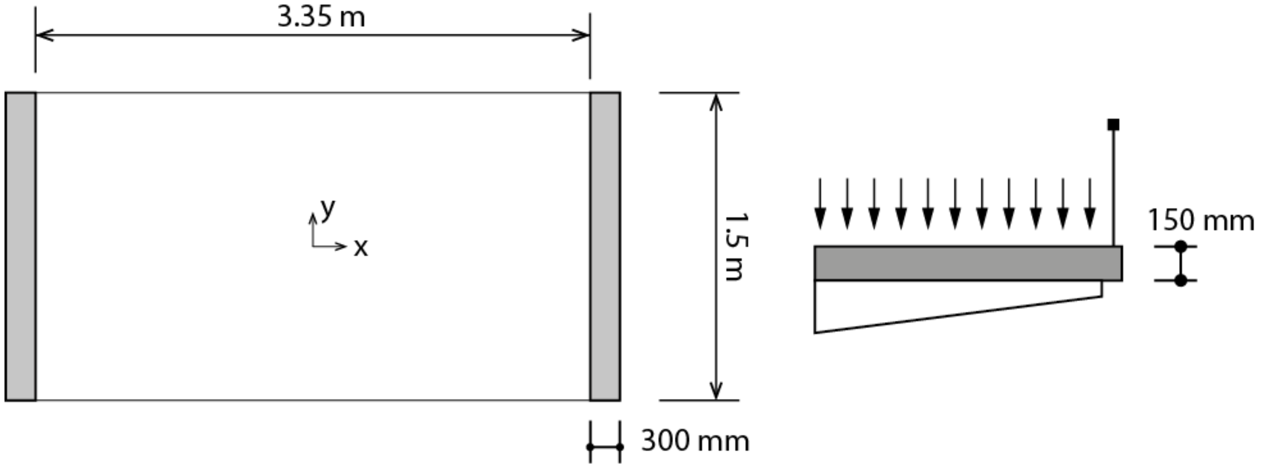


Figure 3.1 Balcony slab.

Parameters:

Concrete strength class : C20/25
weight density concrete : γ = 25 kN/m^3
mean concrete tensile strength : f_{ctm} = 2,2 N/mm^2
modulus of elasticity : E_{cm} = 30000 N/mm^2
Poison's ratio : v = 0,2

Reinforcement class B500B : f_{yd} = 435 N/mm^2
modulus of elasticity : E_s = 200000 N/mm^2

Concrete slab thickness : 150 mm
floor finishing : q_s = 0,4 kN/m^2
Variable load : q_{Qk} = 4,0 kN/m^2

Additional information:

Crack width control:

$$w_{max} = \frac{1}{2}\frac{f_{ctm}}{\tau_{bm}}\frac{Ø}{\rho_{s,eff}}\frac{1}{E_s}(\sigma_s - \alpha\,\sigma_{sr})$$

$$\sigma_{sr} = \frac{M_{crack}}{zA_s}$$

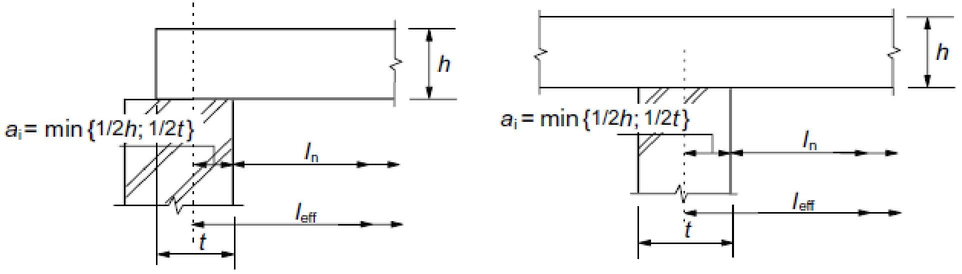
Member loaded in bending:
σ_{sr} follows from the cracking bending moment

Table 15,III Values for τ_{bm}, α and β from eq. (15.15) for various conditions. The values for α between brackets are the recalibrated values as applied in the Eurocode by means of the coefficient k_t (EN 1992-1-1 eq. (7.9))

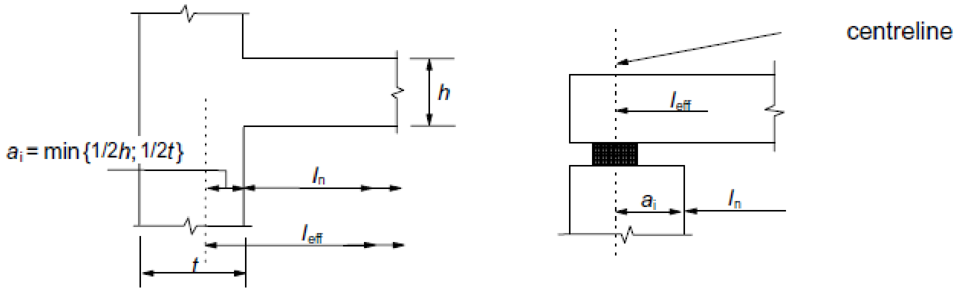
	crack formation stage	stabilized cracking stage
Short term loading	α = 0,5 (0,6) β = 0 τ_{bm} = 2,0 f_{ctm}	α = 0,5(0,6) β = 0 τ_{bm} = 2,0 f_{ctm}
long term or dynamic loading	α = 0,5 (0,6) β = 0 τ_{bm} = 1,6 f_{ctm}	α = 0,3 (0,4) β = 1 τ_{bm} = 2,0 f_{ctm}

Question 3.1

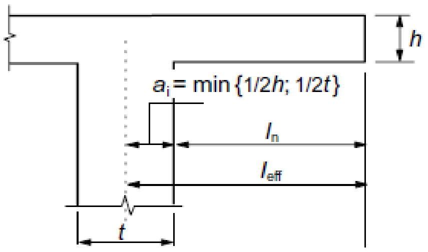
Determine the effective span of the balcony slab. Use the figure below (EN 1992-1-1 fig. 5.4).



(a) Non-continuous members (b) Continuous members



(c) Supports considered fully restrained (d) Bearing provided



(e) Cantilever

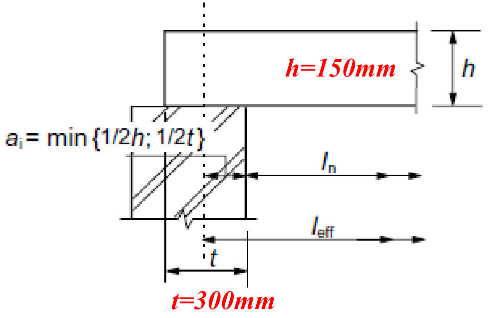
Figure 5.4: Effective span (l_{eff}) for different support conditions

Answer 3.1

The effective span, l_{eff}, of a member can be calculated as follows:

$$l_{eff} = l_n + a_1 + a_2$$

where l_n is the clear distance between the faces of the supports and a_1 and a_2 corresponds to case (a) non-continuous members from EN 1992-1-1, fig 5.4:



(a) Non-continuous members

$$a_1 = a_2 = \min\left\{\frac{1}{2}h; \frac{1}{2}t\right\} = \min\left\{\frac{1}{2}\cdot 0{,}15; \frac{1}{2}\cdot 0{,}3\right\} = 0{,}075 \text{ m}$$

$$l_{eff} = 3{,}35 + 2\cdot 0{,}075 = 3{,}5 \text{ m}$$

Question 3.2

Determine the bending moment $M_{\mathrm{E,x}}$ at SLS (assuming that all load factors are 1,0).

Answer 3.2

self weight:	$h_{\mathrm{slab}} \times \gamma_c = 0.15 \times 25$	$= 3.75$	kN/m^2
floor finishing:		$= 0.4$	kN/m^2
dead weight:	q_{Gk}	$= 4.15$	kN/m^2
variable load:	q_{Qk}	$= 4.0$	kN/m^2

SLS: $q = q_{Gk} + \psi_2 q_{Qk} \qquad = 4{,}15 + 1{,}0 \times 4{,}0 \qquad = 8{,}15\ \mathrm{kN/m^2}$

$$M_{\mathrm{E,x}} = \frac{q l_{eff}{}^2}{8} = \frac{8{,}15 \cdot 3{,}5^2}{8} = 12{,}48\ \mathrm{kNm/m}$$

Question 3.3

Verify whether or not the slab cracks when subjected to the bending moment $M_{\mathrm{E,x}}$. Use the flexural tensile strength of concrete.
(*Note:* the flexural tensile strength is $f_{\mathrm{ctm,fl}} = (1{,}6 - h/1000) f_{\mathrm{ctm}}$ according to the Eurocode).

Answer 3.3

The flexural tensile strength is:
$$f_{\mathrm{ctm,fl}} = (1{,}6 - h/1000) f_{\mathrm{ctm}} = (1{,}6 - 150/1000) \cdot 2{,}2 = 3{,}19\ \mathrm{MPa}$$

The cracking bending moment of the slab is:
$$M_{\mathrm{cr}} = \frac{bh^2}{6} f_{\mathrm{ctm,fl}} = \frac{1000 \cdot 150^2}{6} \cdot 3{,}19 = 11{,}96 \cdot 10^6\ \mathrm{Nmm/m} = 11{,}96\ \mathrm{kNm/m} \leq M_{\mathrm{E,x}} = 12{,}48\ \mathrm{kNm/m}$$
At the SLS bending moment M_{Ex} the **slab will crack**.

Question 3.4

When the slab cracks, determine the height of the compressive zone x and calculate the steel stress σ_{sr} directly after cracking, which is at the cracking bending moment M_{cr}.

Answer 3.4

The height of the compression zone can be calculated as follows:
$$\frac{x}{d} = -\alpha_e \rho + \sqrt{(\alpha_e \rho)^2 + 2\alpha_e \rho} \qquad \text{where} \qquad \alpha_e = \frac{E_s}{E_c} = \frac{200000}{30000} = 6.67$$

$$\rho = \frac{A_s}{bd} = \frac{\frac{1}{4} Ø^2 \pi \cdot 1000 / s}{bd}, \qquad d = 150 - 15 - 10/2 = 130\ \mathrm{mm}$$

$$\rho = \frac{\frac{1}{4} \cdot 10^2 \cdot \pi \cdot 1000 / 200}{1000 \cdot 130} = 0,3\%$$

$$\frac{x}{d} = -6,67 \cdot 0,003 + \sqrt{(6,67 \cdot 0,003)^2 + 2 \cdot 6,67 \cdot 0,003} = 0,182$$

The height of the compression zone is:
$x = 0,182 \cdot 130 = 23,6\text{ mm}$

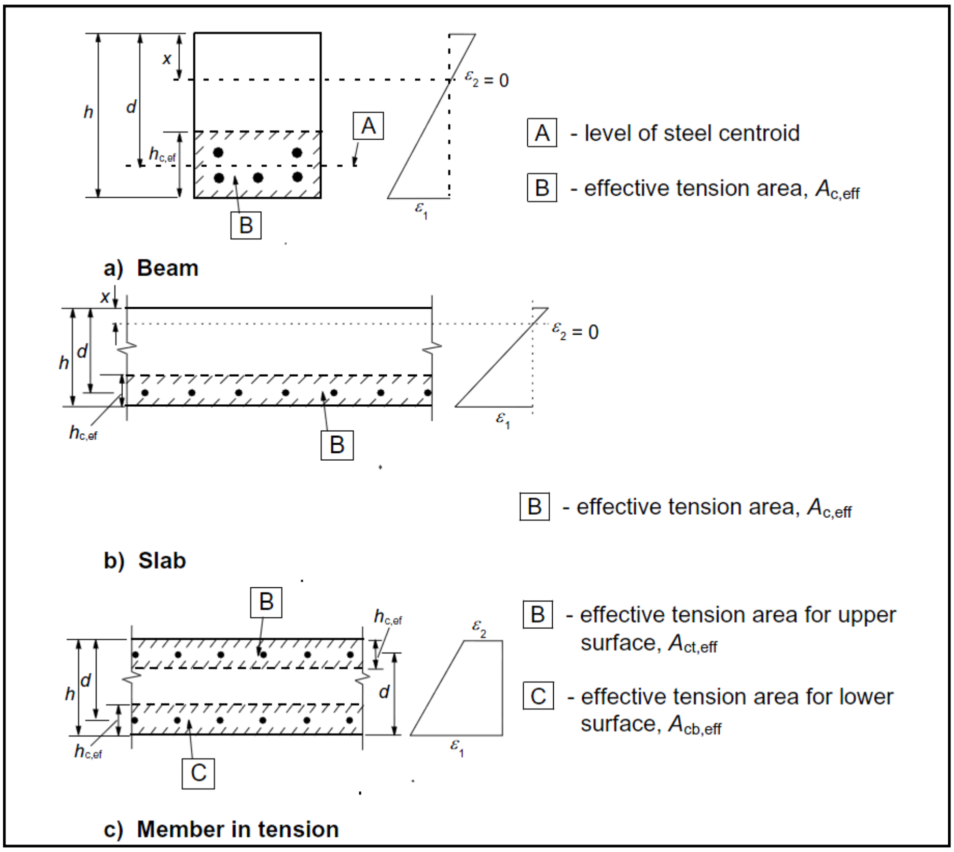
The stress in the steel can be calculated as:

$$\sigma_{\text{sr}} = \frac{M_{\text{cr}}}{A_{\text{s}} z}, \qquad z = d - \frac{x}{3} = 130 - \frac{23,6}{3} = 122,1\text{ mm}$$

$$\sigma_{\text{sr}} = \frac{11,96 \cdot 10^6}{392,7 \cdot 122,1} = 249\text{ MPa}$$

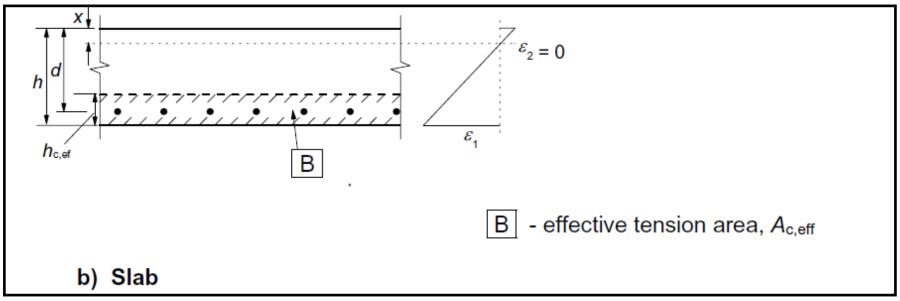
Question 3.5

Indicate the height of the effective tensile area around the tensile reinforcement of the slab $h_{\text{c,eff}}$. Based on that, calculate the effective reinforcement ratio $\rho_{\text{s,eff}}$.
Hint: Use the figure below: Effective concrete area [15.4] (EN 1992-1-1 fig. 7.1).



Answer 3.5

The depth of effective tensile area around the tensile reinforcement $h_{\text{c,eff}}$ can be calculated as follows (EN 1992-1-1, Section 7.3.2):



$$h_{\text{c,eff}} = \min\begin{Bmatrix} 2{,}5\,(h-d) \\ (h-x)/3 \end{Bmatrix} = \min\begin{Bmatrix} 2{,}5\cdot(150-130) \\ (150-23{,}6)/3 \end{Bmatrix} = \min\begin{Bmatrix} 50 \\ 42{,}13 \end{Bmatrix} = 42{,}13\text{ mm}$$

$$\rho_{\text{s,eff}} = \frac{A_{\text{s}}}{bh_{\text{c,eff}}} = \frac{392{,}7}{1000\cdot 42{,}13} = 0{,}93\%$$

Question 3.6

Calculate the maximum crack width w_{max} at a SLS bending moment of 12,48 kNm/m (long term loading). Use the tensile member model.

Maximum crack width is calculated as follows (tensile member model):

$$w_{\text{max}} = \frac{1}{2}\frac{f_{\text{ctm}}}{\tau_{\text{bm}}}\frac{Ø}{\rho_{\text{s,eff}}}\frac{1}{E_{\text{s}}}(\sigma_{\text{s}} - \alpha\sigma_{\text{sr}})$$

where σ_{sr} is the steel stress directly after cracking and σ_s is the steel stress at SLS.

Table 15,III Values for τ_{bm}, α and β from eq. (15.15) for various conditions. The values for α between brackets are the recalibrated values as applied in the Eurocode by means of the coefficient k_t (EN 1992-1-1 eq. (7.9))

	crack formation stage	stabilized cracking stage
Short term loading	α = 0,5 (0,6) β = 0 $\tau_{bm} = 2{,}0\,f_{ctm}$	α = 0,5(0,6) β = 0 $\tau_{bm} = 2{,}0\,f_{ctm}$
long term or dynamic loading	α = 0,5 (0,6) β = 0 $\tau_{bm} = 1{,}6\,f_{ctm}$	α = 0,3 (0,4) β = 1 $\tau_{bm} = 2{,}0\,f_{ctm}$

Answer 3.6

Maximum crack width:

$$w_{\text{max}} = \frac{1}{2}\frac{f_{\text{ctm}}}{\tau_{\text{bm}}}\frac{Ø}{\rho_{\text{s,eff}}}\frac{1}{E_{\text{s}}}(\sigma_{\text{s}} - \alpha\sigma_{\text{sr}})$$ where σ_{sr} is the steel stress directly after cracking.

Steel stress in a crack at SLS (cracked cross-section loaded in pure bending):

$$\sigma_s = \frac{12,48 \cdot 10^6}{392,7 \cdot 122,1} = 260,3 \text{ MPa}$$

$$\left.\begin{matrix} \alpha = 0.3 \\ \tau_{bm} = 2f_{ctm} \end{matrix}\right\} \rightarrow w_{\max} = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{10}{0,0093} \cdot \frac{1}{200000} \cdot (260,3 - 0,3 \cdot 249) = 0,25 \text{ mm}$$

Additional information

Note that σ_{sr} is the steel stress directly after cracking, which is calculated from the cracking bending moment and not from the tensile member model. Using bending moment for the calculation of σ_{sr} results in more consistent results in case it is a prestressed element.

Question 3.7

Based on the reinforcement configuration in *x* direction, determine the minimum reinforcement area in *y* direction.

Answer 3.7

Based on the design bending moment in *x* direction and Poisson's ratio, the bending moment in *y* direction is $M_{\text{Ed,y}} = \nu M_{\text{Ed,x}} \approx 0,2 M_{\text{Ed,x}}$.

Amount of secondary transverse reinforcement required to resist this bending moment:

$$A_{\text{sy}} = \frac{0,2 M_{\text{Ed,x}}}{z_y f_{\text{yd}}}$$

Additional information:

In this example, the reinforcement in the *y* direction has **not** to fulfil the requirement of minimum reinforcement according to the design code (i.e. after concrete cracking, the steel can resist the full tensile force, so that after cracking, the slab/beam does not directly fail). This requirement makes that brittle failure does not occur. For pure bending, the result would be:

$$A_{\text{sy}} = \frac{M_{\text{cr}}}{z_y f_{\text{yk}}} = \frac{\frac{bh^2}{6} f_{\text{ctm,fl}}}{z_y f_{\text{yk}}}$$

(Note that EN 1992-1-1 uses the axial concrete tensile strength f_{ctm}, whereas other codes might use the flexural tensile strength).

In this example, however, it is a one way slab. In that case, the secondary transverse reinforcement should not be less than 20% of the principal reinforcement.

Example 4 – Slab and punching shear resistance

A building uses the outrigger system. The structural system is illustrated in Figure 4.1. The thickness of the concrete floor is 200 mm. The reinforcement mesh in the slab is Ø8 mm – 200 mm in both directions. The concrete cover is 15 mm. The cross-sections of the square columns are 175 mm × 175 mm.

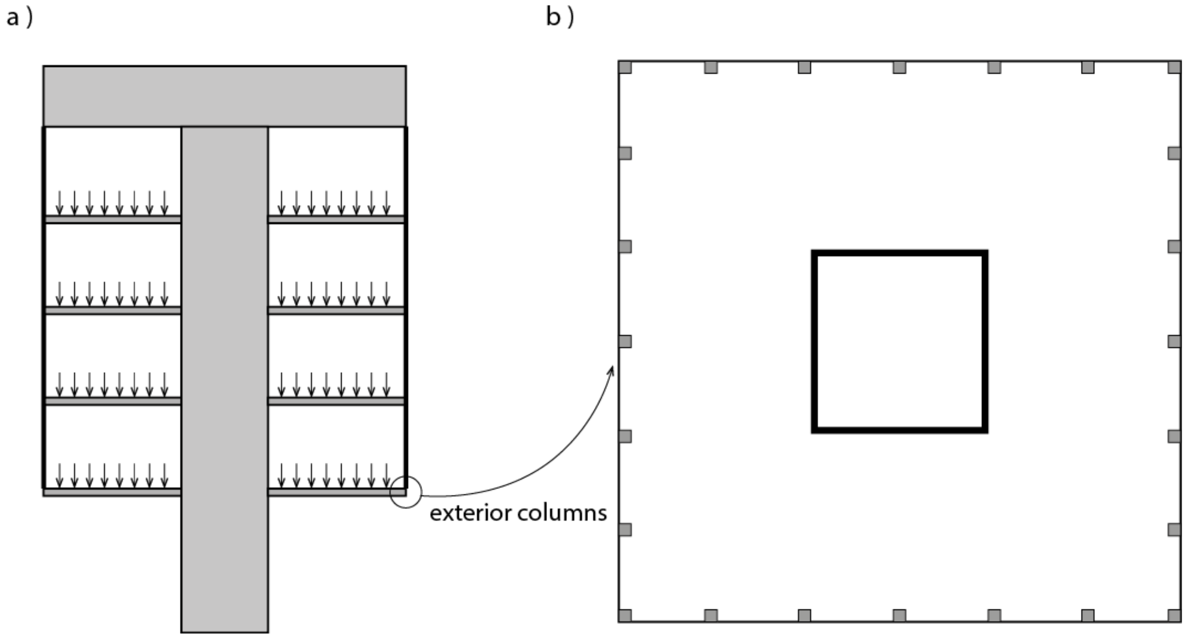


Figure 4.1 Design of a building. (a) side view, (b) plan view.

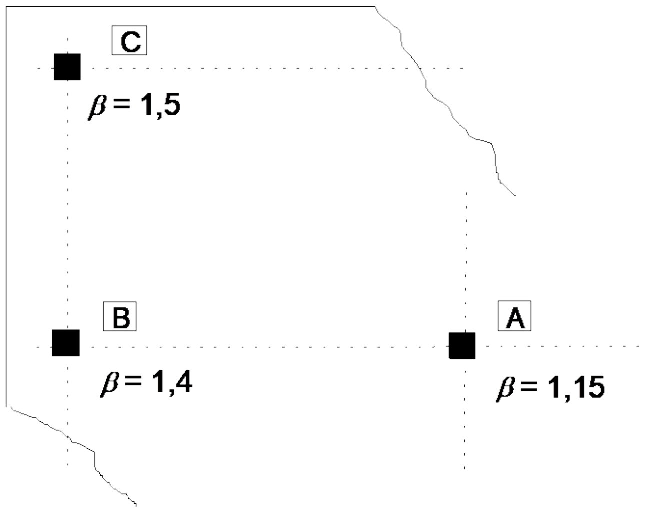
Parameters:

Concrete strength class : C25/30
Material factor : $\gamma_c = 1.5$

Concrete Column : 175 mm × 175 mm
Concrete slab
thickness : 200 mm
concrete cover : 15 mm

Additional information

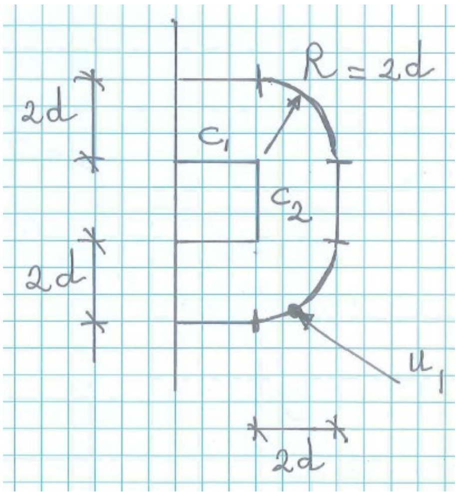
Approximated safe values for β according to the Eurocode:



Question 4.1

The punching capacity of the concrete floor close to the exterior column is verified (see fig 4.1 for the position of the exterior column). Draw and calculate the control perimeter of the side column.

Answer 4.1



u_1 is the basic control perimeter and d is the effective slab depth.

$$\left.\begin{array}{l} d_x = 200 - 15 - \frac{8}{2} = 181mm \\ d_y = 200 - 15 - 8 - \frac{8}{2} = 173mm \end{array}\right\} \rightarrow d = \frac{1}{2}\left(d_x + d_y\right) = 177mm$$

$$2d = 2 \times 177 = 254\ mm$$

$$c_1 = c_2 = 175\ mm$$

$$u_1 = 2c_1 + c_2 + \frac{2 \times 2d \times \pi}{2} = 3c + 2d \times \pi = 3 \times 175 + 2 \times 177 \times \pi = 1637\ \text{mm}$$

Question 4.2

The design load of the column is 150 kN under ULS. Calculate the value of v_{Ed}.

Answer 4.2

$$v_{Ed} = \beta \frac{V_{Ed}}{u_1 d} = 1.4 \times \frac{150 \times 10^3}{1637 \times 177} = 0.72\ \text{MPa}$$

Question 4.3

Calculate $v_{Rd,c}$, (take $v_{\min}$ into account). Does the exterior column punch through the slab?

Answer 4.3

The punching shear resistance stress of the edge column is:

$$v_{Rd,c} = 0.12 \times k \times (100 \rho f_{ck})^{\frac{1}{3}} + k_1 \sigma_{cp} \text{ with a minimum of } v_{\text{Rd,c}} = v_{\min} = 0.035 \times k^{3/2} \sqrt{f_{\text{ck}}} + k_1 \sigma_{cp}$$

where σ_{cp} is the concrete compressive stress in cross section due to axial loading and/or prestressing. In this case σ_{cp}=0.

Reinforcement mesh Ø8 mm – 200 mm is applied:

$$A_{sx} = A_{sy} = \frac{d^2\pi}{4} \times \frac{1000}{s} = \frac{8^2\pi}{4} \times \frac{1000}{200} = 251.33 mm^2 / m$$

$$\left.\begin{aligned} \rho_x = \frac{A_{sx}}{d_x} = \frac{251.32}{181 \times 1000} = 0.14\% \\ \rho_y = \frac{A_{sy}}{d_y} = \frac{251.32}{173 \times 1000} = 0.15\% \end{aligned}\right\} \rightarrow \rho = \sqrt{\rho_x \times \rho_y} = 0.14\%$$

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2,0$$

$$k = 1 + \sqrt{\frac{200}{177}} > 2,0 \rightarrow k = 2,0$$

$$\mathrm{C25/30} \rightarrow f_{\mathrm{ck}} = 25 \mathrm{~MPa}$$

$$v_{Rd,c} = 0.12 \times 2,0 \times (100 \times 0.14 / 100 \times 25)^{\frac{1}{3}} = 0.37 MPa$$

$$v_{\min} = 0.035 \times 2^{3/2} \sqrt{25} = 0.49 MPa$$

$v_{Rd,c} = \max\{v_{Rd,c}, v_{\min}\} = 0.49 MPa < v_{Ed} = 0.72 MPa$ → The exterior column punches through the slab.

Question 4.4

Assumed that the punching shear capacity of the slab is not sufficient, describe the options to increase the punching shear capacity. Check whether all the proposed strengthening methods can be applied in this building.

Answer 4.4

In case structural measures should be taken, these can consist of the:

- application of punching shear reinforcement (hooks, stirrups or dowels). This can be done even after the structure is constructed. However, one should check $v_{\mathrm{Rd,max}}$. Only when v_{Ed} is smaller than $v_{\mathrm{Rd,max}}$ it is possible.

- increase the thickness of the slab locally by a drop panel or column head. This has to be done at the bottom of the slab since the compression zone is at the bottom. u_1 (which is a function of, amongst other, d_H) has to be calculated to determine the size of the head/panel, from which the additional height of the panel can be calculated ($h_{\mathrm{H}} = d_{\mathrm{H}} - d_{\mathrm{eff}}$):

  $$v_{\mathrm{Ed}} = \beta \frac{V_{\mathrm{Ed}}}{u_1 d_{\mathrm{H}}} \leq v_{\mathrm{Rd,c}}$$

- other approaches such as an increase of the compressive strength, increase of the reinforcement ratio, increase of the column size can only be done before the building is constructed.

Example 5 – Slab and crack width

A continuous one-way spanning slab is considered. The floor is intended to be an office area. The slab is supported on 200 mm wide load-bearing block walls at 6000 mm centres, see fig. 5.1.

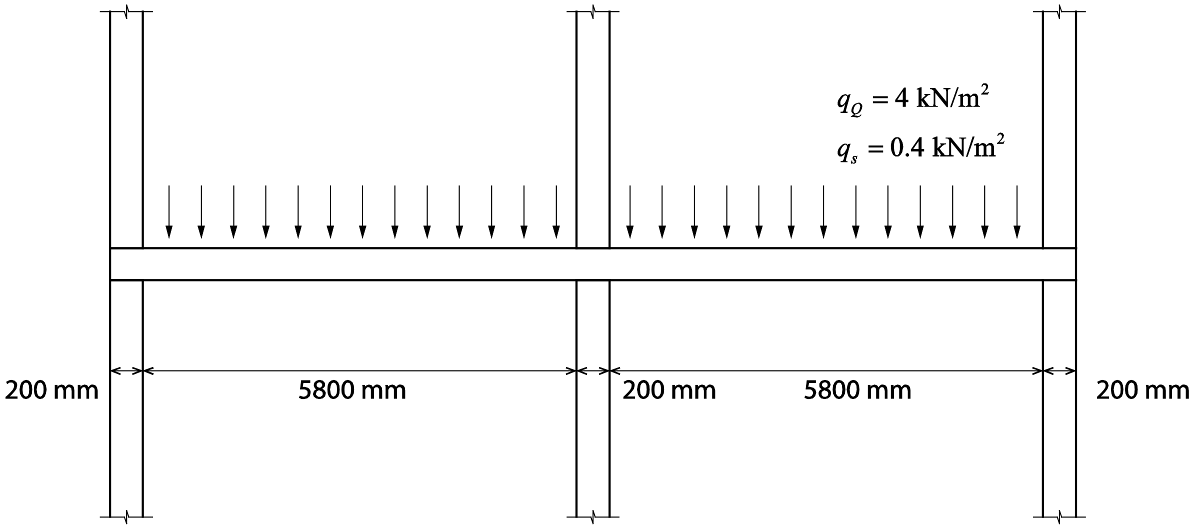


Figure 5.1 Side view of a concrete floor.

Parameters:

Concrete strength class : C20/25
weight density concrete : $\gamma = 25$ kN/m^3
mean concrete tensile strength : $f_{ctm} = 2.2$ N/mm^2
modulus of elasticity : $E_{cm} = 30000$ N/mm^2
Poison's ratio : $v = 0.2$

Reinforcement class B500B : $f_{yd} = 435$ N/mm^2 $f_{yk} = 500$ N/mm^2
modulus of elasticity : $E_s = 200000$ N/mm^2

Design moment of the critical section in the primary load transfer direction : $M_{Ed} = 42$ kNm/m (ULS)
$M_{E,freq} = 25$ kNm/m (SLS)

Additional information:

Concrete Cover:

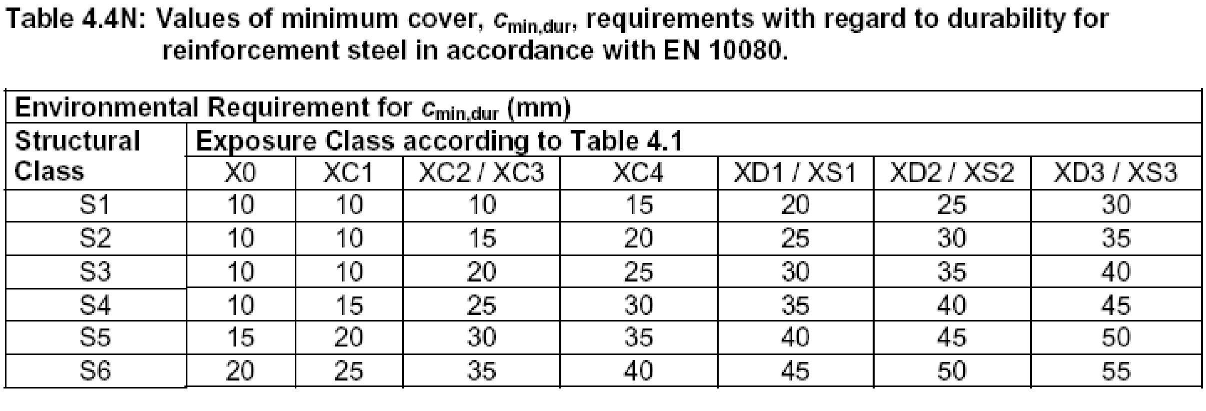
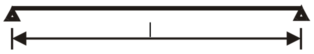
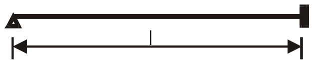
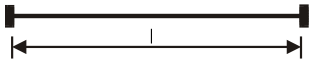
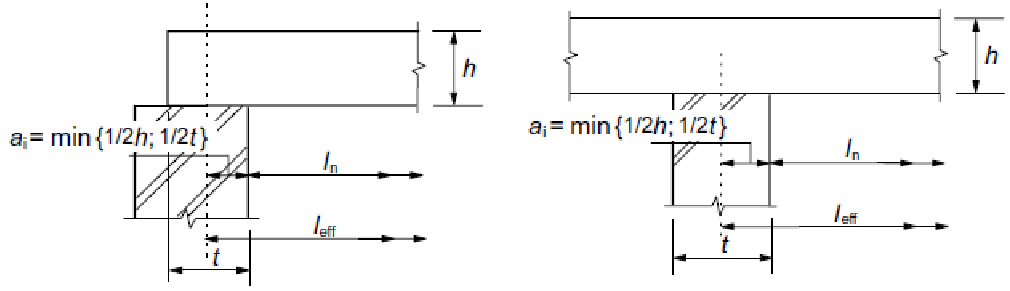


Table 4.4N: Values of minimum cover, $c_{min,dur}$, requirements with regard to durability for reinforcement steel in accordance with EN 10080.

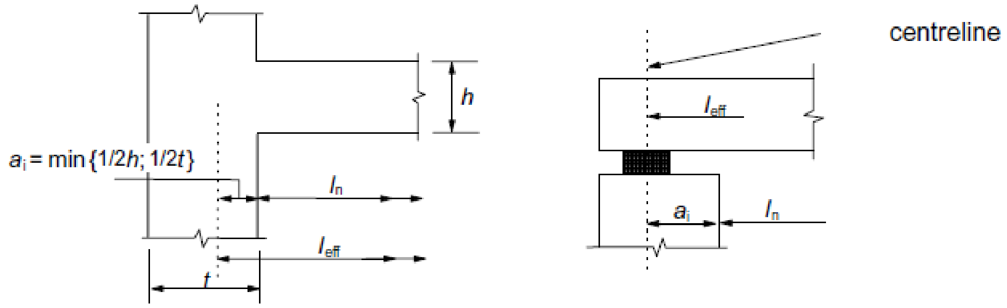
Environmental Requirement for $c_{min,dur}$ (mm)							
Structural Class	Exposure Class according to Table 4.1						
	X0	XC1	XC2 / XC3	XC4	XD1 / XS1	XD2 / XS2	XD3 / XS3
S1	10	10	10	15	20	25	30
S2	10	10	15	20	25	30	35
S3	10	10	20	25	30	35	40
S4	10	15	25	30	35	40	45
S5	15	20	30	35	40	45	50
S6	20	25	35	40	45	50	55

Slab thickness estimation:

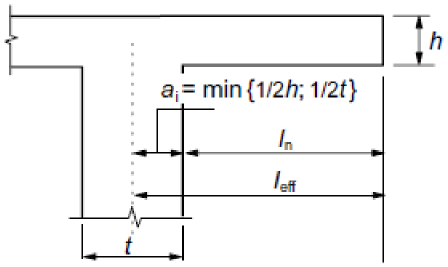
Scheme	l / d ($l \leq$ 7,0 m)	l / d ($l \geq$ 7,0 m)
 slabs simply supported at both sides (pinned supports).	25	175 / l
 slabs simply supported at one side and fixed or continuous at the other side	32	225 / l
 slabs fixed or continuous at both sides	35	245 / l



(a) Non-continuous members (b) Continuous members



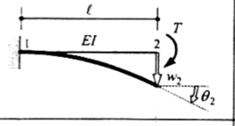
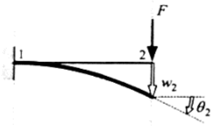
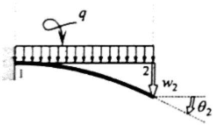
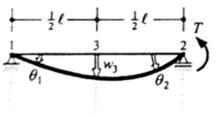
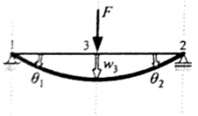
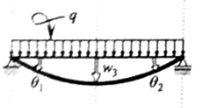
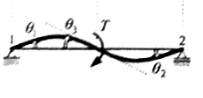
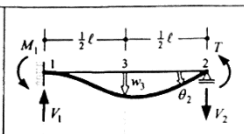
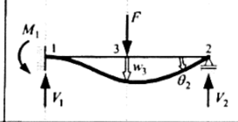
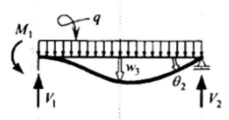
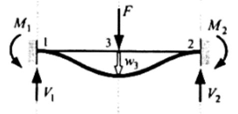
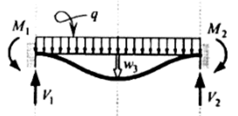
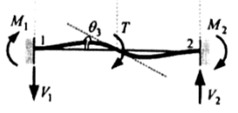
(c) Supports considered fully restrained (d) Bearing provided



(e) Cantilever

Figure 5.4: Effective span (l_{eff}) for different support conditions

Bending moment table of beams with different boundary conditions:

forget-me-nots	(1)		$\theta_2 = \frac{T\ell}{EI}$; $w_2 = \frac{T\ell^2}{2EI}$
	(2)		$\theta_2 = \frac{F\ell^2}{2EI}$; $w_2 = \frac{F\ell^3}{3EI}$
	(3)		$\theta_2 = \frac{q\ell^3}{6EI}$; $w_2 = \frac{q\ell^4}{8EI}$
simply supported beam (statically determinate)	(4)		$\theta_1 = \frac{1}{6}\frac{T\ell}{EI}$; $\theta_2 = \frac{1}{3}\frac{T\ell}{EI}$; $w_3 = \frac{1}{16}\frac{T\ell^2}{EI}$
	(5)		$\theta_1 = \theta_2 = \frac{1}{16}\frac{F\ell^2}{EI}$; $w_3 = \frac{1}{48}\frac{F\ell^3}{EI}$
	(6)		$\theta_1 = \theta_2 = \frac{1}{24}\frac{q\ell^3}{EI}$; $w_3 = \frac{5}{384}\frac{q\ell^4}{EI}$
	(a)		$\theta_1 = \theta_2 = \frac{1}{24}\frac{T\ell}{EI}$; $\theta_3 = \frac{1}{12}\frac{T\ell}{EI}$; $w_3 = 0$
statically indeterminate beam (one fixed end)	(7)		$\theta_2 = \frac{1}{4}\frac{T\ell}{EI}$, $w_3 = \frac{1}{32}\frac{T\ell^2}{EI}$ $M_1 = \frac{1}{2}T$; $V_1 = V_2 = \frac{3}{2}\frac{T}{\ell}$
	(8)		$\theta_2 = \frac{1}{32}\frac{F\ell^2}{EI}$; $w_3 = \frac{7}{768}\frac{F\ell^3}{EI}$ $M_1 = \frac{3}{16}F\ell$; $V_1 = \frac{11}{16}F$; $V_2 = \frac{5}{16}F$
	(9)		$\theta_2 = \frac{1}{48}\frac{q\ell^3}{EI}$; $w_3 = \frac{1}{192}\frac{q\ell^4}{EI}$ $M_1 = \frac{1}{8}q\ell^2$; $V_1 = \frac{5}{8}q\ell$; $V_2 = \frac{3}{8}q\ell$
statically indeterminate beam (two fixed ends)	(10)		$w_3 = \frac{1}{192}\frac{F\ell^3}{EI}$ $M_1 = M_2 = \frac{1}{8}F\ell$; $V_1 = V_2 = \frac{1}{2}F$
	(11)		$w_3 = \frac{1}{384}\frac{q\ell^4}{EI}$ $M_1 = M_2 = \frac{1}{12}q\ell^2$; $V_1 = V_2 = \frac{1}{2}q\ell$
	(b)		$\theta_3 = \frac{1}{16}\frac{T\ell}{EI}$; $w_3 = 0$ $M_1 = M_2 = \frac{1}{4}T$; $V_1 = V_2 = \frac{3}{2}\frac{T}{\ell}$

Some formulae for prismatic beams with bending stiffness EI.

T, F and q represent the load by a couple, force and uniformly distributed load respectively.

M_i and V_i represent the bending moment and shear force on the end i of the beam, due to the support reactions.

Question 5.1

Determine the concrete cover of the floor slab. Assume that Δc_{dev} = 10 mm. The structural class is S1. Also estimate the thickness of the floor.

Answer 5.1

The floor is intended to be the office area (indoor). According to EN 1992-1-1 Table 4.1, the corresponding exposure class is XC1.
The structural class is S1.
The nominal cover can be determined using the following equation:

$$c_{nom} = c_{min} + \Delta c_{dev}$$

$$c_{min} = \min \begin{cases} c_{\mathrm{min},b} \\ c_{\mathrm{min,dur}} + \Delta c_{dur,\gamma} - \Delta c_{dur,\mathrm{st}} - \Delta c_{dur,\mathrm{add}} \\ 10 \text{ mm} \end{cases}$$

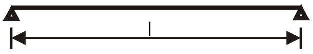
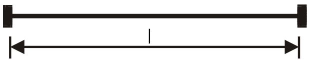
where:

- $c_{\mathrm{min},b}$ is the minimum cover requirement with regard to bond and should be ≥ Ø (bar diameter, or in case of bundled reinforcement, the equivalent diameter)
- $c_{\mathrm{min,dur}}$ takes into account the exposure classes and the structural classes, and can be determined from EN 1992-1-1, Table 4.4N. For exposure class X1 and structural class S1 $c_{\mathrm{min},dur}$ = 10 mm
- $\Delta c_{dur,\gamma}$ is the additive safety element (the recommended value is 0)
- $\Delta c_{dur,\mathrm{st}}$ is reduction of minimum cover for use of stainless steel (the recommended value is 0)
- $\Delta c_{dur,\mathrm{add}}$ is reduction of minimum cover for use of additional protection (the recommended value is 0)

Therefore, c_{min} = 10 mm, and for the recommended value of Δc_{dev} = 10 mm:

$$c_{nom} = c_{min} + \Delta c_{dev} = 10 + 10 = 20 \text{ mm}$$

Slab thickness can be estimated from the following table:

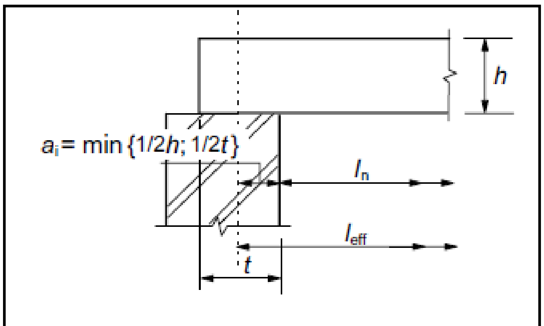
Scheme	***l / d* (*l* ≤ 7,0 m)**	***l / d* (*l* ≥ 7,0 m)**
 slabs simply supported at both sides (pinned supports).	25	175 / *l*
 slabs simply supported at one side and fixed or continuous at the other side	32	225 / *l*
 slabs fixed or continuous at both sides	35	245 / *l*

$l / d = 32$

The effective span, l_{eff}, of a member can be calculated as follows:

$$l_{eff} = l_n + a_1 + a_2$$

where l_n is the clear distance between the faces of the supports and a_1 and a_2 correspond to case "(a) non-continuous members" from EN 1992-1-1 fig 5.4:



(a) Non-continuous members

Assuming that $h \geq t$, $h \geq 200mm$

$$l_{eff} = l_n + 2 \times \frac{t}{2} = 5800 + 200 = 6000 \text{ mm}$$

$$d = l / 32 = l_{eff} / 32 = 6000 / 32 = 187.5 \text{ mm}$$

$$h = d + c + \O / 2$$

Assuming that $\O = 12$ mm :

$h = 187.5 + 20 + 12 / 2 = 213.5 \approx 220$ mm which is in accordance to the intial assumption $h \geq 200mm$

Question 5.2

Assume that the thickness of the floor is 220 mm. Design the primary reinforcement and the secondary reinforcement at the bottom of the floor. Check your design with the minimum reinforcement ratio.

Reminder: Use the design moment given in the question properly. The minimum reinforcement ratio can be estimated by $\rho_{\min} = 0{,}26 f_{\text{ctm}}/f_{\text{yk}}$, with $s_{\max,\text{slab}} \leq 450$ mm; you may assume that $z = 0.9d$.

Answer 5.2

With $h = 220mm$ and assuming that $Ø = 12mm$ $\quad \rightarrow d_x = 220 - 20 - 12/2 = 194mm$

$\rightarrow z_x = 0.9 \times d_x = 174.6mm$

In this assignment, the design moment at the critical section in the primary load transfer direction is given: $M_{\text{Ed}} = 42$ kNm/m (ULS) and $M_{\text{E,freq}} = 25$ kNm/m (SLS).

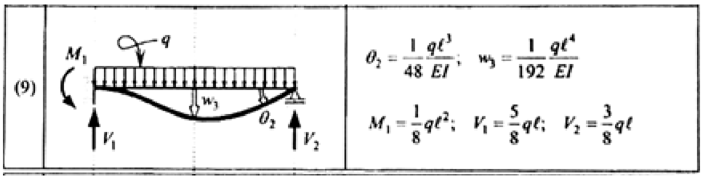
Additional information

If it was not given, it would have to be calculated in the following way:

self-weight:	$h_{\text{slab}} \times \gamma_c = 0.22 \times 25$	$= 5.5$	kN/m^2
floor finishing:		$= 0.4$	kN/m^2
dead weight:	q_{Gk}	$= 5.9$	kN/m^2
variable load:	q_{Qk}	$= 4.0$	kN/m^2

ULS: $\quad q_{Ed} = \gamma_G \times q_{Gk} + \gamma_Q \times q_{Qk} \quad = 1.2 \times 5.9 + 1.5 \times 4.0 \quad = 13.08\,\text{kN/m}^2$

This is one way span, which maximum moment can be calculated by using the strip method. Note that the maximum positive bending moment is not at midspan position, but at $^3/_8 l$ from an end support, see figure below ($M_{\text{span,max}}$ at the position where the shear force $V = 0$).



Result:

$$M_{\max} = \frac{3}{8}ql \times \frac{3}{8}l - \frac{1}{2}q(\frac{3}{8}l)^2 = \frac{9}{128}ql^2$$

$$M_{\text{Ed}} = \frac{9ql_{eff}^{\;2}}{128} = \frac{9 \times 13.08 \times 6^2}{128} = 33.1\,\text{kNm/m}$$

The primary reinforcement can be determined as follows:

$$A_{s,x} = \frac{M_{Ed,x}}{z_x f_{yd}} = \frac{42}{174.6 \times 435} 10^6 = 553 \text{ mm}^2/\text{m}$$

For one way slabs, the secondary transverse reinforcement can be determined from the bending moment in transverse direction:

$M_{Ed,y} = \nu M_{Ed,x} \approx 0,2 M_{Ed,x}$ (Poisson's ratio for uncracked concrete is about 0.2).

$$M_{Ed,y} \approx 0.2 \times 42 = 8.4 kNm / m$$

Assuming that $Ø = 12mm$, $d_y = 194 - 12 = 182mm \rightarrow z_y = 0.9 \times d_y = 163.8mm$

$$A_{s,y} = \frac{M_{Ed,y}}{z_y \times f_{yd}} = \frac{8.4}{163.8 \times 435} 10^6 = 117.9 mm^2 / m$$

For the primary reinforcement:

$$A_{s,x} > A_{s,\min} \rightarrow A_{s,x} = 553 \text{ mm}^2/\text{m}$$

$$s_x \leq \frac{1000}{\frac{4 \times A_{s,x}}{Ø^2 \pi}}$$

With $Ø = 12mm \rightarrow s_x \leq \frac{1000}{\frac{4 \times 553}{12^2 \pi}} = 204.5mm \rightarrow Ø12 - 200mm$ $\left(A_s = \frac{Ø^2 \pi}{4} \times \frac{1000}{s} = 565.5 mm^2 / m \right)$

For the secondary transverse reinforcement:

$\rightarrow A_{s,y} = 118 \text{ mm}^2/\text{m}$

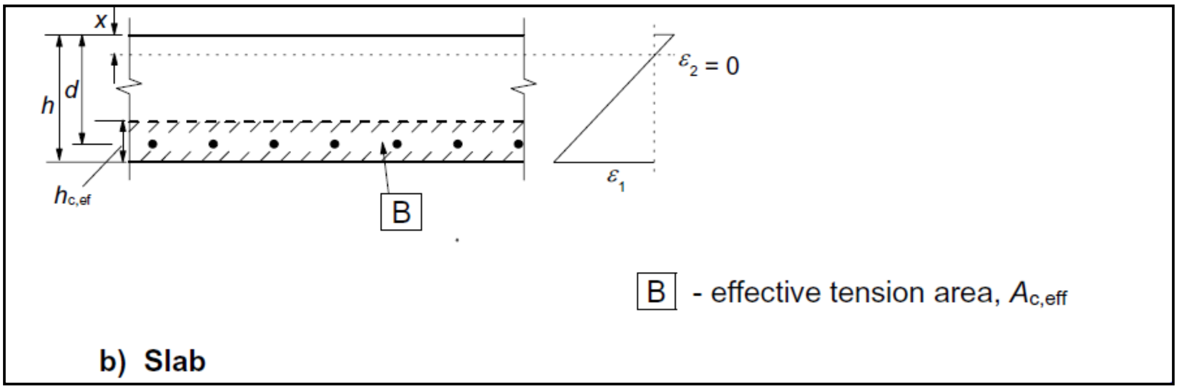
With $Ø = 12mm \rightarrow s_x \leq \frac{1000}{\frac{4 \times 118}{12^2 \pi}} = 960 \text{ mm}$

Since $s_{\max,slab} \leq 450mm \rightarrow Ø12 - 450mm$ $\left(A_s = \frac{Ø^2 \pi}{4} \times \frac{1000}{s} = 251 \text{ mm}^2/\text{m} \right)$

Or with $Ø = 8mm \rightarrow s_x \leq \frac{1000}{\frac{4 \times 118}{8^2 \pi}} = 426 \text{ mm} \rightarrow Ø8 - 425 \text{ mm}$ $\left(A_s = \frac{Ø^2 \pi}{4} \times \frac{1000}{s} = 118 \text{ mm}^2/\text{m} \right)$

Question 5.3

The crack width has to be evaluated at the bottom surface of the floor. Estimate the effective depth of the equivalent tensile member. (Assuming a reinforcement design of Ø12 – 200 mm in the primary direction).



b) Slab

$$h_{\text{c,eff}} = \min \begin{Bmatrix} 2{,}5\,(h-d) \\ (h-x)/3 \end{Bmatrix}$$

where x is the height of the concrete compression zone.

Answer 5.3

Calculate the concrete compression zone height in the cracked stage:

$$\frac{x}{d} = -\alpha_e \rho + \sqrt{(\alpha_e \rho)^2 + 2\alpha_e \rho}$$

where

$$\alpha_e = \frac{E_s}{E_c} = \frac{200000}{30000} = 6.67$$

$$\rho = \frac{A_s}{bd} = \frac{\frac{1}{4}Ø^2 \pi \cdot 1000/s}{bd} = \frac{\frac{1}{4} \cdot 12^2 \cdot \pi \cdot 1000/200}{1000 \cdot 194} = 0{,}29\%$$

$$\frac{x}{d} = -6{,}67 \cdot 0{,}003 + \sqrt{(6{,}67 \cdot 0{,}0029)^2 + 2 \cdot 6{,}67 \cdot 0{,}0029} = 0{,}179$$

The height of the compression zone is:

$$x = 0{,}179 \cdot 194 = 34{,}7 \text{ mm}$$

$$h_{\text{c,eff}} = \min \begin{Bmatrix} 2{,}5\,(h-d) \\ (h-x)/3 \end{Bmatrix} = \min \begin{Bmatrix} 2{,}5 \cdot (220-194) \\ (220-34.7)/3 \end{Bmatrix} = \min \begin{Bmatrix} 65 \\ 61.8 \end{Bmatrix} = 61.8 \text{ mm}$$

Question 5.4

Determine the maximum crack width of the floor (long term loading; do not take into account shrinkage). Use the tensile member model.

Crack width control:

$$w_{\max} = \frac{1}{2}\frac{f_{\text{ctm}}}{\tau_{\text{bm}}}\frac{Ø}{\rho_{\text{s,eff}}}\frac{1}{E_{\text{s}}}(\sigma_{\text{s}} - \alpha\,\sigma_{\text{sr}})$$

$$\sigma_{\text{sr}} = \frac{M_{\text{crack}}}{zA_{\text{s}}}$$

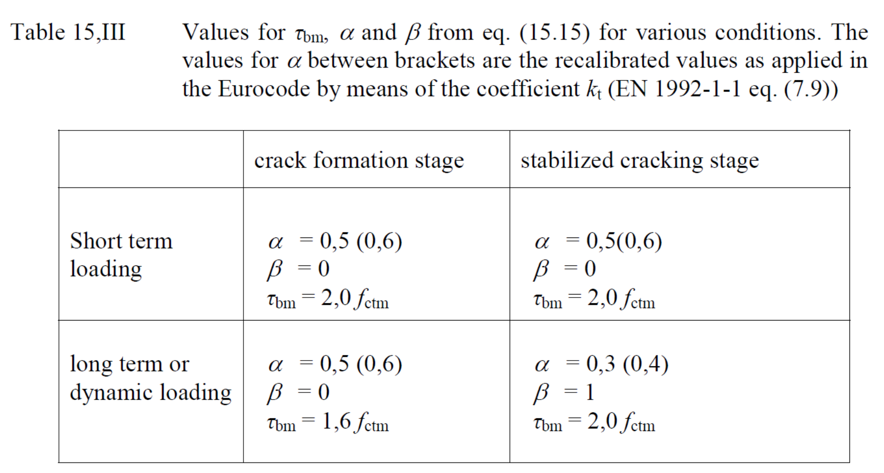


Table 15,III Values for τ_{bm}, α and β from eq. (15.15) for various conditions. The values for α between brackets are the recalibrated values as applied in the Eurocode by means of the coefficient k_t (EN 1992-1-1 eq. (7.9))

	crack formation stage	stabilized cracking stage
Short term loading	α = 0,5 (0,6) β = 0 $\tau_{\text{bm}} = 2{,}0\,f_{\text{ctm}}$	α = 0,5(0,6) β = 0 $\tau_{\text{bm}} = 2{,}0\,f_{\text{ctm}}$
long term or dynamic loading	α = 0,5 (0,6) β = 0 $\tau_{\text{bm}} = 1{,}6\,f_{\text{ctm}}$	α = 0,3 (0,4) β = 1 $\tau_{\text{bm}} = 2{,}0\,f_{\text{ctm}}$

Answer 5.4

First, it is important to determine whether cracking occurs when the SLS load $M_{E,\text{freq}} = 25$ kNm/m is applied and, if so, what the cracking stage is (either crack formation stage or stabilized cracking stage).

The cracking bending moment of the slab is (calculated based on the flexural tensile strength):

$$M_{cr} = W \times f_{ctm,fl}$$

where $f_{ctm,fl}$ is the flexural tensile strength:

$$f_{\text{ctm,fl}} = (1{,}6 - h/1000)f_{\text{ctm}} = (1{,}6 - 220/1000)\cdot 2{,}2 = 3{,}04 \text{ MPa}$$

$$M_{\text{cr}} = W \times f_{\text{ctm,fl}} = \frac{bh^2}{6} \times f_{\text{ctm,fl}} = \frac{1000\cdot 220^2}{6}\cdot 3{,}04 = 24{,}49 \text{ kNm/m} < M_{\text{E,freq}} = 25kNm/m$$

Result:
The member **cracks** at SLS. Furthermore, when the SLS load is applied, the member is in the **stabilized cracking stage**.

Maximum crack width is calculated as follows (tensile member model):

$$w_{\max} = \frac{1}{2}\frac{f_{\text{ctm}}}{\tau_{\text{bm}}}\frac{Ø}{\rho}\frac{1}{E_{\text{s}}}(\sigma_{\text{s}} - \alpha\sigma_{\text{sr}} + \beta\varepsilon_{sc}\,\text{E}_s)$$

where σ_{sr} is the steel stress directly after cracking, σ_s is the steel stress at SLS and α and β can be determined using the following table:

Table 15,III Values for τ_{bm}, α and β from eq. (15.15) for various conditions. The values for α between brackets are the recalibrated values as applied in the Eurocode by means of the coefficient k_t (EN 1992-1-1 eq. (7.9))

	crack formation stage	stabilized cracking stage
Short term loading	α = 0,5 (0,6) β = 0 $\tau_{\mathrm{bm}} = 2{,}0 f_{\mathrm{ctm}}$	α = 0,5(0,6) β = 0 $\tau_{\mathrm{bm}} = 2{,}0 f_{\mathrm{ctm}}$
long term or dynamic loading	α = 0,5 (0,6) β = 0 $\tau_{\mathrm{bm}} = 1{,}6 f_{\mathrm{ctm}}$	α = 0,3 (0,4) β = 1 $\tau_{\mathrm{bm}} = 2{,}0 f_{\mathrm{ctm}}$

σ_{sr} is the steel stress directly after cracking:

$$\sigma_{\mathrm{sr}} = \frac{M_{\mathrm{crack}}}{A_{\mathrm{s}}\, z}$$

$$z = d - \frac{x}{3} = 192 - \frac{34.7}{3} = 182.4mm$$

$$\sigma_{\mathrm{sr}} = \frac{24,49 \times 10^6}{565.5 \times 182.4} = 237MPa$$

Steel stress in a crack at SLS (cracked cross-section loaded in pure bending):

$$\sigma_{\mathrm{s}} = \frac{M_{\mathrm{E,freq}}}{A_{\mathrm{s}} \times z}$$

Note that here, z is not the same as in **Question 5.2** where the ULS was investigated and where it was stated that it was allowed to assume that $z = 0.9d$. Now, the member is at the SLS. Therefore:

$$z = d - \frac{x}{3} = 192 - \frac{34.7}{3} = 182.4mm$$

$$\sigma_{\mathrm{s}} = \frac{25 \times 10^6}{565.5 \times 182.4} = 242MPa$$

According to table 15,III $\rightarrow \alpha = 0.3$ and asuming $\varepsilon_{sc} = 0$ and $\tau_{\mathrm{bm}} = 2 f_{\mathrm{ctm}}$

Finally, maximum crack width in the stabilized crack stage can be calculated as follows:

$$w_{\mathrm{max}} = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{12}{0,0091} \cdot \frac{1}{200 \times 10^3} \cdot (242 - 0,3 \cdot 237) = 0,28 \text{ mm}$$

Example 6 – Slab and punching shear resistance

Consider the following two-way slab, simply supported at four edges as rigid line supports (no rotational fixity) and a column in the centre.

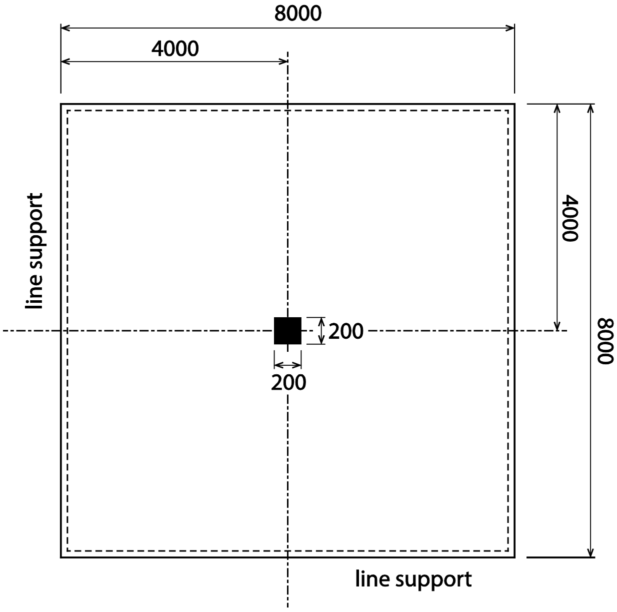


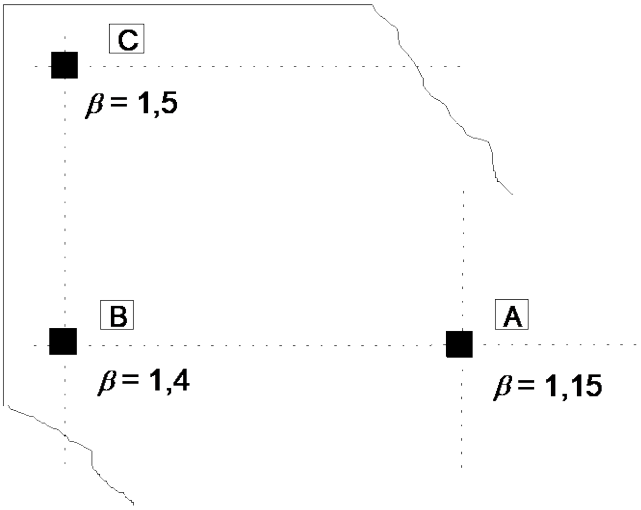
Figure 6.1 Top view of the slab

Parameters:

Concrete strength class : C30/35
Floor thickness : $h = 180$ mm
Column size : 200×200 mm^2
Concrete cover : c = 20 mm
Rebar configurations : $\rho_{lx} = 0.26\%$, $\rho_{ly} = 0.24\%$, with Ø12 rebars in both directions
Design value of the punching shear force : $V_{Ed} = 150$ kN

Additional information

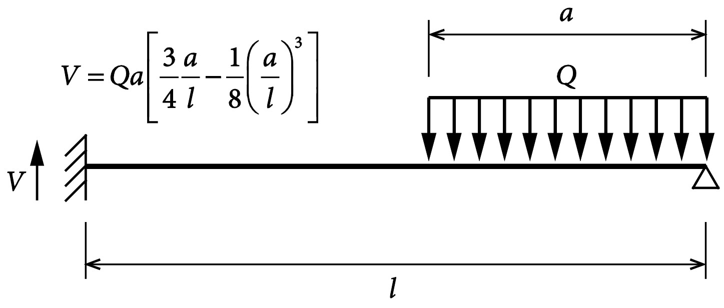
Approximated safe values for β according to the Eurocode:



Question 6.1

Estimate the force that is carried by the column with the strip method. There is a uniformly distributed load *q* on the slab, thus the value of the force is in terms of *q*.

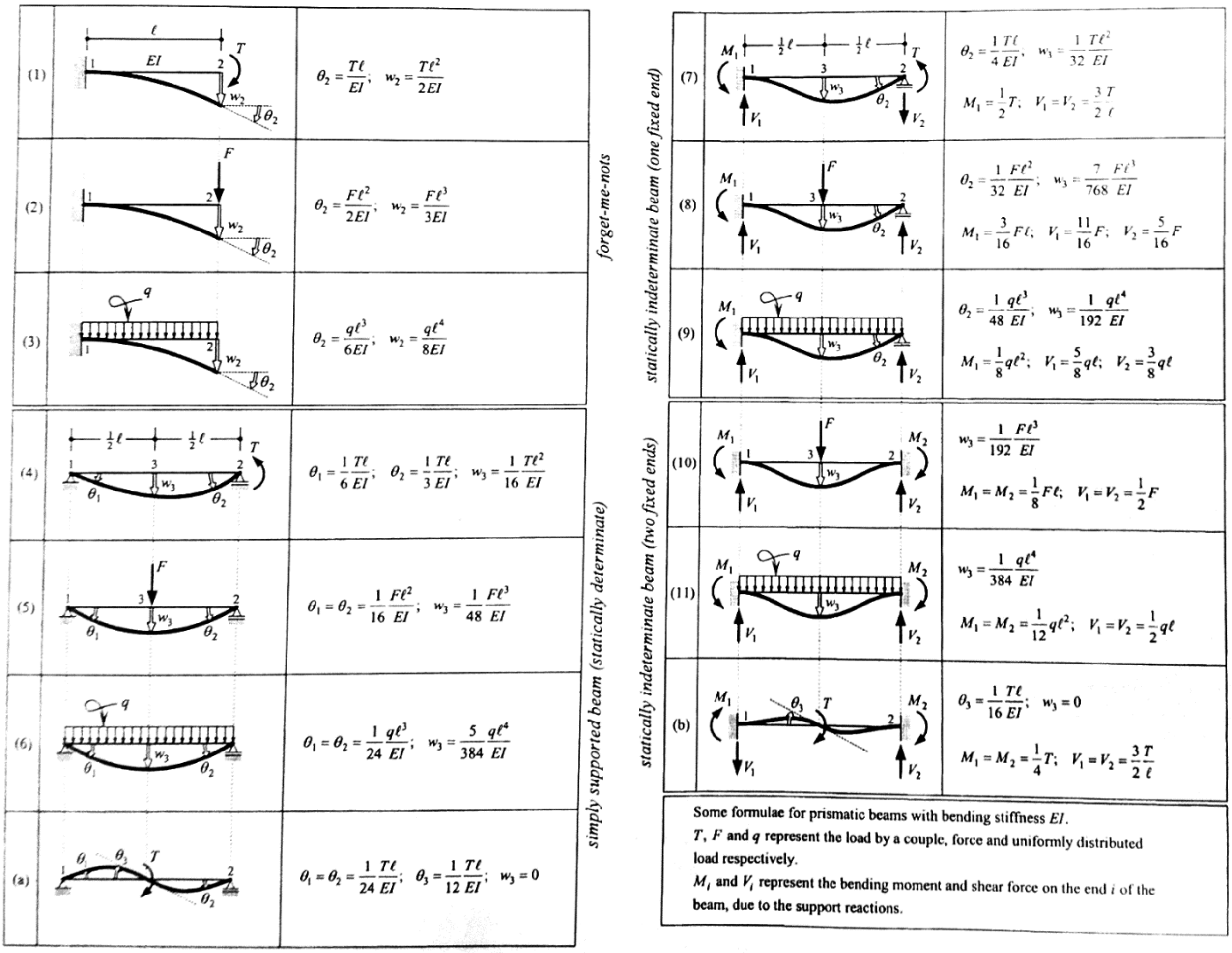
Hint: You may assume that the width of the column strip is $^{1}/_{4}l$, or simply choose a load carrying mechanism which you think is appropriate. Figure 6.2 is only an addition to the following formulae table.



$$V = Qa\left[\frac{3}{4}\frac{a}{l} - \frac{1}{8}\left(\frac{a}{l}\right)^3\right]$$

Figure 2.2 A complimentary formula for a shear force calculation.

Bending moment and shear force table of beams with different boundary conditions:

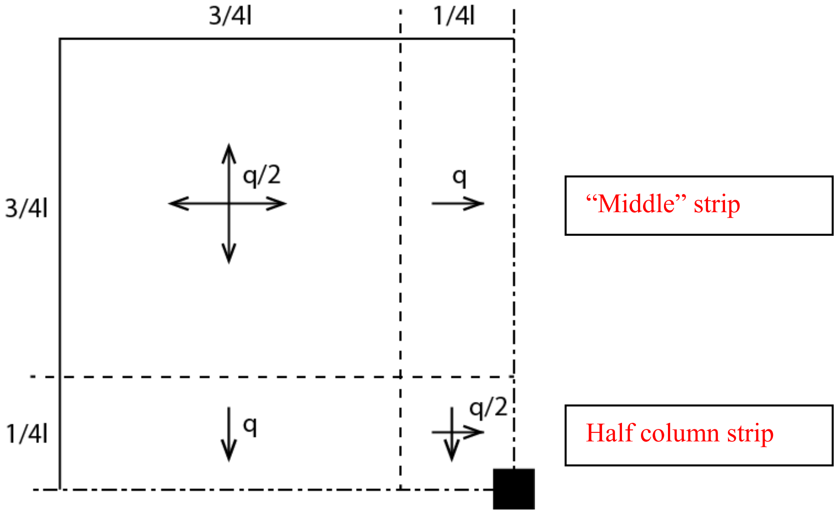


forget-me-nots	(1)	$\theta_2 = \frac{T\ell}{EI}$; $w_2 = \frac{T\ell^2}{2EI}$
	(2)	$\theta_2 = \frac{F\ell^2}{2EI}$; $w_2 = \frac{F\ell^3}{3EI}$
	(3)	$\theta_2 = \frac{q\ell^3}{6EI}$; $w_2 = \frac{q\ell^4}{8EI}$
simply supported beam (statically determinate)	(4)	$\theta_1 = \frac{1}{6}\frac{T\ell}{EI}$; $\theta_2 = \frac{1}{3}\frac{T\ell}{EI}$; $w_3 = \frac{1}{16}\frac{T\ell^2}{EI}$
	(5)	$\theta_1 = \theta_2 = \frac{1}{16}\frac{F\ell^2}{EI}$; $w_3 = \frac{1}{48}\frac{F\ell^3}{EI}$
	(6)	$\theta_1 = \theta_2 = \frac{1}{24}\frac{q\ell^3}{EI}$; $w_3 = \frac{5}{384}\frac{q\ell^4}{EI}$
	(a)	$\theta_1 = \theta_2 = \frac{1}{24}\frac{T\ell}{EI}$; $\theta_3 = \frac{1}{12}\frac{T\ell}{EI}$; $w_3 = 0$
statically indeterminate beam (one fixed end)	(7)	$\theta_2 = \frac{1}{4}\frac{T\ell}{EI}$; $w_3 = \frac{1}{32}\frac{T\ell^2}{EI}$ $M_1 = \frac{1}{2}T$; $V_1 = V_2 = \frac{3}{2}\frac{T}{\ell}$
	(8)	$\theta_2 = \frac{1}{32}\frac{F\ell^2}{EI}$; $w_3 = \frac{7}{768}\frac{F\ell^3}{EI}$ $M_1 = \frac{3}{16}F\ell$; $V_1 = \frac{11}{16}F$; $V_2 = \frac{5}{16}F$
	(9)	$\theta_2 = \frac{1}{48}\frac{q\ell^3}{EI}$; $w_3 = \frac{1}{192}\frac{q\ell^4}{EI}$ $M_1 = \frac{1}{8}q\ell^2$; $V_1 = \frac{5}{8}q\ell$; $V_2 = \frac{3}{8}q\ell$
statically indeterminate beam (two fixed ends)	(10)	$w_3 = \frac{1}{192}\frac{F\ell^3}{EI}$ $M_1 = M_2 = \frac{1}{8}F\ell$; $V_1 = V_2 = \frac{1}{2}F$
	(11)	$w_3 = \frac{1}{384}\frac{q\ell^4}{EI}$ $M_1 = M_2 = \frac{1}{12}q\ell^2$; $V_1 = V_2 = \frac{1}{2}q\ell$
	(b)	$\theta_3 = \frac{1}{16}\frac{T\ell}{EI}$; $w_3 = 0$ $M_1 = M_2 = \frac{1}{4}T$; $V_1 = V_2 = \frac{3}{2}\frac{T}{\ell}$

Some formulae for prismatic beams with bending stiffness EI.
T, F and q represent the load by a couple, force and uniformly distributed load respectively.
M_i and V_i represent the bending moment and shear force on the end i of the beam, due to the support reactions.

Answer 6.1

Take a quarter of the slab. Sub-dividing this part of the slab into strips could result in the following sketch:



The horizontal half column strip is now considered.

Boundary conditions:

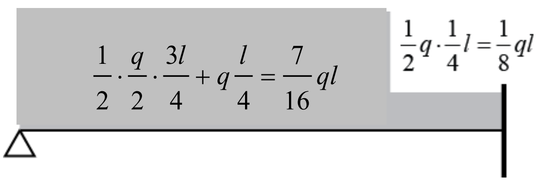
At the left, there is a rigid line support without rotational fixity. The result is a vertical support only.
At the right, symmetry makes that the column strip can’t rotate. Therefore, there’s a fixed end.

The column strip functions both as a part of the slab and as a beam (it carries part of the load from the middle strip and it carries its own load part too).

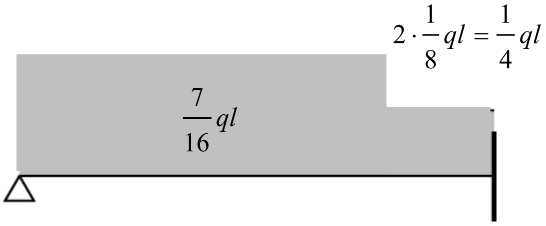
In the left hand side part of the middle strip, 50% of the load is transferred in horizontal direction; 50% in vertical direction. The result is 4 times *q/4* transferred. Two times *q/4* is transferred to the line supports; two times *q/4* is transferred to the two half column strips (*q/4* to the horizontal half column strip; *q/4* to the vertical half column strip;

First consider loads transferred in → direction. There is *q/2* (→) at the right hand side, directly loading the strip (over half column strip width):

Now, loads transferred in ↓ direction are considered. There is *q/2* from the middle strip (width $^{3}/_{4}l$). Half of this load is being transferred to the edge rigid line support and half to the half column strip. This load acts over a $^{3}/_{4}l$ part (length) of the half column strip. Moreover, there is *q* from the half column strip itself (width $^{1}/_{4}l$). This load also acts over a $^{3}/_{4}l$ (length) part of the half column strip. Finally, there’s q/2 from the half column strip itself (width $^{1}/_{4}l$). This load acts over a $^{1}/_{4}l$ (length) part of the half column strip. The result is:

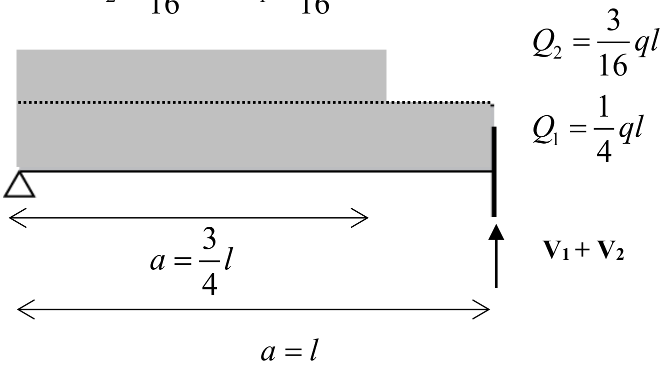


The total load on the half column strip is:



To be able to use this in the equation in figure 6.2, the load is presented as:

$Q_1=\frac{1}{4}ql$ and $Q_2=\frac{7}{16}ql-Q_1=\frac{3}{16}ql$



Use the equation given in figure 6.2:

$$V=Qa\left[\frac{3}{4}\frac{a}{l}-\frac{1}{8}\left(\frac{a}{l}\right)^3\right]$$

The support reaction at the column edge becomes:

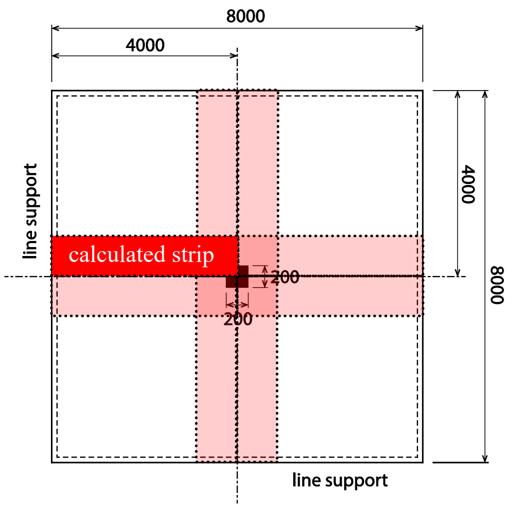
For load $Q_1 \rightarrow\ V_1=Q_1l\left[\frac{3}{4}-\frac{1}{8}\right]=\frac{1}{4}ql\frac{5}{8}l=\frac{5}{32}ql^2$ $\quad(a=l)$

For load $Q_2 \rightarrow\ V_1=Q_2\frac{3}{4}l\left[\frac{3}{4}\cdot\frac{3}{4}-\frac{1}{8}\cdot\left(\frac{3}{4}\right)^3\right]=\frac{3}{16}ql\frac{3}{4}l\frac{261}{512}=0.072ql^2$ $\quad(a=\frac{3}{4}l)$

The total force carried by the column is (l = 4 m) (8 half column strips; 4 in both horizontal and vertical direction, see following figure):

$$V=8\left(V_1+V_2\right)=8\cdot 0.23\cdot ql^2=29.2q \qquad (l=4m)$$

($\frac{29.2q}{64q}\approx 0.46 \rightarrow$ this is around 46% of the total vertical force carried by the structure)



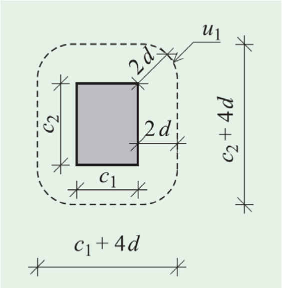
Question 6.2

Assume that the design value of the total force transferred to the column (result from Question 6.1) is V_{Ed} = 150 kN. Check the punching shear capacity of the floor around the column with the provided information.

Answer 6.2

For a rectangular cross-section of a column the size of the basic control perimeter is:

$$u_1 = 2(c_1 + c_2) + 2d \cdot 2\pi$$



where d is the effective slab depth.

$$\left.\begin{array}{l} d_x = 180 - 20 - \dfrac{12}{2} = 154mm \\ d_y = 154 - 12 = 142mm \end{array}\right\} \rightarrow d = \frac{1}{2}\left(d_x + d_y\right) = 148mm$$

The length of the basic control perimeter is:

$$u_1 = 4 \times 200 + 4 \times 148 \times \pi = 2659.82mm$$

According to Eurocode, the load eccentricity factor for inner columns is $\beta = 1.15$, so the design shear stress is:

$$v_{Ed} = \beta \frac{V_{Ed}}{u_1 \times d} = 1.15 \times \frac{150}{2659.82 \times 148} \times 10^3 = 0.44MPa$$

The punching shear resistance stress of the slab is:

$v_{Rd,c} = 0.12 \times k \times (100 \rho f_{ck})^{\frac{1}{3}} + k_1 \sigma_{cp}$ with a minimum of $v_{\mathrm{Rd,c}} = v_{\min} = 0.035 \times k^{3/2} \sqrt{f_{\mathrm{ck}}} + k_1 \sigma_{cp}$

where σ_{cp} is the concrete compressive stress in cross section due to axial loading and/or prestressing. In this case σ_{cp}=0.

According to the provided information:

$$\left.\begin{aligned} \rho_x &= 0.26\% \\ \rho_y &= 0.24\% \end{aligned}\right\} \rightarrow \rho = \sqrt{\rho_x \times \rho_y} = 0.25\%$$

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2,0$$

$$k = 1 + \sqrt{\frac{200}{148}} > 2,0 \rightarrow k = 2,0$$

$$\mathrm{C30/35} \rightarrow f_{\mathrm{ck}} = 30\ \mathrm{MPa}$$

$$v_{Rd,c} = 0.12 \times 2 \times (100 \times 0.25/100 \times 30)^{\frac{1}{3}} = 0.47 MPa$$

$$v_{\min} = 0.035 \times 2^{3/2} \sqrt{30} = 0.54 MPa$$

$v_{Rd,c} = \max\left\{ v_{Rd,c}, v_{\min} \right\} = 0.54 MPa > v_{Ed} = 0.44 MPa \rightarrow$ The column will not punch through the slab so the slab is safe for punching shear failure.

Example 7 – Slab and crack width

The ceiling of a cut-and-cover tunnel segment is considered. The cross section of the tunnel is given in Figure 7.1.The ceiling is a reinforced concrete slab with a thickness of 600 mm. One may assume the clear span of the ceiling slab is 10 m. All the joints between the walls and the slabs are assumed to be hinges.

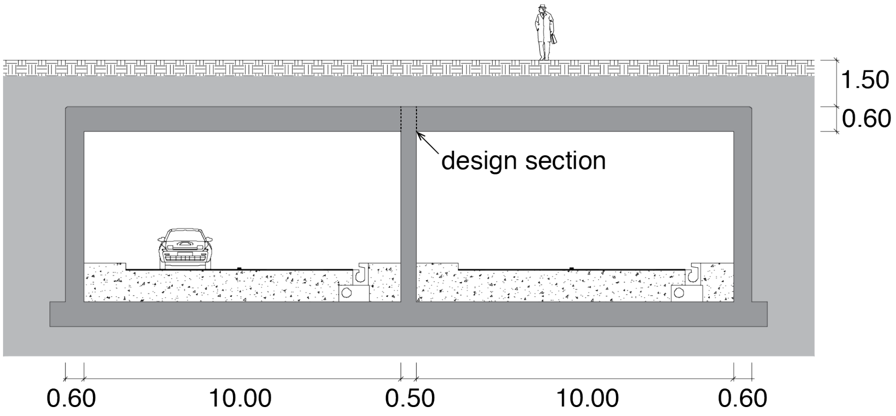


Figure 7.1. Cross section of the cut-and-cover tunnel segment. (unit [m])

Parameters:

Concrete strength class : C40/50
density of concrete : $\gamma_c = 25$ kN/m^3
mean concrete tensile strength : $f_{ctm} = 3.5$ N/mm^2
modulus of elasticity : $E_{cm} = 35000$ N/mm^2
Poison's ratio : $v = 0.2$
Slab thickness : 600 mm
Concrete cover : $c = 30$ mm

Reinforcement class B500B : $f_{yd} = 435$ N/mm^2 $f_{yk} = 500$ N/mm^2
modulus of elasticity : $E_s = 200000$ N/mm^2

Soil density : $\gamma_{soil} = 20$ kN/m^3
Depth of soil cover : 1500 mm
Traffic load on the ground : 5 kN/m^2

Quasi-permanent combination (SLS) : $\psi_2 = 0.5$ for live load
Partial load factors (ULS) : $\gamma_G = 1.35$
$\gamma_Q = 1.50$

Question 7.1

Determine the design hogging moment at the intermediate support of the tunnel ceiling slab at the ULS. Consider the following actions: traffic load, soil weight, self-weight of the structure. The simplified diagram from Figure 7.2 can be used.

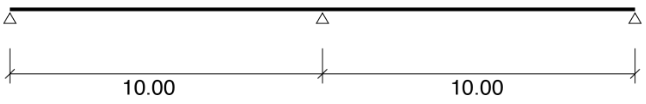
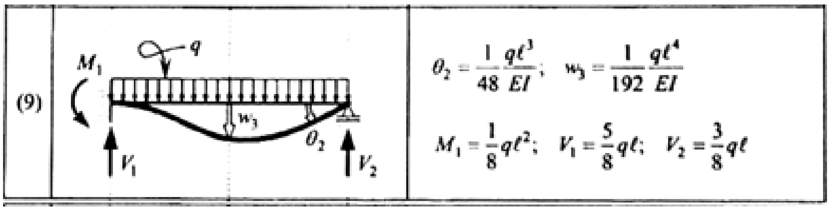


Figure 7.2. Simplified static scheme of the ceiling slab.



$\theta_2 = \frac{1}{48}\frac{q\ell^3}{EI}$; $w_3 = \frac{1}{192}\frac{q\ell^4}{EI}$

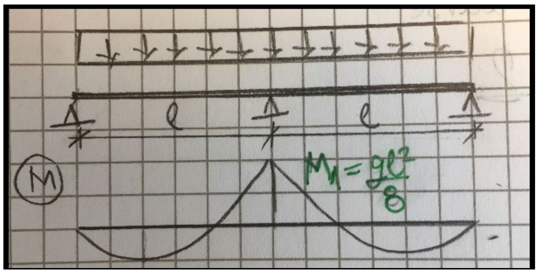
$M_1 = \frac{1}{8}q\ell^2$; $V_1 = \frac{5}{8}q\ell$; $V_2 = \frac{3}{8}q\ell$

Answer 7.1

self-weight:	$h_{\text{slab}} \times \gamma_c = 0.6 \times 25$	$= 15.0$ kN/m^2
soil weight:	$h_{\text{soil}} \times \gamma_s = 1.5 \times 20$	$= 30.0$ kN/m^2
dead weight:	q_{Gk}	$= 45.0$ kN/m^2
variable load:	q_{Qk}	$= 5.0$ kN/m^2

ULS: $q_{Ed} = \gamma_G \times q_{Gk} + \gamma_Q \times q_{Qk} \quad = 1.35 \times 45 + 1.5 \times 5.0 \; = 68.25\,\text{kN/m}^2$

The hogging moment at the intermediate support of the tunnel ceiling slab at ULS can be determined by using the following system:



$$M_{Ed} = \frac{q_{Ed} l^2}{8} = \frac{68.25 \times 10^2}{8} = 853.13\,\text{kNm/m}$$

Question 7.2

Determine the design moment at the SLS. And check if the slab cracks or not at the intermediate support at the SLS.

Hint: You may use f_{ctm} instead of the flexural tensile strength $f_{\text{ctm,fl}}$.

Answer 7.2

SLS: $q = q_{Gk} + \psi_2 \times q_{Qk} \quad = 45 + 0.5 \times 5 \; = 47.5\,\text{kN/m}^2$

$$M = \frac{ql^2}{8} = \frac{47.5 \times 10^2}{8} = 593.75 kNm/m$$

The flexural tensile strength is:

$f_{\text{ctm,fl}} = (1,6 - h/1000) f_{\text{ctm}} = (1,6 - 600/1000) \cdot 3.5 = 3.5\text{MPa}$ (or $f_{ctm,fl} = f_{ctm}$, as given by hint)

The cracking bending moment of the slab is:

$$M_{\mathrm{cr}} = \frac{bh^2}{6} f_{\mathrm{ctm,fl}} = \frac{1000 \cdot 600^2}{6} \cdot 3{,}5 = 210 \cdot 10^6 \text{ Nmm/m} = 210 \text{ kNm/m} < M = 593.75 \text{ kNm/m}$$

At the SLS the slab at the intermediate support is cracked.

Question 7.3

Calculate the maximum crack width (long term loading) of the slab if it cracks. The structural engineer decides to apply Ø25 – 125 mm reinforcement in the top layer of the governing cross-section.

Hint: You may assume that the height of the concrete compressive zone $x = 0{,}25d$.

Crack width control:

$$w_{\max} = \frac{1}{2}\frac{f_{\mathrm{ctm}}}{\tau_{\mathrm{bm}}}\frac{Ø}{\rho_{\mathrm{s,eff}}}\frac{1}{E_{\mathrm{s}}}(\sigma_{\mathrm{s}} - \alpha\,\sigma_{\mathrm{sr}})$$

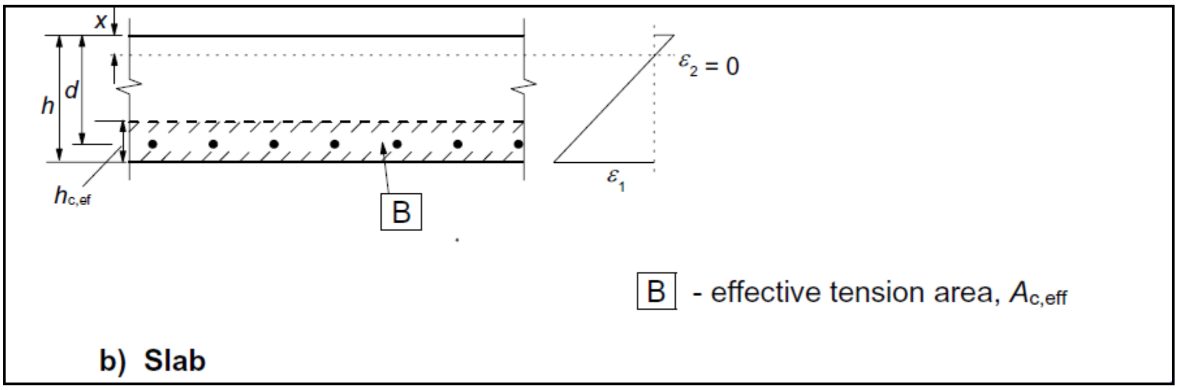
$$\sigma_{\mathrm{sr}} = \frac{M_{crack}}{zA_{\mathrm{s}}}$$

Member loaded in bending:

σ_{sr} follows from the cracking bending moment

Table 15,III Values for τ_{bm}, α and β from eq. (15.15) for various conditions. The values for α between brackets are the recalibrated values as applied in the Eurocode by means of the coefficient k_{t} (EN 1992-1-1 eq. (7.9))

	crack formation stage	stabilized cracking stage
Short term loading	$\alpha = 0{,}5\ (0{,}6)$ $\beta = 0$ $\tau_{\mathrm{bm}} = 2{,}0 f_{\mathrm{ctm}}$	$\alpha = 0{,}5(0{,}6)$ $\beta = 0$ $\tau_{\mathrm{bm}} = 2{,}0 f_{\mathrm{ctm}}$
long term or dynamic loading	$\alpha = 0{,}5\ (0{,}6)$ $\beta = 0$ $\tau_{\mathrm{bm}} = 1{,}6 f_{\mathrm{ctm}}$	$\alpha = 0{,}3\ (0{,}4)$ $\beta = 1$ $\tau_{\mathrm{bm}} = 2{,}0 f_{\mathrm{ctm}}$



b) Slab

$$h_{\mathrm{c,eff}} = \min\begin{Bmatrix} 2{,}5\,(h-d) \\ (h-x)/3 \end{Bmatrix}$$ where d is the effective depth and x is the height of the compressive zone

Answer 7.3

Maximum crack width is calculated as follows (tensile member model):

$$w_{\max} = \frac{1}{2}\frac{f_{\mathrm{ctm}}}{\tau_{\mathrm{bm}}}\frac{Ø}{\rho_{s,eff}}\frac{1}{E_{\mathrm{s}}}(\sigma_{\mathrm{s}} - \alpha\sigma_{\mathrm{sr}} + \beta\varepsilon_{sc}\,\mathrm{E}_s)$$

where σ_{sr} is the steel stress directly after cracking, σ_s is the steel stress at SLS and α and β follow from the table.

where $\rho_{\text{s,eff}} = \dfrac{A_\text{s}}{bh_{\text{c,eff}}}$

and $h_{\text{c,eff}}$ is the depth of effective tensile area around the tensile reinforcement and can be calculated as follows (EN 1992-1-1, Section 7.3.2):

$$h_{\text{c,eff}} = \min\begin{Bmatrix} 2,5\,(h-d) \\ (h-x)/3 \end{Bmatrix}$$

where d is the effective depth and x is the height of the compressive zone

$$d = 600 - 30 - 25/2 = 557.5 \text{ mm}$$

$x = 0.25d = 0.25 \times 557.5 = 139.4$ mm (see the hint)

$$h_{\text{c,eff}} = \min\begin{Bmatrix} 2.5\,(h-d) \\ (h-x)/3 \end{Bmatrix} = \min\begin{Bmatrix} 2.5\cdot(600-575.5) \\ (600-139.3)/3 \end{Bmatrix} = \min\begin{Bmatrix} 106.2 \\ 153.5 \end{Bmatrix} = 106.2 \text{ mm}$$

$$A_s = \frac{1}{4}Ø^2\pi \cdot \frac{1000}{s} = \frac{25^2\pi}{4}\frac{1000}{150} = 3927 mm^2 / m$$

$$\rho_{\text{s,eff}} = \frac{A_\text{s}}{bh_{\text{c,eff}}} = \frac{3927}{1000\cdot 106.2} = 3.7\%$$

σ_{sr} is the steel stress directly after cracking:

$$\sigma_{\text{sr}} = \frac{M_{cr}}{A_\text{s}\,z} \text{ where } z = d - \frac{x}{3} = 557.5 - \frac{139.4}{3} = 511 mm$$

$$\sigma_{\text{sr}} = \frac{210\times10^6}{511\times3927} = 105 MPa$$

Additional information

In case the hint was not given, the height of the compression zone could be calculated as follows:

$$\frac{x}{d} = -\alpha_e \rho + \sqrt{(\alpha_e \rho)^2 + 2\alpha_e \rho}$$

where

$$\alpha_e = \frac{E_s}{E_c} = \frac{200000}{35000} = 5.71$$

$$\rho = \frac{A_s}{bd} = \frac{\frac{1}{4}Ø^2 \pi \cdot 1000 / s}{bd} = \frac{\frac{1}{4} \cdot 25^2 \cdot \pi \cdot 1000 / 125}{1000 \cdot 557.5} = 0{,}7\%$$

$$\frac{x}{d} = -5.71 \cdot 0.007 + \sqrt{(5.71 \cdot 0.007)^2 + 2 \cdot 5.71 \cdot 0.007} = 0.25$$

The height of the compression zone is: $x = 0{,}25 \cdot 575.5 = 139.4$ mm

Steel stress in a crack at SLS (cracked cross-section loaded in pure bending):

$$\sigma_s = \frac{M}{A_s \times z} \text{ where } z = d - \frac{x}{3} = 557.5 - \frac{139.4}{3} = 511 mm$$

$$\sigma_s = \frac{593.75 \times 10^6}{511 \times 3927} = 296 MPa$$

According ot table 15,III $\rightarrow \alpha = 0.3$ and asuming $\varepsilon_{sc} = 0$ and $\tau_{bm} = 2 f_{ctm}$, the maximum crack width in the stabilized crack stage can be calculated as follows:

$$w_{\max} = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{25}{0.037} \cdot \frac{1}{200000}(296 - 0.3 \cdot 105) = 0{,}22 \, \text{mm}$$

Question 7.4

Since the tunnel is covered by soil, the pressure of soil and water at the side surfaces of the tunnel has to be taken into account. An illustration of the pressures acting on the side surfaces of the tunnel is given in figure 7.3. Taking the information from this picture into account, check whether the ceiling slab cracks or not.

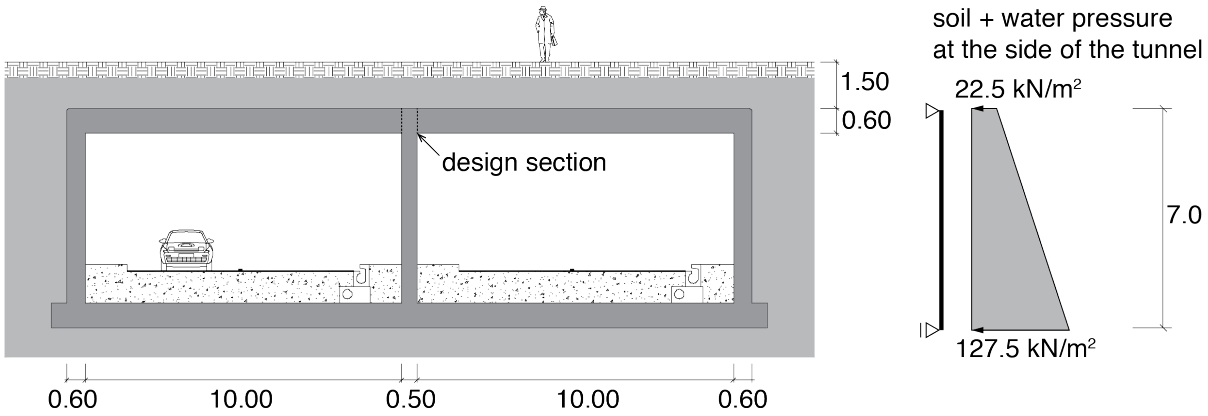
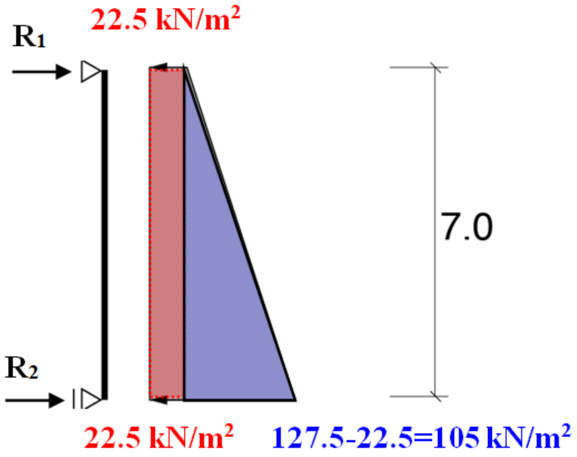


Figure 7.3. Indication of the soil and water pressure acting at the side surfaces of the tunnel segment.

Answer 7.4



Reaction force (in the ceiling slab) due to pressure of soil and water can be calculated as follows:

$$R_1 = \frac{22.5 \times 7}{2} + \frac{105 \times 7}{2} \times \frac{1}{3} = 201.5 \text{ kN/m}$$

In-plane compressive stress in the ceiling slab caused by the pressure of soil and water:

$$\sigma_p = \frac{R}{h \times b} = \frac{201.5}{1000 \times 600} \times 1000 = 0.33 MPa$$

This stress acts as prestressing in the slab (increasing the cracking bending moment of the slab):

$$M_{\text{cr}} = \frac{bh^2}{6}(f_{\text{ctm,fl}} + \sigma_p) = \frac{1000 \cdot 600^2}{6} \cdot (3{,}5 + 0.33) = 229.8\text{kNm/m} < M = 593.75 \text{ kNm/m}$$

Still, the slab will be cracked at the SLS load. Note that for the cracking moment, the flexural tensile strength is used.

Example 8 – Slab and punching shear resistance

Consider the floor plan of a two-way flat slab indicated in Figure 8.1.

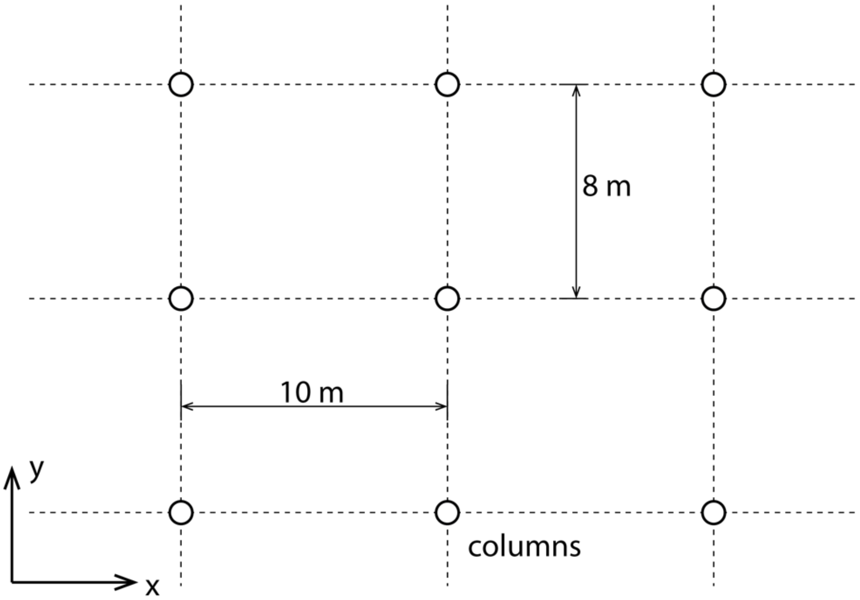


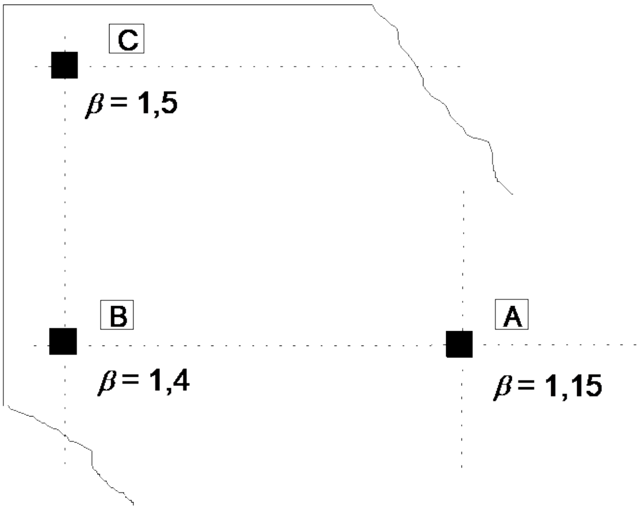
Figure 8.1 Top view of a flat slab floor plan

Parameters:

Concrete strength class	: C30/35
Floor thickness	: h = 180 mm
Concrete cover	: c = 20 mm
Rebar configurations	: ρ_{lx} = 0.26%, ρ_{ly} = 0.24%, with Ø12 rebars in both directions
Design value of the punching shear force	: V_{Ed} = 250 kN
Column diameter	: 200 mm

Additional information

Approximated safe values for β according to the Eurocode:



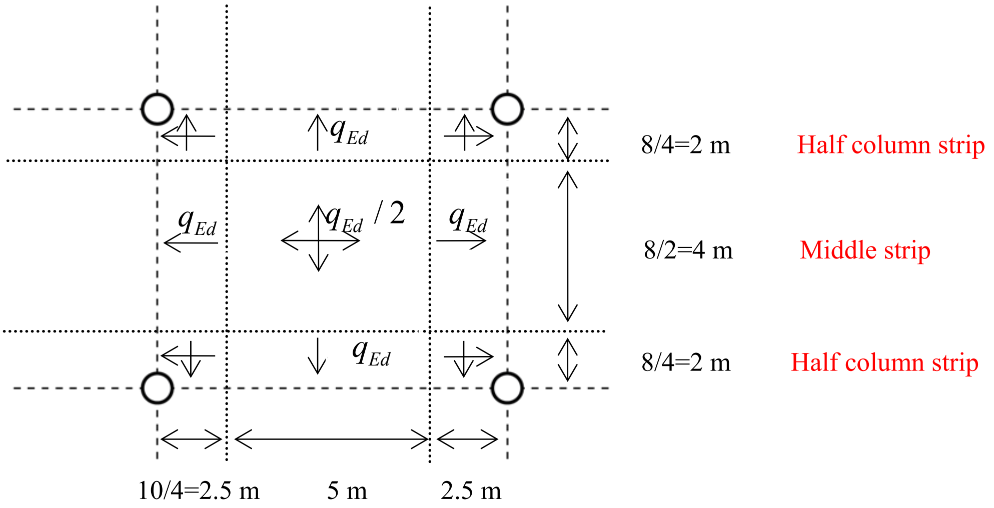
Question 8.1

Strip method
Assume that the dead load q_G and the live load q_Q are applied over the full floor plan area (all the slab spans). Consider the loading condition for the bottom reinforcement. Draw the boundary conditions and load distributions of the **Middle – Strip** in the *x* direction.

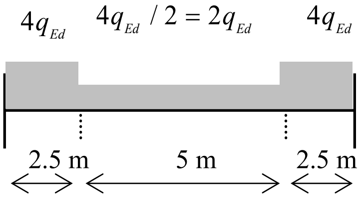
Hint: you **DON'T** need to calculate the moment distribution.

Answer 8.1

Dead load and live load are applied to the whole floor plan.



The boundary conditions of the middle strip are shown in the following figure. Loading symmetry makes that the single slab span can be assumed to have no rotational flexibility at both its ends. The load transferred by the middle strip in x-direction is *q/2* over 5 m and *q* at both its ends (2,5 m each). The total line load on the middle strip is the load q multiplied with the middle strip width (4 m).



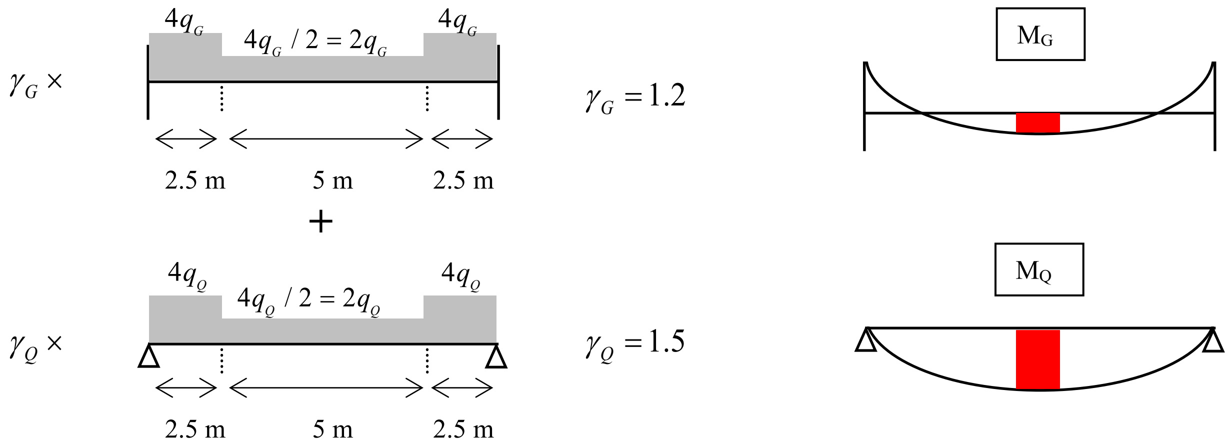
where

$$q_{Ed}=\gamma_G\times q_G+\gamma_Q\times q_Q=1.2\times q_G+1.5\times q_Q$$

Question 8.2

Assume that the dead load q_G is applied to the whole floor plan (all the slab spans), but the live load q_Q is only applied on a single slab span. Considering the loading condition for the bottom reinforcement, draw the boundary conditions and load distributions of the **Middle – Strip** in the *x* direction. Compare the results of 8.1 and 8.2, and give your opinion on which one is more critical for the design of the bottom reinforcement.

Answer 8.2



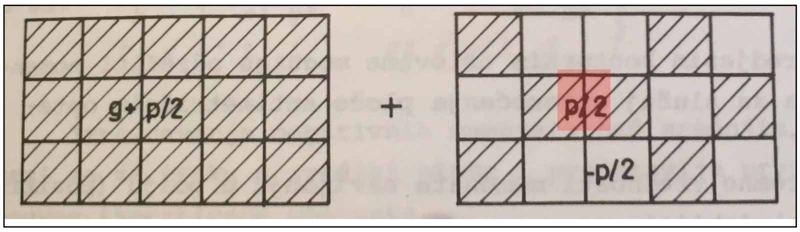
The G load transfer and scheme have already been presented in Answer 8.1. The Q load now is present over one span only. It is now almost as if the slab can freely rotate at a support. Therefore, the strip scheme is now as if it is a simply supported element.

For the design of the bottom reinforcement, more critical is the result from 8.2b because it causes a higher sagging moment (smaller negative support bending moments). However, for the design of the top reinforcement, more critical is the result from 8.2a as it causes higher hogging moments.

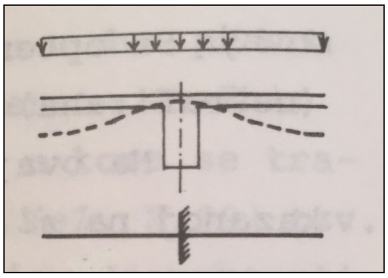
Additional information

Why the different boundary conditions are valid for the two situations (permanent, G and variable, Q load) in answer 8.2?

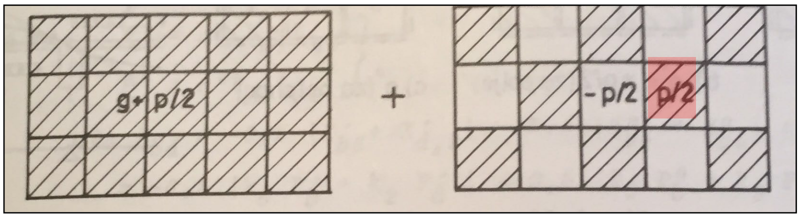
Maximum negative bending moment:
In order to get the largest hogging moment (zone marked red), the load distribution can be presented as follows. In this image, the variable load is marked as the “p” load.



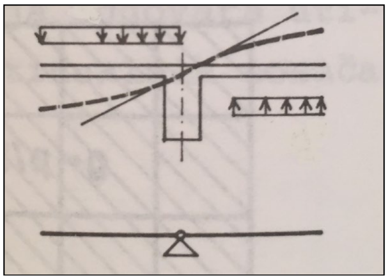
The following boundary conditions are then valid for both the permanent and the variable load and they should be used for determining the largest hogging moment:



Maximum positive bending moment:
In order to get the largest sagging moment (zone marked red), the load distribution can be divided as follows:



The following boundary conditions are now valid for the right hand side P/2 variable load parts, which should be used for determining the largest sagging moment (and design of the bottom reinforcement):



Question 8.3

The punching shear capacity of a floor is considered. Assume that the design force in columns is V_{Ed} = 250 kN. The engineer found that the punching shear capacity of the floor is not sufficient. Therefore it was suggested to apply a circular drop panel under the floor. The thickness of the drop panel ensures that the punching shear capacity of the drop panel itself is **sufficient**. The dimension of the panel l_H indicated in Figure 2.2 has to be determined. Please calculate the value of l_H for the engineer.

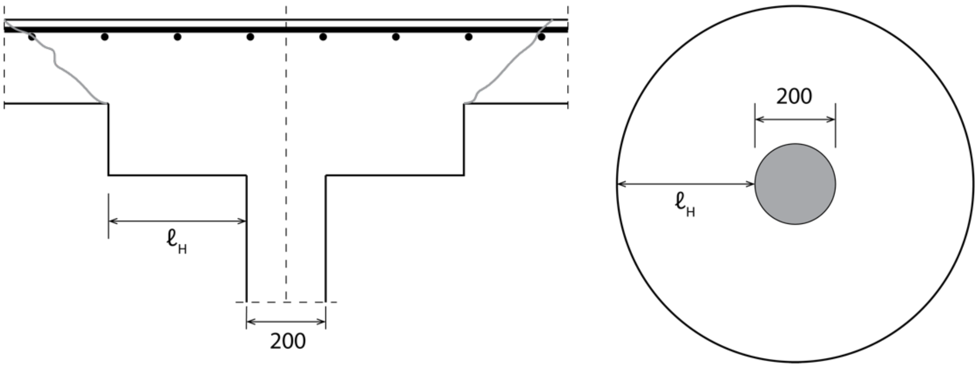
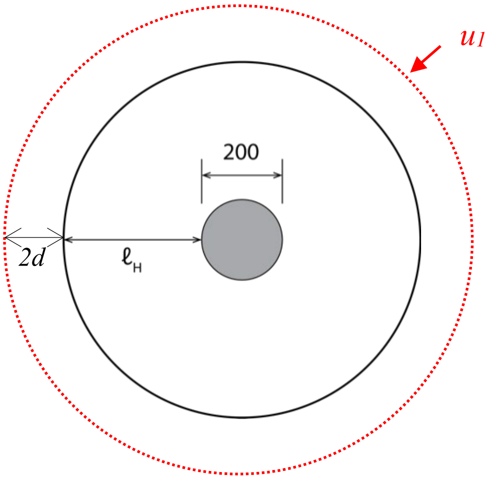


Figure 8.2. Side and bottom view of the drop panel.

Answer 8.3

For a circular cross-section of a column the size of the basic control perimeter is:

$$u_1 = (2(2d + l_H) + 200)\pi$$



where d is the effective slab depth.

$$\left.\begin{array}{l} d_x = 180 - 20 - \dfrac{12}{2} = 154mm \\ d_y = 154 - 12 = 142mm \end{array}\right\} \rightarrow d = \frac{1}{2}\left(d_x + d_y\right) = 148mm$$

The length of the basic control perimeter is:

$$u_1 = (4\times148 + 2l_H + 200)\pi = (792 + 2l_H)\pi$$

According to the Eurocode, the eccentricity factor for inner columns is $\beta = 1.15$, so the design shear stress is:

$$v_{Ed} = \beta\frac{V_{Ed}}{u_1 \times d} = 1.15\times\frac{250}{(792+2l_H)\pi\times148}\times10^3$$

The punching shear resistance stress of the column is:

$v_{Rd,c} = 0.12\times k\times(100\rho f_{ck})^{\frac{1}{3}} + k_1\sigma_{cp}$ with a minimum of $v_{\mathrm{Rd,c}} = v_{\min} = 0.035\times k^{3/2}\sqrt{f_{\mathrm{ck}}} + k_1\sigma_{cp}$

where σ_{cp} is the concrete compressive stress in a cross section from axial loading and/or prestressing. In this case σ_{cp}=0.

According to the provided information:

$$\left.\begin{aligned}\rho_x &= 0.26\%\\ \rho_y &= 0.24\%\end{aligned}\right\} \rightarrow \rho = \sqrt{\rho_x\times\rho_y} = 0.25\%$$

$$k = 1+\sqrt{\frac{200}{d}} \le 2,0$$

$$k = 1+\sqrt{\frac{200}{148}} > 2,0 \rightarrow k = 2,0$$

$$\mathrm{C30/35} \rightarrow f_{\mathrm{ck}} = 30\ \mathrm{MPa}$$

$$v_{Rd,c} = 0.12\times2,0\times(100\times0.25/100\times30)^{\frac{1}{3}} = 0.47MPa$$

$$v_{\min} = 0.035\times2^{3/2}\sqrt{30} = 0.54MPa$$

$$\rightarrow v_{Rd,c} = \max\left\{v_{Rd,c}, v_{\min}\right\} = 0.54MPa > v_{Ed} = 1.15\times\frac{250}{(792+2l_H)\pi\times148}\times10^3$$

$$l_H > \frac{\frac{1.15\times250\times10^3}{0.54\times148\times\pi} - 792}{2} = 177\ \mathrm{mm}$$

Example 9 – Reservoir and crack width

A water tank has a foundation slab and a circular reinforced concrete tank wall, see Figure 9.1. The wall thickness is 200 mm. The inner diameter of the wall is 20 meters; the wall height is 12 meters. There is a sliding connection - without friction - between the tank wall and the foundation slab.

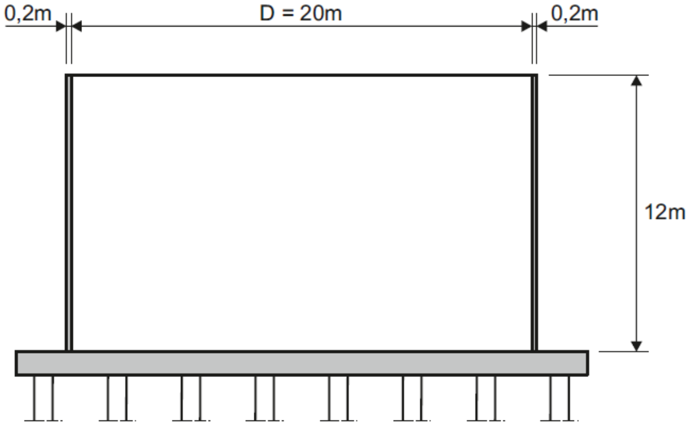


Figure 9.1 Cross-section of the water tank

Concrete strength class : C25/30
Density concrete : $\rho = 25$ kN/m^3
Reinforcement class B500B : $f_{yd} = 435$ N/mm^2
Modulus of elasticity : $E_s = 200000$ N/mm^2
Concrete wall
  thickness : 200 mm
  concrete cover : 25 mm
  allowable crack width : 0,20 mm
  : $h_{c.eff} = 2,5\ (h - d)$
  mean concrete tensile strength: $f_{ctm} = 2,6$ N/mm^2
  bond strength : $\tau_{bm} = 2\,f_{ctm}$
Concrete modulus of elasticity : $E_{cm} = 31000$ N/mm^2
Partial load factors : $\gamma_G = 1,2$
: $\gamma_Q = 1,5$

Question 9.1

Calculate the maximum design force (N_{Ed}) in the tank wall given that the tank is completely filled with water.

Answer 9.1

Hoop force (ring force) from the hydrostatic water pressure: $N = qR$.
Maximum design hoop force is at the bottom: $N_{Ed,max} = \gamma_Q\ h\ \rho_{water}\ R$.

$$N_{Ed,max} = 1,5 \cdot 12 \cdot 10 \cdot 10 = 1800 \text{ kN/m}$$

Question 9.2

Calculate the required amount of reinforcement in the tank wall in mm^2 per 1 meter of height, assuming that the maximum tensile force is constant over this 1 meter.

Answer 9.2

It is an ULS check. Therefore, use f_{yd}.

$$A_s = \frac{N_{Ed,max}}{f_{yd}} = \frac{1800 \cdot 10^3}{435} = 4138 \text{ mm}^2/\text{m}$$

Question 9.3

Design the required reinforcement by choosing a bar diameter and a number of bars per meter.

Answer 9.3

Apply bars Ø16: $n_s = \frac{A_s}{\frac{1}{4}\pi \cdot 16^2} = \frac{4138}{201} = 21 / \text{m}$

At each of both sides (inside & outside): 21/2 = 10,5 bars/m = Ø16 spaced 1000/10,5 = Ø16-95.

Question 9.4

Calculate crack width w_{max} and check whether the calculated crack width is smaller than 0,20 mm.
Hint: calculate $\rho_{s,eff}$.

Crack width control:

$$w_{max} = \frac{1}{2}\frac{f_{ctm}}{\tau_{bm}}\frac{Ø}{\rho_{s,eff}}\frac{1}{E_s}(\sigma_s - \alpha\, \sigma_{sr})$$

$$\sigma_{sr} = \frac{N_{crack}}{A_s}$$

Table 15,III Values for τ_{bm}, α and β from eq. (15.15) for various conditions. The values for α between brackets are the recalibrated values as applied in the Eurocode by means of the coefficient k_t (EN 1992-1-1 eq. (7.9))

	crack formation stage	stabilized cracking stage
Short term loading	α = 0,5 (0,6) β = 0 $\tau_{bm} = 2{,}0\, f_{ctm}$	α = 0,5(0,6) β = 0 $\tau_{bm} = 2{,}0\, f_{ctm}$
long term or dynamic loading	α = 0,5 (0,6) β = 0 $\tau_{bm} = 1{,}6\, f_{ctm}$	α = 0,3 (0,4) β = 1 $\tau_{bm} = 2{,}0\, f_{ctm}$

Answer 9.4

Concrete cover = 25 mm
$h - d$ = 25 + Ø/2 = 25 + 16/2 = 33 mm
$h_{c.eff} = 2{,}5\,(h - d)$ = 83 mm

Bars: Ø16-95 → $\rho_{\mathrm{s,eff}} = \dfrac{\frac{1}{4}\pi \cdot 16^2}{95 \cdot 83} = 0{,}0255$

Hoop force (ring force) from the hydrostatic water pressure: $N = qR$.
SLS check: Maximum hoop force is at the bottom: $N_{\mathrm{E,max}} = h\ \rho_{\mathrm{water}}\ R$.

$N_{\mathrm{E,max}} = 12 \cdot 10 \cdot 10 = 1200$ kN/m

Steel stress in SLS: $\sigma_{\mathrm{s,max}} = \dfrac{N_{\mathrm{E,max}}}{A_{\mathrm{s}}} = \dfrac{1200 \cdot 10^3}{\frac{1}{4}\pi \cdot 16^2 \cdot 21} = 284\ \mathrm{N/mm^2}$

Note: This result is as expected. It's SLS, so no load factor applied. ULS steel stress 435 N/mm^2 now is 435/1,5 = 290 N/mm^2 ≈ 284 N/mm^2 ($A_{\mathrm{s,applied}} \approx A_{\mathrm{s,required\ in\ ULS}}$).

Steel stress directly after cracking:

$$\rho = \frac{\frac{1}{4}\pi \cdot 16^2}{95 \cdot (200/2)} = 0{,}0212$$

$$\sigma_{\mathrm{sr}} = f_{\mathrm{ctm}}\left(1 + \frac{E_{\mathrm{s}}}{E_{\mathrm{c}}}\rho\right)\frac{1}{\rho} = 2{,}6 \cdot \left(1 + \frac{200}{31} \cdot 0{,}0212\right) \cdot \frac{1}{0{,}0212} = 139\ \mathrm{N/mm^2}$$

Maximum crack width:

Assume long term loading:

$$w_{\max} = \frac{1}{2}\frac{f_{\mathrm{ctm}}}{\tau_{\mathrm{bm}}}\frac{Ø}{\rho_{\mathrm{s,eff}}}\frac{1}{E_{\mathrm{s}}}\left(\sigma_{\mathrm{s}} - \alpha\,\sigma_{\mathrm{sr}}\right) = \frac{1}{2} \cdot \frac{2{,}6}{2 \cdot 2{,}6} \cdot \frac{16}{0{,}0255} \cdot \frac{1}{200 \cdot 10^3} \cdot \left(284 - 0{,}3 \cdot 139\right) = 0{,}19\ \mathrm{mm}$$

Smaller than 0,2 mm; **OK**.

Question 9.5

Is the calculated crack width related to the phase "fully developed crack pattern" or not?

Answer 9.5

$N_{\mathrm{E,max}} = 12 \cdot 10 \cdot 10 = 1200$ kN/m

Concrete tensile stress assuming that the concrete is uncracked in SLS:

$$\sigma_{\text{c,max}} = \frac{N_{\text{E,max}}}{A_c} = \frac{1200\cdot10^3}{1000\cdot200} = 6{,}0\ \text{N/mm}^2 >> \sigma_{\text{cr}}$$

Fully developed crack pattern (or: stabilized cracking stage) since $N_{\text{E}} > N_{\text{crack}}$.

Question 9.6

It is now assumed that the tank is filled with water. At which water level (height in meters) will the first crack in the wall occur?

Answer 9.6

SLS check:

Maximum hoop force is at the bottom: $N_{\text{E,max}} = h_{\text{water}}\ \rho_{\text{water}}\ R$.

$N_{\text{E,max}} = h_{\text{water}}\ 10\cdot10 = 100\ h_{\text{water}}$ [units kN & m]

$$\sigma_{\text{c}} = \sigma_{\text{cr}} \rightarrow \frac{N_{\text{E,max}}}{a_c} = \frac{100\cdot10^{-6}\cdot10^3 h_{\text{water}}}{1\cdot200} = 2{,}6\ \text{N/mm}^2 \quad \text{[units N \& mm]}$$

$$\rightarrow h_{water} = 5200\ \text{mm}$$

This result also follows directly from: (2,6 N/mm^2 / 6,0 N/mm^2)·12 m = 5,2 m.

When also taking into account the impact of the reinforcement on the cracking force:

$$N_{\text{crack}} = A_{\text{c}} f_{\text{ctm}}\left(1+\frac{E_{\text{s}}}{E_{\text{c}}}\rho\right) = 200\cdot1\cdot2{,}6\cdot\left(1+\frac{200}{31}\cdot\frac{2\cdot\frac{1}{4}\pi\cdot16^2}{200\cdot95}\right) =$$

$$= 200\cdot1\cdot2{,}6\cdot(1+0{,}137) = 591\ \text{N/mm}$$

bottom: $N_{\text{E,max}} = h_{\text{water}}\ \rho_{\text{water}}\ R$.

$h_{\text{water}} = 591/(10\cdot10^{-6}\cdot10\cdot10^3) = 5910$ mm [units N & mm]

Example 10 – Prestressed concrete

A post-tensioned beam has a 35 m span, see figure 10.1. The beam has a **rectangular** cross-section; width b = 0,75 m and height h = 1,8 m. The beam is prestressed using curved tendons (strands in ducts). The fictitious tendon profile consists of two parabolas, namely parabola 1 with a radius of curvature R_1 = 62 m over ℓ_1 = 10 m and parabola 2 with R_2 = 330 m over ℓ_2 = 25 m

The bottom of both parabolas is at position A (they joint at this position) in Figure 3.1, which is at ℓ_1 = 10 m from the left support. The bottom of the parabolas is at 0,2 m from the bottom fiber of the beam.

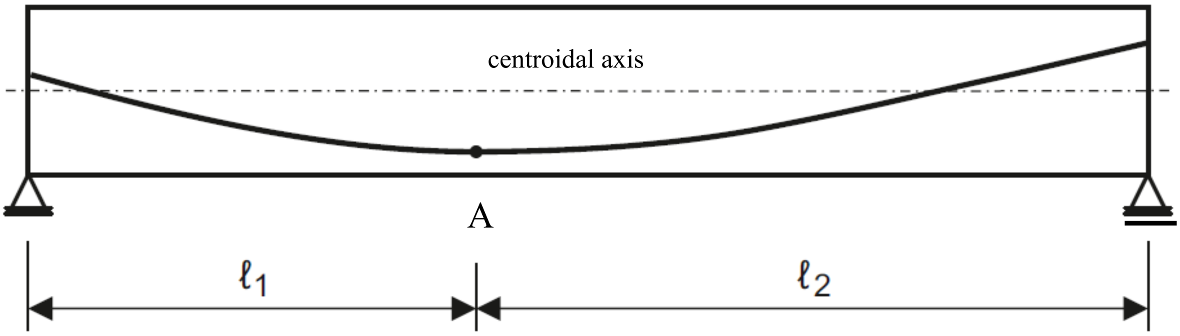


Figure 10.1 Side view of the girder including fictitious tendon profile (not to scale).

Parameters:

Density concrete	: ρ = 25 kN/m^3
Variable load	: q_{Qk} = 10 kN/m
Strength class of concrete	: C50/60
Strength class of prestressing steel	: Y1860S7
Elastic modulus prestressing steel	: E_p = 195000 N/mm^2

Question 10.1

Calculate the tendon eccentricities relative to the centroidal axis at the two anchors.

Answer 10.1

Parabola: $y = \dfrac{x^2}{2R}$

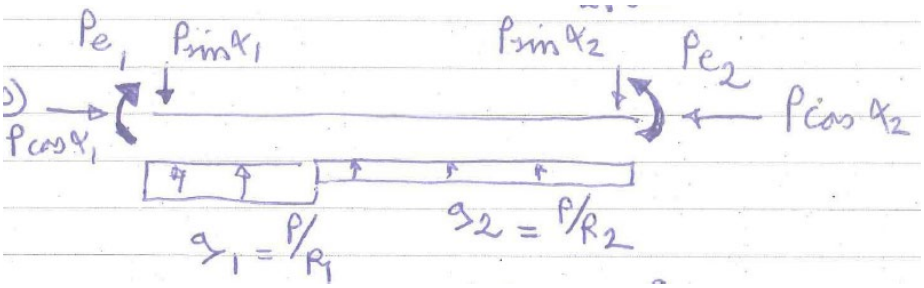
ℓ_1 = 10 m: $y_1 = \dfrac{10^2}{2 \cdot 62} = 0,806 \text{ m}$ => 806 + 200 - 900 = 106 mm

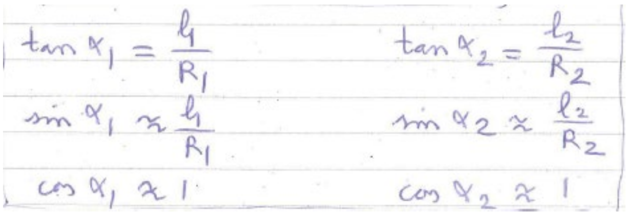
ℓ_2 = 25 m: $y_2 = \dfrac{25^2}{2 \cdot 330} = 0,947 \text{ m}$ =>947 + 200 - 900 = 247 mm

Question 10.2

Show in a figure all the loads on the beam from a prestressing force P, assuming that there is no friction and no wedge set.

Answer 10.2





$\tan\alpha_1 = \frac{l_1}{R_1}$ $\tan\alpha_2 = \frac{l_2}{R_2}$

$\sin\alpha_1 \approx \frac{l_1}{R_1}$ $\sin\alpha_2 \approx \frac{l_2}{R_2}$

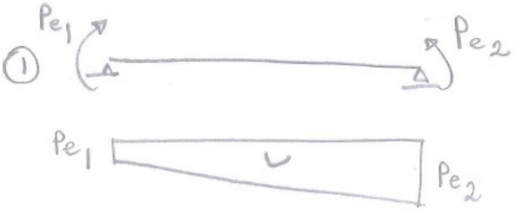
$\cos\alpha_1 \approx 1$ $\cos\alpha_2 \approx 1$

Question 10.3

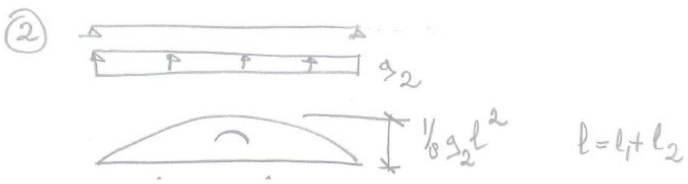
Calculate and draw the bending moment diagram resulting from a prestressing force *P*. Assume that there is no friction and no wedge set.

Answer 10.3

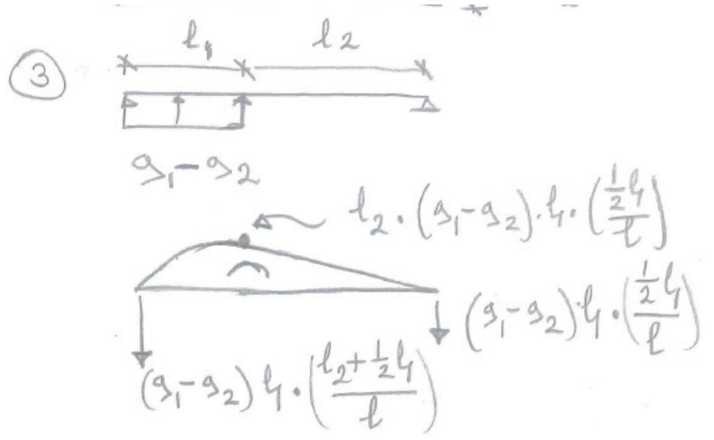
1: From anchor eccentricities:



2: Uniformly distributed load



3: $q_1 > q_2$; contribution left after applying q_2:



Total = 1 + 2 + 3:

Midspan (units: mm):

$$M = P\left(\frac{e_1+e_2}{2}\right)-\frac{1}{8}q_2 l^2-(q_1-q_2)l_1\left(\frac{\frac{1}{2}l_1}{l}\right)\frac{1}{2}l$$

$$M = P\left(\frac{106+247}{2}\right)-\frac{1}{8}\frac{P}{330\cdot10^3}\cdot\left(35\cdot10^3\right)^2-\left(\frac{P}{62\cdot10^3}-\frac{P}{330\cdot10^3}\right)\cdot10\cdot10^3\cdot\left(\frac{\frac{1}{2}\cdot10\cdot10^3}{35\cdot10^3}\right)\frac{1}{2}\cdot35\cdot10^3$$

$$M = P(176{,}5-464{,}0-327{,}5)=-615P$$

Additional info:

Calculate M at l_1:

$$M = P\left(e_1-\frac{l_1}{l}(e_1-e_2)\right)-\frac{1}{2}q_2 l_1(l-l_1)-l_2(q_1-q_2)l_1\left(\frac{\frac{1}{2}l_1}{l}\right)$$

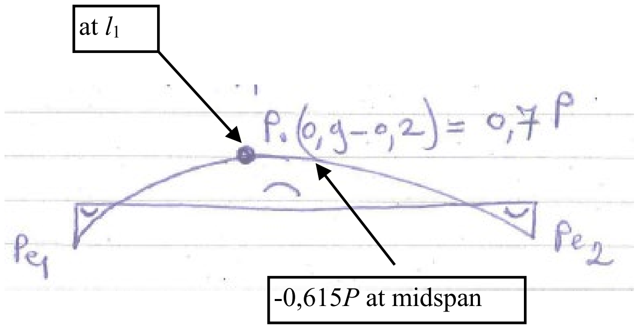
$$M = P\left(106-\frac{10}{35}\cdot(106-247)\right)$$

$$-\frac{1}{2}\frac{P}{330\cdot10^3}\cdot10\cdot10^3\left(35\cdot10^3-10\cdot10^3\right)$$

$$-25\cdot10^3\cdot\left(\frac{P}{62\cdot10^3}-\frac{P}{330\cdot10^3}\right)\cdot10\cdot10^3\cdot\left(\frac{\frac{1}{2}\cdot10\cdot10^3}{35\cdot10^3}\right)$$

$$M = P(146{,}3-378{,}8-467{,}8)=-700P$$

unit [mm]



at l_1

-0,615P at midspan

unit [m]

Check:

$$\text{Midspan tendon eccentricity} = \;= 0,7 - \frac{(17,5-10)^2}{2\cdot 330} = 0,7 - 0,085 = 0,616 \text{ m}$$

Effective height: d_p = 1800/2 + 615 = 1515 mm

Question 10.4

Calculate the **maximum** initial prestressing force P_{m0} [kN] **allowed** to have no tension in the **midspan** cross-section when stressing the tendons.

Answer 10.4

$G = 0{,}75 \cdot 1{,}8 \cdot 25 = 33{,}75$ kN/m
$M_G = Gl^2/8 = 5168$ kNm

Check top fiber level.

Units [kN, m]:

$$-\frac{P}{0,75\cdot 1,8}+\frac{0,615P}{\frac{1}{6}\cdot 0,75\cdot 1,8^2}-\frac{5168}{\frac{1}{6}\cdot 0,75\cdot 1,8^2}\le 0$$
$$P \le 16406 \text{ kN}$$

Question 10.5

Calculate the **minimum** working prestressing force $P_{m\infty}$ [kN] **required** to have no tension in the **midspan** cross-section, at maximum SLS load.

Answer 10.5

$G + Q = (33{,}75 + 10)$ kN/m
$M_{G+Q} = (G+Q)l^2/8 = 6700$ kNm

Check bottom fiber level.

Units [kN, m]:

$$-\frac{P}{0,75\cdot 1,8}-\frac{0,615P}{\frac{1}{6}\cdot 0,75\cdot 1,8^2}+\frac{6700}{\frac{1}{6}\cdot 0,75\cdot 1,8^2}\le 0$$
$$P \ge 7322 \text{ kN}$$

Question 10.6

The tendons are stressed from the left hand side support. Sketch the profile of the prestressing force $P_{m,x}/P_{m,x=0}$ at $t = 0$ over the full length of the beam. Mark important points/values. Assume that the friction coefficient $\mu = 0{,}20$ and the wobble factor $k = 0{,}01$ rad/m.

Prestressing force including frictional loss:

$$P_m(x) = P_m(x=0)\cdot e^{-\mu(\theta+kx)}$$

friction coefficient $\mu = 0{,}2$
wobble factor $k = 0{,}01$ rad/m

Answer 10.6

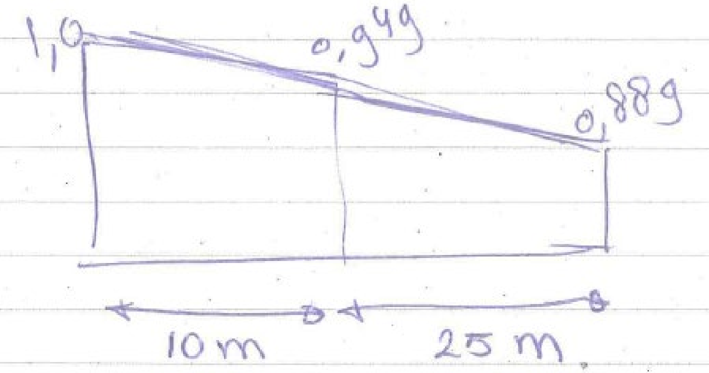
Use $\Delta\theta = \left|\frac{\Delta x}{R}\right|$

$x = l_1 = 10$ m:

$$P_m(x=10) = P_m(x=0)\cdot e^{-\mu(\theta+kx)} = P_m(x=0)\cdot e^{-0{,}2\cdot\left(\frac{10}{62}+10\cdot 0{,}01\right)} = 0{,}949 P_m(x=0)$$

$x = l = 35$ m:

$$P_m(x=35) = P_m(x=0)\cdot e^{-\mu(\theta+kx)} = P_m(x=0)\cdot e^{-0{,}2\cdot\left(\frac{10}{62}+\frac{25}{330}+35\cdot 0{,}01\right)} = 0{,}889 P_m(x=0)$$



Question 10.7

The tendons are assumed to be stressed to a stress $\sigma_{p,max} = 1395$ N/mm^2. Wedge set is $w_{set} = 3$ mm. Calculate the wedge set influence length and sketch the profile of the prestressing steel stress immediately after anchoring. Use the results from question 10.6**f**.

Influence length of wedge set:

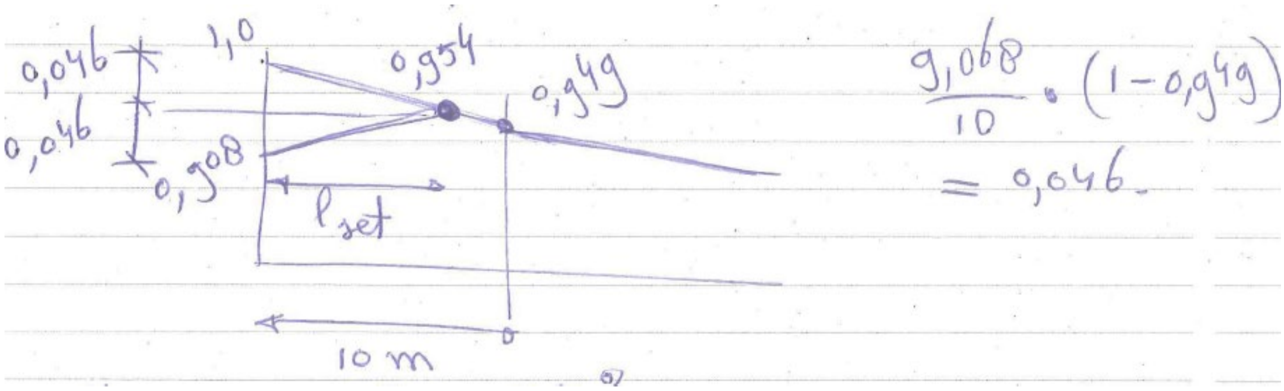
$$l_{set} = \sqrt{\frac{w_{set} E_p}{\Delta\sigma_{p,\mu}/\Delta x}}$$

Answer 10.7

Units: N & mm:

$$\Delta\sigma_{\mathrm{p,\mu}} \big/ \Delta x = \frac{(1-0,949)\cdot 1395}{10\cdot 10^3} = 7,11\cdot 10^{-3}\ \mathrm{N/mm^3}$$

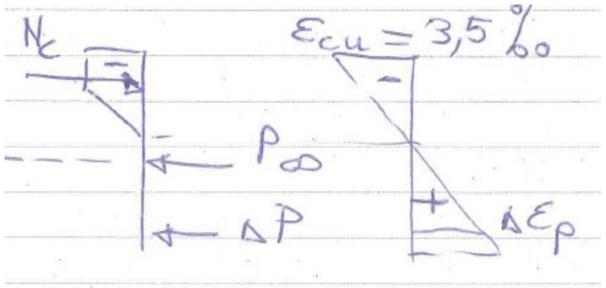
$$l_{\mathrm{set}} = \sqrt{\frac{w_{\mathrm{set}}E_{\mathrm{p}}}{\Delta\sigma_{\mathrm{p,\mu}} \big/ \Delta x}} = \sqrt{\frac{3\cdot 195000}{7,11\cdot 10^{-3}}} = 9068\ \mathrm{mm}$$



Question 10.8

The **midspan** cross-section of the beam is now analyzed in **ULS** at $t = \infty$. A structural engineer decides to apply A_{p} = 4000 mm^2 and assumes that the working prestressing stress $\sigma_{\mathrm{pm}\infty}$ = 1200 N/mm^2. Bending moment capacity needs to be calculated. Sketch the strain distribution over the height of the **midspan** cross-section and show the equilibrium between external and internal forces. Use the bi-linear ULS stress - strain diagram of concrete and mark important points/values.

Answer 10.8



Question 10.9

Determine the height of the concrete compression zone.

Answer 10.9

Assume that a prestressing steel stress $0,95f_{\mathrm{pk}}/\gamma_{\mathrm{s}}$ is reached.

$0,95f_{\mathrm{pk}}/\gamma_{\mathrm{s}} = 0,95\cdot 1860/1,1 = 1606$ N/mm^2

$$x_\mathrm{u} = \frac{A_\mathrm{p}\sigma_\mathrm{p,ULS}}{\alpha b f_\mathrm{cd}} = \frac{4000 \cdot 1606}{0,75 \cdot 750 \cdot \dfrac{50}{1,5}} = 343 \text{ mm}$$

Question 10.10

Check whether the actual prestressing steel stress equals the assumed stress and, if not, explain (in words, no calculation) which next step must be taken to arrive at the correct answer. Indicate the impact on, e.g., concrete compression zone height and prestressing steel stress.

Answer 10.10

Increase of prestressing steel strain:

$$\Delta\varepsilon_\mathrm{p} = \left(\frac{d_\mathrm{p} - x_\mathrm{u}}{x_\mathrm{u}}\right) \cdot 3,5 \cdot 10^{-3} = \left(\frac{1515-343}{343}\right) \cdot 3,5 \cdot 10^{-3} = 12,0 \cdot 10^{-3}$$

$$\varepsilon_{\mathrm{p}\infty} = \frac{1200}{195000} = 6,2 \cdot 10^{-3}$$

total strain: 18,2 ‰

$$\sigma_\mathrm{p} = 1522 + \left(\frac{18,2-7,8}{35-7,8}\right) \cdot (1691-1522) = 1522 + 65 = 1587 \text{ N/mm}^2$$

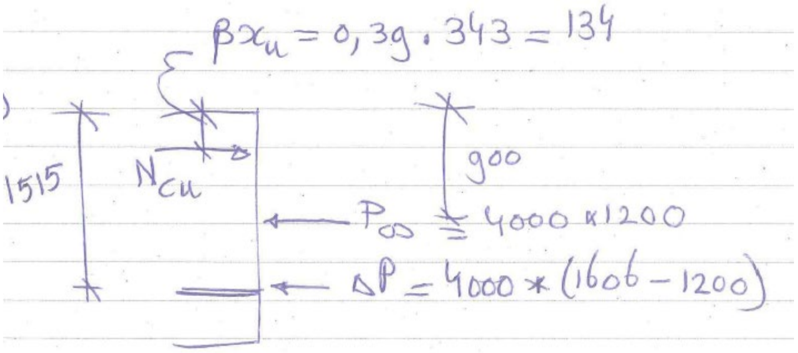
Assumed was: 1606 N/mm^2. That's too high => reduce the assumed $\sigma_\mathrm{p,ULS}$ => reduction of x_u => increase of prestressing strain increase => increase of total prestressing steel strain => increase of prestressing steel stress

Question 10.11

Calculate the bending moment resistance M_Rd at mid-span (use the assumed prestressing steel stress).

Answer 10.11

Units [N, mm]



$$M_\mathrm{Rd} = (900-134) \cdot 400 \cdot 1200 + (1515-134) \cdot 4000 \cdot 406 = 5920 \cdot 10^6 \text{ Nmm}$$

Question 10.12

Check whether there is sufficient rotational capacity at the mid-span cross-section (use the concrete compression zone height from question 10.9.

Rotational capacity:

$$\frac{x_\mathrm{u}}{d} \leq \frac{500}{500+f}$$

where

$$f = \frac{\left(\frac{f_\mathrm{pk}}{\gamma_\mathrm{s}} - \sigma_{\mathrm{pm},\infty}\right)A_\mathrm{p} + f_\mathrm{yd}A_\mathrm{s}}{A_\mathrm{p} + A_\mathrm{s}}$$

Answer 10.12

$A_\mathrm{s} = 0$

$$f = \frac{\left(\frac{f_\mathrm{pk}}{\gamma_\mathrm{s}} - \sigma_{\mathrm{pm},\infty}\right)A_\mathrm{p} + f_\mathrm{yd}A_\mathrm{s}}{A_\mathrm{p} + A_\mathrm{s}} = \frac{\left(\frac{1860}{1,1} - 1200\right)\cdot 4000 + 0}{4000 + 0} = 491\ \mathrm{N/mm^2}$$

$$\frac{343}{1515} ? \leq ? \frac{500}{500+491}$$

$$0,23 ! \leq ! 0,50$$

OK

Example 11 - Prestressed concrete

A post-tensioned beam has a span of $\ell = 20$ m, see figure 11.1. The beam has a **rectangular** cross-section; width $b = 0{,}9$ m and height $h = 1{,}5$ m. The beam is prestressed using bonded tendons (strands in ducts). The fictitious tendon profile consists of a parabola with a radius of curvature R over $0{,}5\ell = 10$ m (A-C), and a linear, horizontal part over $0{,}5\ell = 10$ m (C-B).

The bottom of the parabola (tendon profile A-C) is at position C in Figure 11.1, which is at $0{,}5\ell = 10$ m from the left support (support A).

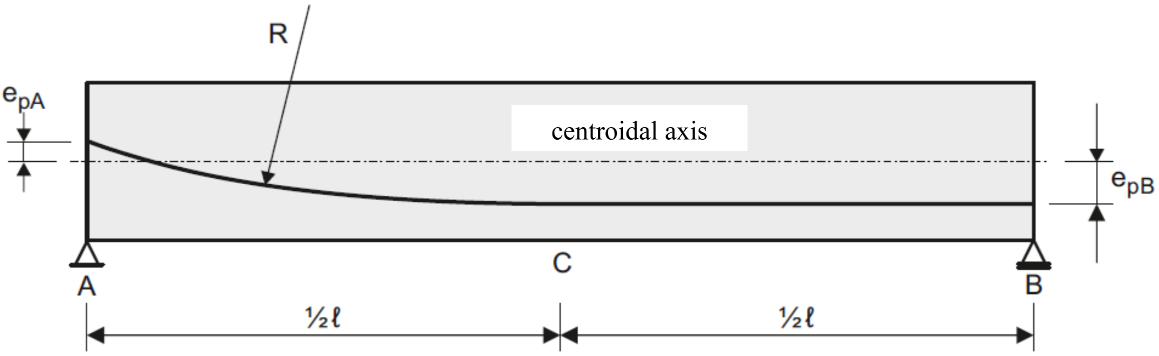


Figure 11.1 Side view of the beam including fictitious tendon profile (not to scale).

Parameters:

Density concrete : $\rho = 25$ kN/m^3
Variable load : $q_{Qk} = 15$ kN/m
Load factors ULS: : $\gamma_G = 1{,}2$ $\gamma_Q = 1{,}5$ $\gamma_P = 1{,}0$
Concrete strength class : C50/60

Strength class of prestressing steel : Y1860
Elastic modulus prestressing steel : $E_p = 195000$ N/mm^2

Question 11.1

Calculate the maximum allowable tendon eccentricities (e_{pA} (↑) and e_{pB} (↓)) relative to the centroidal axis at the two anchors such that no tensile stresses at sections A and B occur.

Answer 11.1

No bending moments from G and Q; prestressing only.

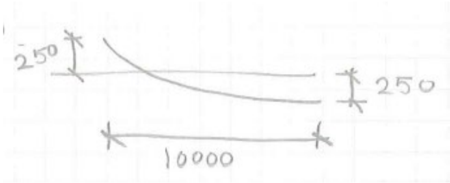
Kern area: $h/6 = 1500/6 = 250$ mm.
Max. eccentricities:

$e_{pA} = 250$ mm (↑) and $e_{pB} = 250$ mm (↓)

Question 11.2

Calculate the radius of curvature R of the parabola based on e_{pA}, e_{pB} determined in question 11.1.

Answer 11.2

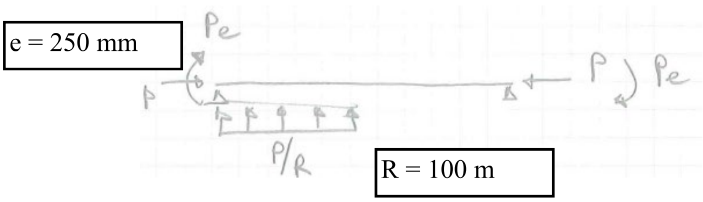


parabola expression: $R = \frac{l^2}{8f} = \frac{(2 \cdot 10000)^2}{8 \cdot (250 + 250)} = 100 \cdot 10^3$ mm

Question 11.3

Show in a figure all the loads on the beam from a prestressing force *P*, assuming that there is no friction or wedge set.

Answer 11.3



Question 11.4

Calculate and draw the bending moment diagram resulted from a prestressing force *P*. Assume that there is no friction and no wedge set.

Answer 11.4

Two components:

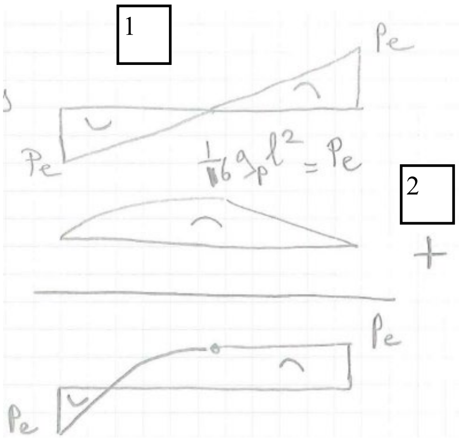
1
External bending moments at A and B from tendon eccentricities.

2
Upward curvature pressure from A to C; bending moment at midspan = 0,5·(ql^2 / 8).

$$R = \frac{l^2}{8f} = \frac{l^2}{8 \cdot 2e}$$

Parabola expression: $q_{\mathrm{P}} = \frac{P}{R} = \frac{16Pe}{l^2}$

$$\frac{1}{16} q_{\mathrm{P}} l^2 = Pe$$



Question 11.5

Calculate the **minimum** working prestressing force $P_{\mathrm{m}\infty}$ [kN] **required** to have no tension in the **midspan** cross-section, at maximum SLS load.

Answer 11.5

G = 0,9·1,5·25 = 33,75 kN/m
Q = 15 kN/m

Total: 48,75 kN/m

Bending moment at midspan: ql^2 / 8 = 48,75·20^2 / 8 = 2438 kNm

Check bottom fiber level.

Units [kN, m]:

$$-\frac{P}{0,9\cdot 1,5}-\frac{0,25P}{\frac{1}{6}\cdot 0,9\cdot 1,5^2}+\frac{2438}{\frac{1}{6}\cdot 0,9\cdot 1,5^2}\leq 0$$

$P \geq 4876$ kN

Question 11.6

The tendons are stressed from support A. Sketch the profile of the prestressing force $P_{\mathrm{m,x}}/P_{\mathrm{m,x=0}}$ at $t = 0$ over the full length of the beam. Mark important points/values. Assume that the friction coefficient μ = 0,24 and the wobble factor k = 0,015 rad/m.

Prestressing force including frictional loss:

$$P_{\mathrm{m}}(x)=P_{\mathrm{m}}(x=0)\cdot e^{-\mu(\theta+kx)}$$

friction coefficient $\mu = 0,24$
wobble factor $k = 0,015$ rad/m

Answer 11.6

Use $\Delta\theta=\left|\frac{\Delta x}{R}\right|$

at C:

A => C: $\Delta\theta = \left|\frac{\Delta x}{R}\right| = \frac{10}{100} = 0{,}1$

$$P_{\mathrm{m}}(C) = P_{\mathrm{m}}(A)\cdot e^{-\mu(\theta+kx)} = P_{\mathrm{m}}(A)\cdot e^{-0{,}24\cdot(0{,}1+10\cdot 0{,}015)} = 0{,}942P_{\mathrm{m}}(A)$$

at B:

C => B: $\Delta\theta = \left|\frac{\Delta x}{R}\right| = \frac{10}{\infty} = 0$ (no curvature)

$$P_{\mathrm{m}}(C) = P_{\mathrm{m}}(A)\cdot e^{-\mu(\theta+kx)} = P_{\mathrm{m}}(A)\cdot e^{-0{,}24\cdot(0{,}1+0+20\cdot 0{,}015)} = 0{,}908P_{\mathrm{m}}(A)$$

Question 11.7

A prestressing force P_{m0} = 5800 kN (section A) is used. Calculate the time-dependent prestress losses due to shrinkage, creep and relaxation in N/mm^2 at $t = \infty$ at midspan (i.e. in the midspan cross-section, section C). The tendons can assumed to have an initial stress σ_{pi} = 1200 N/mm^2 at section C. Assume that the beam is always fully loaded (maximum SLS load).

Concrete modulus of elasticity : 36000 N/mm^2
Creep coefficient : $\varphi = 2{,}1$
Deformation caused by shrinkage : $\varepsilon_{\mathrm{cs}} = 0{,}25$ ‰

Answer 11.7

P_{m0} = 5800 kN at A

friction: $P_{\mathrm{C}} = 0{,}942\ P_{\mathrm{A}}$ => P_{C} = 5464 kN

Concrete stress at prestressing steel level:

Bending moment from G and Q: 2438 kNm

Units [kN, m]:

$$\sigma_{\mathrm{c}} = -\frac{5464}{0{,}9\cdot 1{,}5} - \frac{(5464\cdot 0{,}25)\cdot 0{,}25}{\frac{1}{12}\cdot 0{,}9\cdot 1{,}5^3} + \frac{2438\cdot 0{,}25}{\frac{1}{12}\cdot 0{,}9\cdot 1{,}5^3} = -4047 + 1059 = -2988\ \mathrm{kN/m^2} = -2{,}99\ \mathrm{N/mm^2}$$

Concrete strain at prestressing steel level:

ε_{c} = -2,99/36000

Creep:

$$2{,}1\cdot\left(\frac{-2{,}99}{36000}\right)\cdot E_{\mathrm{p}} = -34\ \mathrm{N/mm^2}$$

Shrinkage:

$$-0,25 \cdot 10^{-3} \cdot E_{\mathrm{p}} = -49 \text{ N/mm}^2$$

Relaxation:

$$\sigma_{\mathrm{pi}} = 1200 \text{ N/mm}^2$$

$$\mu = \frac{1200}{1860} = 0,65$$

$$\frac{\Delta\sigma_{\mathrm{pr}}}{\sigma_{\mathrm{pi}}} = 0,66 \cdot 2,5 \cdot e^{9,1 \cdot 0,65} \cdot 500^{0,75 \cdot (1-0,65)} \cdot 10^{-5} = 3,13 \cdot 10^{-5}$$

$$\Delta\sigma_{\mathrm{pr}} = 38 \text{ N/mm}^2$$

Total loss: 34 + 49 + 0,8·38 = 113 N/mm^2

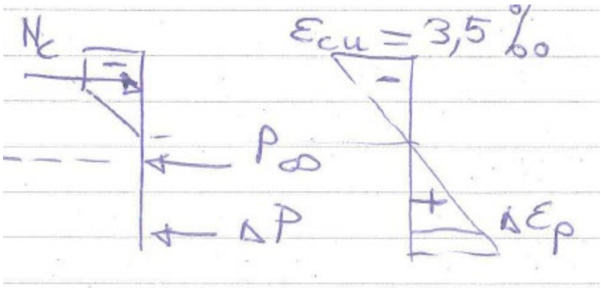
Question 11.8

The **midspan** cross-section of the beam is now analyzed in **ULS** at $t = \infty$. A structural engineer decides to apply A_{p} = 4500 mm^2 and assumes that the working prestressing stress $\sigma_{\mathrm{pm}\infty}$ = 1100 N/mm^2. The engineer wants to determine the bending moment capacity at ULS.

Sketch the strain distribution over the height of the **midspan** cross-section and show the external and internal forces.

Use the bi-linear ULS stress - strain diagram of concrete and mark important points/values.

Answer 11.8



Question 11.9

Calculate the bending moment resistance M_{Rd} and the design bending moment M_{Ed} at mid-span (section C).

Answer 11.9

Assume that a prestressing steel stress $0,95 f_{\mathrm{pk}}/\gamma_{\mathrm{s}}$ is reached.

$0,95 f_{\mathrm{pk}}/\gamma_{\mathrm{s}} = 0,95 \cdot 1860/1,1 = 1606$ N/mm^2

$$x_{\mathrm{u}} = \frac{A_{\mathrm{p}}\sigma_{\mathrm{p,ULS}}}{\alpha b f_{\mathrm{cd}}} = \frac{4500 \cdot 1606}{0,75 \cdot 900 \cdot \dfrac{50}{1,5}} = 322 \text{ mm}$$

Increase of prestressing steel strain:

$$\Delta\varepsilon_{\mathrm{p}} = \left(\frac{d_{\mathrm{p}} - x_{\mathrm{u}}}{x_{\mathrm{u}}}\right) \cdot 3,5 \cdot 10^{-3} = \left(\frac{700 + 250 - 322}{322}\right) \cdot 3,5 \cdot 10^{-3} = 7,4 \cdot 10^{-3}$$

$$\varepsilon_{\mathrm{p}\infty} = \frac{1100}{195000} = 5,6 \cdot 10^{-3}$$

Total strain: 7,4 + 5,6 = 13,0 ‰

Actual prestressing steel stress:

$$\sigma_{\mathrm{p}} = 1522 + \left(\frac{13,0 - 7,8}{35 - 7,8}\right) \cdot \left(1691 - 1522\right) = 1554 \text{ N/mm}^2$$

Note: Assumed was 1606 N/mm^2. That’s too high

Horizontal force equilibrium:

$N_{\mathrm{P}} = N_{\mathrm{c}} \approx 4500 \cdot 1606 = 7227 \cdot 10^3$ N

Bending moment resistance:

$$M_{\mathrm{Rd}} = \left(\frac{1}{2}h - \beta x_{\mathrm{u}}\right) N_{\mathrm{C}} + (250) \cdot \Delta N_{\mathrm{P}}$$
$$= \left(750 - 0,39 \cdot 322\right) \cdot 7227 \cdot 10^3 + \left(250\right) \cdot \left(7227 \cdot 10^3 - 4500 \cdot 1100\right)$$
$$= \left(4513 + 569\right) \cdot 10^6 = 5082 \cdot 10^6 \text{ Nmm}$$

Design bending moment:

$$M_{\mathrm{Ed}} = \frac{1}{8}\left(G + Q\right) l^2 - P_{\infty} e_{\mathrm{P}}$$
$$= \frac{1}{8}\left(33,75 \cdot 1,2 + 15 \cdot 1,5\right) \cdot 20^2 - 4500 \cdot 1100^{-3} \cdot 0,25$$
$$= \left(3150 - 1238\right) = 1912 \text{ kNm}$$

Example 12 - Prestressed concrete

A beam has a **rectangular** cross-section; width b = 0,6 m and height h = 2,0 m. The beam is post-tensioned using curved tendons (strands in ducts) , see figure 12.1. The beam has a 30 m span. The fictitious tendon profile consists of two parabolas, namely parabola 1 with a radius of curvature R_1 = 100 m over 15 m and parabola 2 with R_2 = 120 m over 15 m

The bottom of both parabolas (at 0,2 m from the bottom fiber of the beam) is at mid span position A (they joint at this position).

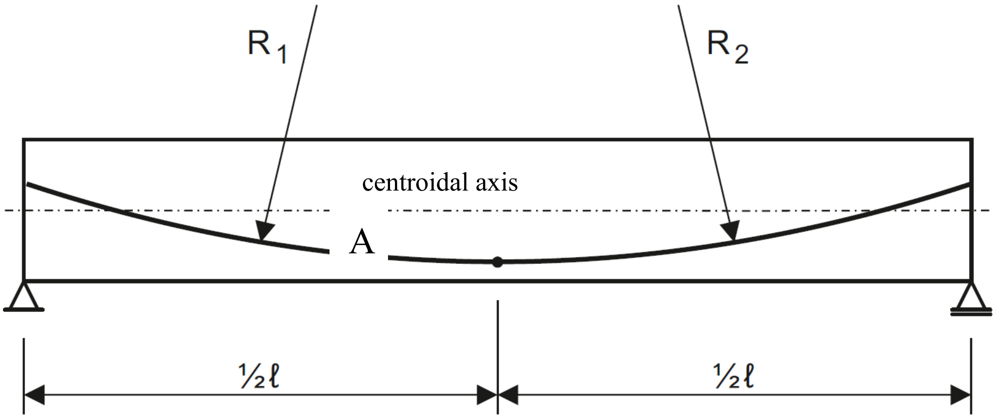


Figure 12.1 Side view of the girder including fictitious tendon profile (not to scale).

Parameters:

Density concrete : $\rho = 25$ kN/m^3
Variable load : $q_{Qk} = 10$ kN/m

Strength class of concrete : C50/60

Strength class of prestressing steel : Y1860S7
Elastic modulus prestressing steel : $E_p = 195000$ N/mm^2

Question 12.1

Calculate the tendon eccentricities relative to the centroidal axis at the two anchors.

Answer 12.1

Parabola expression: $y = \dfrac{x^2}{2R}$

R_1 = 100 m: $y = \dfrac{15^2}{2 \cdot 100} = 1{,}125\text{ m}$

R_2 = 120 m: $y = \dfrac{15^2}{2 \cdot 120} = 0{,}9375\text{ m}$

$e_{\text{left}} = 2/2 - 0{,}2 - 1{,}125 = -0{,}325$ m ↑

$e_{\text{right}} = 2/2 - 0{,}2 - 0{,}9375 = -0{,}1375$ m ↓

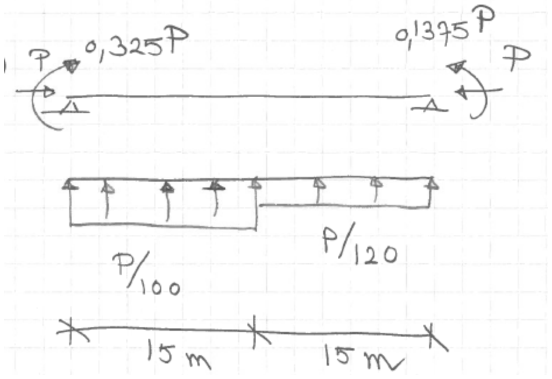
Question 12.2

Show in a figure all the loads on the beam from a prestressing force *P* and calculate and draw the bending moment diagram resulting from a prestressing force *P*. Assume that there is no friction and no wedge set.

Answer 12.2

Loads (engineering model):

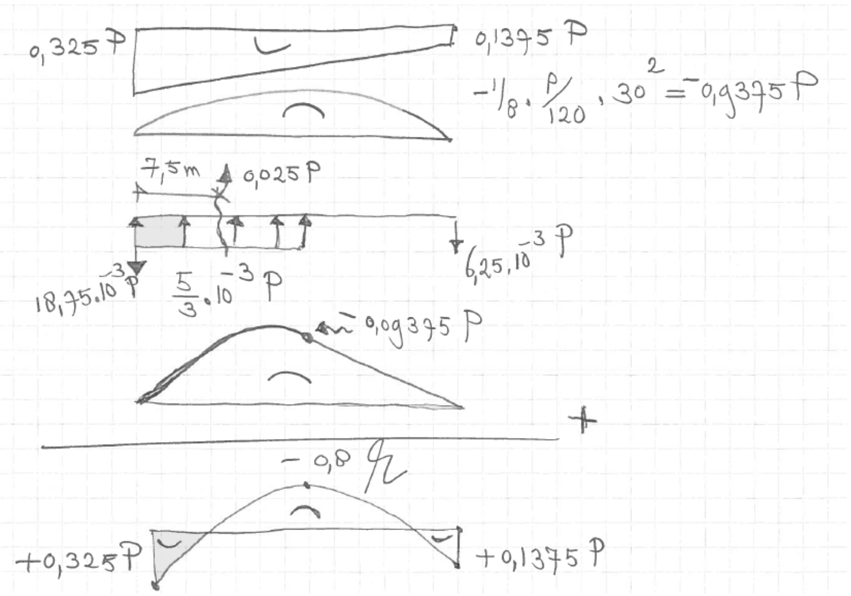
- Axial compressive force P.
- Concentrated bending moments from tendon eccentricity at the anchors.
- Upward curvature pressures.



Additional info:
Exact forces and bending moment at an anchor: Pcosα horizontal force; Psinα vertical force; e·Pcosα (α follows from the tendon profile)

Bending moment from:

- Concentrated bending moments at both beam ends.
- Uniformly distributed upward curvature pressure over full beam length: P/120.
- Uniformly distributed upward curvature pressure over half beam length: P/100 - P/120.



Question 12.3

Calculate the **maximum** initial prestressing force P_{m0} [kN] **allowed** to have no tension in the **midspan** cross-section when stressing the tendons.

Answer 12.3

Directly after stressing the tendons: Selfweight only

G = 0,6·2,0·25 = 30 kN/m

Bending moment at midspan: $ql^2 / 8 = 30 \cdot 30^2 / 8 = 3375$ kNm

Check top fiber level.

Units [kN, m]:

$$-\frac{P}{0,6\cdot 2,0}+\frac{0,8P}{\frac{1}{6}\cdot 0,6\cdot 2,0^2}-\frac{3375}{\frac{1}{6}\cdot 0,6\cdot 2,0^2}\leq 0$$

$$P \leq 7232\ kN$$

Question 12.4

Calculate the **minimum** working prestressing force $P_{m\infty}$ [kN] **required** to have no tension in the **midspan** cross-section, at maximu m SLS load.

Answer 12.4

G = 30 kN/m
Q = 10 kN/m

Bending moment from Q at midspan: $ql^2 / 8 = 10 \cdot 30^2 / 8 = 1125$ kNm

Check bottom fiber level.

Units [kN, m]:

$$-\frac{P}{0,6\cdot 2,0}-\frac{0,8P}{\frac{1}{6}\cdot 0,6\cdot 2,0^2}+\frac{3375+1125}{\frac{1}{6}\cdot 0,6\cdot 2,0^2}\leq 0$$

$$P \geq 3971\ kN$$

Question 12.5

Each tendon is stressed from its both ends; first from the left hand side support, followed by stressing from the right hand side support. Sketch the profile of the prestressing force $P_{m,x}/P_{m,x=0}$ at $t = 0$ over the full length of the beam after stressing from both ends. Mark important points/values. Assume that the friction coefficient $\mu = 0,16$ and the wobble factor $k = 0,012$ rad/m. Assume that there is no wedge set.

Use the following expression and input:

Prestressing force including frictional loss:

$$P_{\mathrm{m}}(x)=P_{\mathrm{m}}(x=0)\cdot e^{-\mu(\theta+kx)}$$

friction coefficient $\mu = 0{,}16$
wobble factor $k = 0{,}012$ rad/m

Answer 12.5

Use $\Delta\theta=\left|\dfrac{\Delta x}{R}\right|$

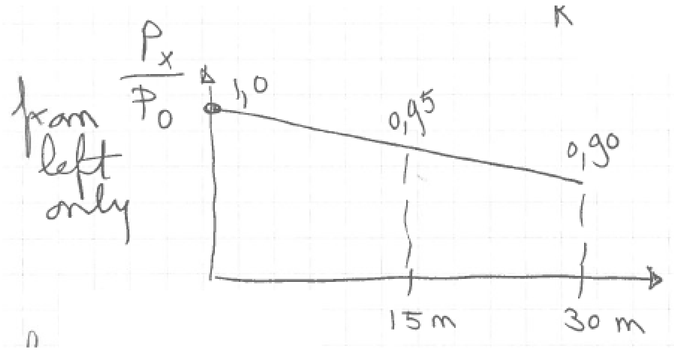
First from left only:

at 15 m (midspan): $\Delta\theta=\left|\dfrac{\Delta x}{R}\right|=\dfrac{15}{100}=0{,}15$

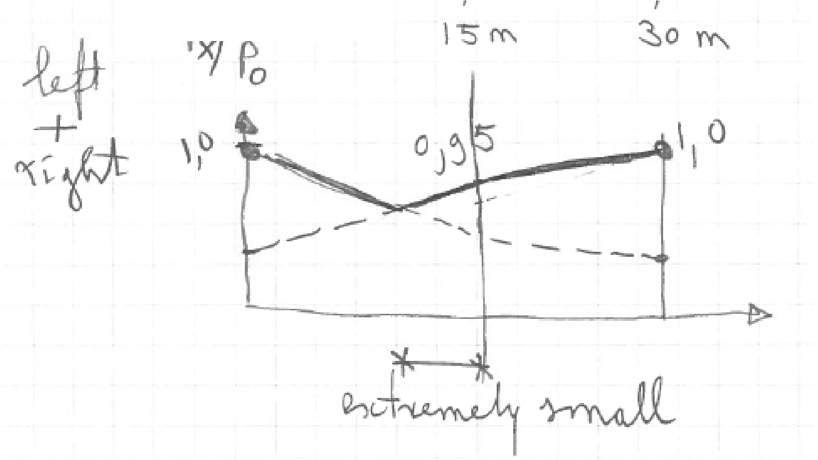
$$P_{\mathrm{m}}\left(x=15\ \mathrm{m}\right)=P_{\mathrm{m}}\left(x=0\right)\cdot e^{-\mu(\theta+kx)}=P_{\mathrm{m}}\left(x=0\right)\cdot e^{-0{,}16\cdot(0{,}15+15\cdot 0{,}02)}=0{,}95P_{\mathrm{m}}\left(x=0\right)$$

at 30 m: $\Delta\theta=\left|\dfrac{\Delta x}{R}\right|=\dfrac{15}{100}+\dfrac{15}{120}$

$$P_{\mathrm{m}}\left(x=30\ \mathrm{m}\right)=P_{\mathrm{m}}\left(x=0\right)\cdot e^{-\mu(\theta+kx)}=P_{\mathrm{m}}\left(x=0\right)\cdot e^{-0{,}16\cdot(15/100+15/120+30\cdot 0.02)}\quad\left(x=0\right)$$



(not to scale)



(not to scale)

Since 15/120 < 10/120, the slope of the right hand side part of the curve is smaller than the slope of the left hand side part. This implies that the two solid lines don't intersect at midspan, but a bit to the left.

Question 12.6

The tendons are assumed to be stressed to a stress $\sigma_{p,max}$ = 1395 N/mm^2. Wedge set now is w_{set} = 2 mm. Calculate the wedge set influence length l_{set} when anchoring a tendon after stressing it from the left hand side support. Sketch the profile of the prestressing steel stress immediately after anchoring. Use the results from question **12.5**.

$$l_{set} = \sqrt{\frac{w_{set}\,E_p}{\Delta\sigma_{p,\mu}/\Delta x}}$$

Answer 12.6

Units: N & mm:

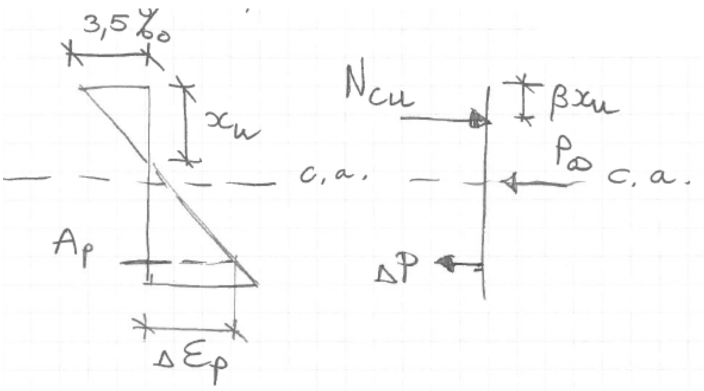
$$\Delta\sigma_{p,\mu}/\Delta x = \frac{(1-0{,}95)\cdot\sigma_{p,max}}{15\cdot 10^3}$$

$$l_{set} = \sqrt{\frac{w_{set}E_p}{\Delta\sigma_{p,\mu}/\Delta x}} = \sqrt{\frac{2\cdot 195000}{\frac{0{,}05\cdot 1395}{15\cdot 10^3}}} = 9{,}16\cdot 10^3 \text{ mm}$$

Question 12.7

The **mid-span** cross-section of the beam is now analyzed in **ULS** at $t = \infty$. A_p = 4500 mm^2 is assumed and the working prestressing stress $\sigma_{pm\infty}$ = 1200 N/mm^2. The bending moment capacity has to be calculated. Use the bi-linear ULS stress - strain diagram of concrete. **First**, sketch strain distribution over beam height, indicate the position of forces and mark important points/values.

Answer 12.7



Question 12.8

Determine the height of the concrete compression zone.

Answer 12.8

Assume that a prestressing steel stress $0{,}95 f_{pk}/\gamma_s$ is reached.

$0{,}95 f_{pk}/\gamma_s = 0{,}95\cdot 1860/1{,}1 = 1606$ N/mm^2

$$x_{\mathrm{u}} = \frac{N_{\mathrm{cu}}}{\alpha b f_{\mathrm{cd}}} = \frac{A_{\mathrm{p}}\sigma_{\mathrm{p,ULS}}}{\alpha b f_{\mathrm{cd}}} = \frac{4500 \cdot 1606}{0,75 \cdot 600 \cdot \frac{50}{1,5}} = 482 \text{ mm}$$

Question 12.9

Check whether the actual prestressing steel stress equals the assumed stress.

Answer 12.9

Increase of prestressing steel strain:

$$\Delta\varepsilon_{\mathrm{p}} = \left(\frac{d_{\mathrm{p}} - x_{\mathrm{u}}}{x_{\mathrm{u}}}\right) \cdot 3,5 \cdot 10^{-3} = \left(\frac{1800 - 482}{482}\right) \cdot 3,5 \cdot 10^{-3} = 9,6 \cdot 10^{-3}$$

$$\varepsilon_{\mathrm{p}\infty} = \frac{1200}{195000} = 6,2 \cdot 10^{-3}$$

total strain: 15,8 ‰

$$\sigma_{\mathrm{p}} = 1522 + \left(\frac{15,8 - 7,8}{35 - 7,8}\right) \cdot (1691 - 1522) = 1572 \text{ N/mm}^2$$

Assumed was: 1606 N/mm^2. That's too high => reduce the assumed $\sigma_{\mathrm{p,ULS}}$ => reduction of x_{u} => increase of prestressing strain increase => increase of total prestressing steel strain => increase of prestressing steel stress

Example 13 - Prestressed concrete

A simply supported post-tensioned beam has an ℓ = 16 m span, see figure 13.1. The beam has a rectangular cross-section (h = 1200 mm; b = 500 mm). The beam is prestressed using bonded tendons. The tendon profile is modeled using two parabolas and a horizontal, linear part. There are no kinks at the positions where the parabola and the linear parts join.

Beam span: $\ell = 16$ m ($\ell_1 = 4$ m, $\ell_2 = 8$ m).
Tendon eccentricities: $e_{p1} = 0{,}2$ m; $e_{p2} = 0{,}5$ m.

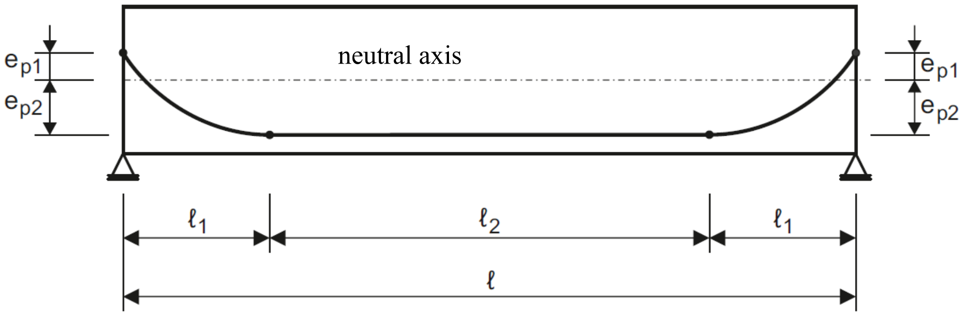


Figure 13.1 Side view of the beam including tendon profile (not to scale).

Parameters:

Density concrete	: $\rho = 25$ kN/m^3
Variable load	: $q_{Qk} = 15$ kN/m
Concrete strength class	: C40/50
Concrete modulus of elasticity	: 35000 N/mm^2
Creep coefficient	: $\varphi = 1{,}5$
Deformation caused by shrinkage	: $\varepsilon_{cs} = 0{,}3$ ‰
Strength class of prestressing steel	: Y1860
Prestressing steel modulus of elasticity	: 195000 N/mm^2 (strands)

Question 13.1

Calculate the radius of curvature of the parabolic parts of the tendon profile.

Answer 13.1

Parabola expression: $y = \dfrac{x^2}{2R}$

$$y = 0{,}2 + 0{,}5 = \frac{4^2}{2R} \Rightarrow R = 11{,}4 \text{ m}$$

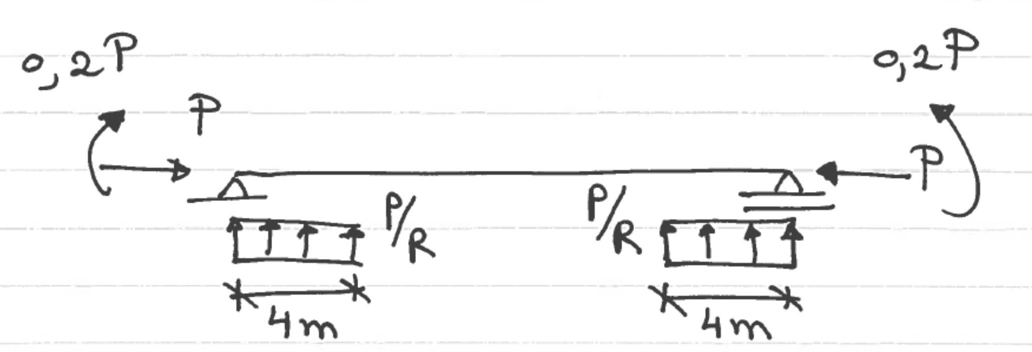
Question 13.2

Show in a sketch all the loads on the beam caused by a prestressing force P. Assume that there is no friction and no wedge set.

Answer 13.2

Loads (engineering model):

- Axial compressive force P.
- Concentrated bending moments from tendon eccentricity at the anchors.
- Upward curvature pressures.



Additional info:
Exact forces and bending moment at an anchor: Pcosα horizontal force; Psinα vertical force; $e_{p1} \cdot P\cos\alpha$ (α follows from the tendon profile).
$\alpha = 2 \cdot (0{,}2+0{,}5)/4 = 0{,}35$ rad
alternative:
$\alpha = 4/11{,}4 = 0{,}35$ rad.
Check vertical force equilibrium:
Psinα ↓ at the anchor
$(P/R) \cdot l_1$ ↑ from curvature pressure
$\alpha = l_1/R$ and $\sin\alpha \approx \alpha$: $(P/R) \cdot l_1 \uparrow = P\alpha \approx P\sin\alpha$; OK.

Question 13.3

Calculate the bending moment from selfweight (*G*) and variable load (*Q*) at midspan (i.e. at the midspan cross-section), at maximum SLS load.

Answer 13.3

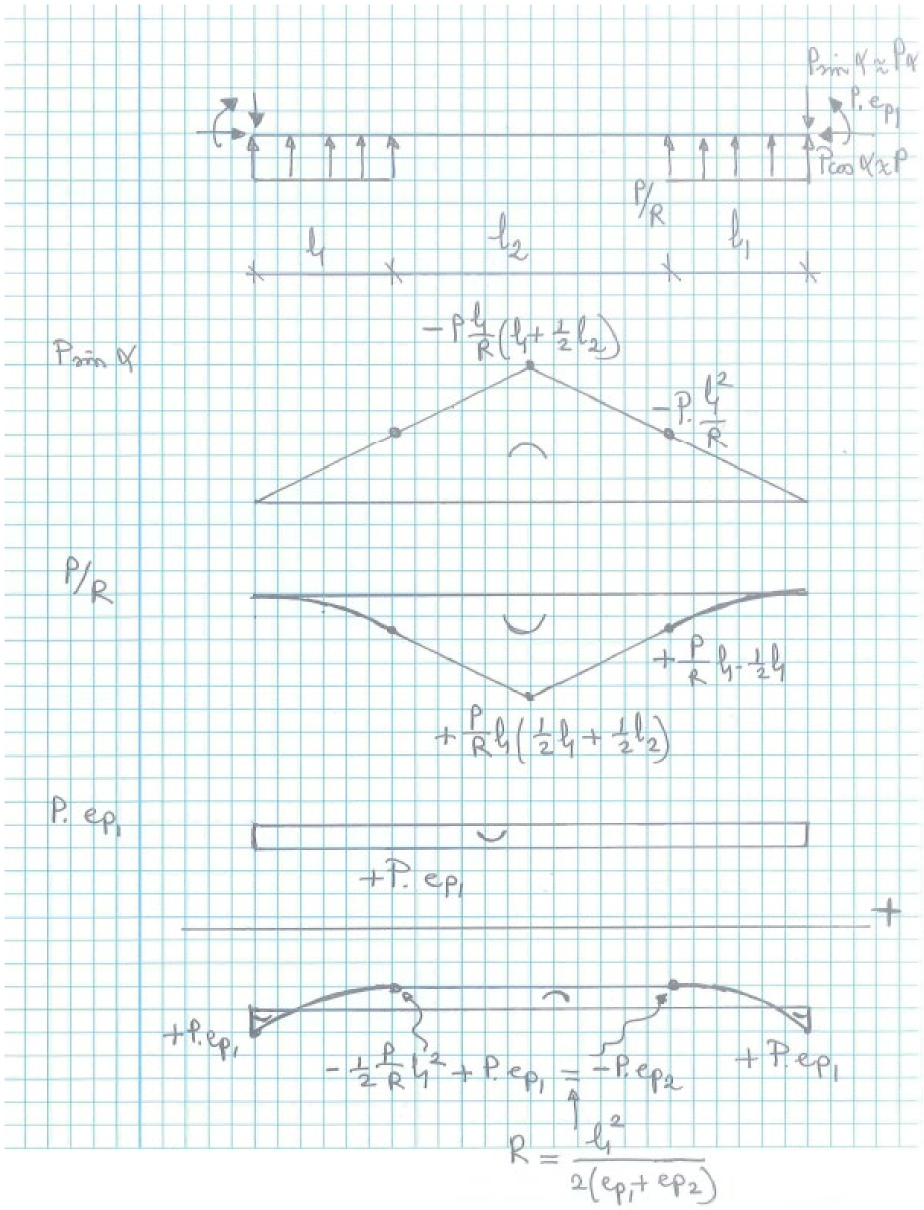
$G = 0{,}5 \cdot 1{,}2 \cdot 25 = 15$ kN/m
$Q = 15$ kN/m

Bending moment from G+Q at midspan: $ql^2 / 8 = 30 \cdot 16^2 / 8 = 960$ kNm

Question 13.4

Calculate the bending moment from prestressing (*P*) at midspan (i.e. at the midspan cross-section) expressed as a function of the prestressing force *P*. Assume that there is no friction and no wedge set.

Answer 13.4



Midspan bending moment from prestressing: $-P \cdot e_{p2}$

Question 13.5

Calculate the **minimum** working prestressing force $P_{m\infty}$ [kN] required to have no tension at midspan (i.e. at the midspan cross-section) at maximum SLS load.

Answer 13.5

Check bottom fiber level.

Units [kN, m]:

$$-\frac{P}{0,5 \cdot 1,2} - \frac{0,5P}{\frac{1}{6} \cdot 0,5 \cdot 1,2^2} + \frac{960}{\frac{1}{6} \cdot 0,5 \cdot 1,2^2} \leq 0$$

$$P \geq 1375 \text{ kN}$$

Question 13.6

A prestressing force P_{m0} = 1650 kN is used. Calculate the concrete stress at the prestressing steel level at midspan (i.e. at the midspan cross-section), at maximum SLS load. Assume that there is no friction and no wedge set.

Answer 13.6

Bending moment: 960 kNm - 0,5P
P = 1650 kN

$z = e_{p2} = 0{,}5$ m

units [kN, m]

$$-\frac{1650}{0,5\cdot 1,2}+\frac{(960-0,5\cdot 1650)\cdot 0,5}{\frac{1}{12}\cdot 0,5\cdot 1,2^3}=-1810\text{ kN/m}^2=-1,81\text{ N/mm}^2$$

Question 13.7

A prestressing force P_{m0} = 1650 kN is used. Calculate the time-dependent prestress losses due to shrinkage, creep and relaxation in N/mm^2 at $t = \infty$ at midspan (i.e. at the midspan cross-section). The tendons are assumed to be stressed to an initial stress σ_{pi} = 1395 N/mm^2.

Assume that there is no friction and no wedge set.

The equation for the calculation of relaxation is:

$$\frac{\Delta\sigma_{pr}}{\sigma_{pi}} = 0,66\ \rho_{1000}\ e^{(9,1\,\mu)}\left(\frac{t}{1000}\right)^{(0,75\,(1-\mu))}\cdot 10^{-5}$$

where:
$\Delta\sigma_{pr}$: is absolute value of the relaxation losses of the prestressing steel
σ_{pi} : is the initial stress in the prestressing steel
t : is the time after tensioning in hours ($t = \infty$ = 500000 hours)
μ : = σ_{pi} / f_{pk}, where f_{pk} is the characteristic value of the tensile strength of the prestressing steel
ρ_{1000} : is the value of relaxation loss (in %), at 1000 hours after tensioning and at a mean temperature of 20°C
ρ_{1000} : = 2,5%

The influence of shrinkage and creep reduces the relaxation. When taking into account shrinkage and creep losses, relaxation loss is: $\Delta\sigma_{\text{pr(incl,c+s)}} = 0,8\ \Delta\sigma_{\text{pr(excl,c+s)}}$

Answer 13.7

P_{m0} = 1650 kN

Concrete strain at prestressing steel level:
ε_c = -1,81/35000

Creep:

$$1{,}5\cdot\left(\frac{-1{,}81}{35000}\right)\cdot E_{\mathrm{p}}=-15\ \mathrm{N/mm^2}$$

Shrinkage:

$$-0{,}30\cdot10^{-3}\cdot E_{\mathrm{p}}=-59\ \mathrm{N/mm^2}$$

Relaxation:

$$\sigma_{\mathrm{pi}}=1395\ \mathrm{N/mm^2}$$

$$\mu=\frac{1395}{1860}=0{,}75$$

$$\frac{\Delta\sigma_{\mathrm{pr}}}{\sigma_{\mathrm{pi}}}=0{,}75\cdot2{,}5\cdot e^{9{,}1\cdot0{,}75}\cdot500^{0{,}75\cdot(1-0{,}75)}\cdot10^{-5}=2{,}72\cdot10^{-2}$$

$$\Delta\sigma_{\mathrm{pr}}=68\ \mathrm{N/mm^2}$$

Total loss: 15 + 59 + 0,8·68 (N/mm^2)

Question 13.8

The tendons are stressed from **one** end, namely the left support. Sketch the profile of the prestressing force (ratio $P_{\mathrm{m}}(x)/P_{\mathrm{m}}(x=0)$) at $t = 0$ over the full length of the beam. Mark important points/values. Use the following expression and input.

Prestressing force including frictional loss:

$$P_{\mathrm{m}}(x)=P_{\mathrm{m}}(x=0)\cdot e^{-\mu(\theta+kx)}$$

friction coefficient $\mu = 0{,}18$
wobble factor $k = 0{,}012$ rad/m

Answer 13.8

Use $\Delta\theta=\left|\frac{\Delta x}{R}\right|$

$R = 11{,}4$ m

at 4 m:

$$\Delta\theta=\left|\frac{\Delta x}{R}\right|=\frac{4}{11{,}4}$$

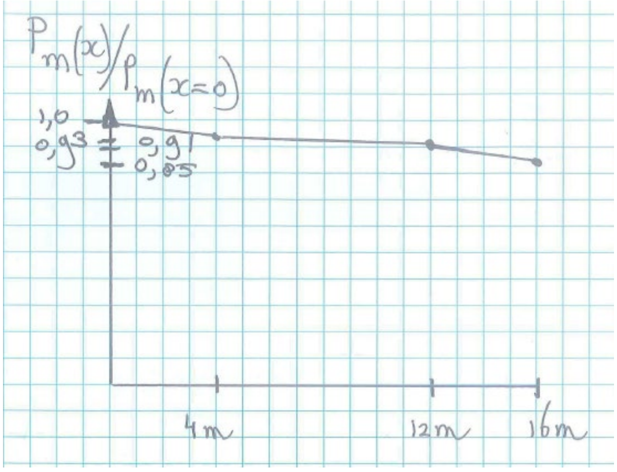
at 16 m:

$$\Delta\theta = \left|\frac{\Delta x}{R}\right| = \frac{4}{11,4} + \frac{4}{11,4}$$

$$P_{\text{m}}\left(x = 4\text{ m}\right) = P_{\text{m}}\left(x = 0\right) \cdot e^{-\mu(\theta + kx)} = P_{\text{m}}\left(x = 0\right) \cdot e^{-0,18 \cdot (4/11,4 + 4 \cdot 0,012)} = 0,93 P_{\text{m}}\left(x = 0\right)$$

$$P_{\text{m}}\left(x = 12\text{ m}\right) = P_{\text{m}}\left(x = 0\right) \cdot e^{-\mu(\theta + kx)} = P_{\text{m}}\left(x = 0\right) \cdot e^{-0,18 \cdot (4/11,4 + 0 + 12 \cdot 0,012)} = 0,91 P_{\text{m}}\left(x = 0\right)$$

$$P_{\text{m}}\left(x = 16\text{ m}\right) = P_{\text{m}}\left(x = 0\right) \cdot e^{-\mu(\theta + kx)} = P_{\text{m}}\left(x = 0\right) \cdot e^{-0,18 \cdot (4/11,4 + 0 + 4/11,4 + 16 \cdot 0,012)} = 0,85 P_{\text{m}}\left(x = 0\right)$$



Question 13.9

The tendons are assumed to be stressed to a stress $\sigma_{\text{p,max}}$ = 1395 N/mm^2. Wedge set is w_{set} = 1,5 mm. Calculate the wedge set influence length and sketch the profile of the prestressing steel stress immediately after anchoring. Use the results from question **13.8**.

Influence length of wedge set:

$$l_{\text{set}} = \sqrt{\frac{w_{\text{set}} E_{\text{p}}}{\Delta\sigma_{\text{p},\mu} / \Delta x}}$$

Answer 13.9

Units: N & mm:

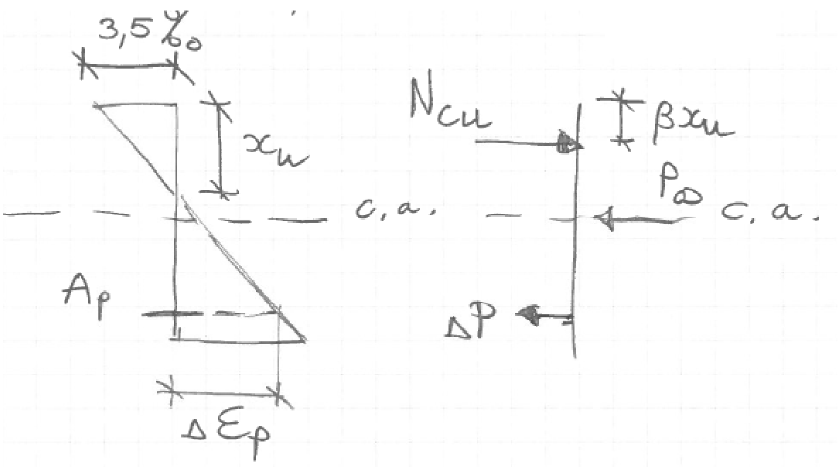
$$\Delta\sigma_{\text{p},\mu} / \Delta x = \frac{\left(1 - 0,93\right) \cdot \sigma_{\text{p,max}}}{4 \cdot 10^3}$$

$$l_{\text{set}} = \sqrt{\frac{w_{\text{set}} E_{\text{p}}}{\Delta\sigma_{\text{p},\mu} / \Delta x}} = \sqrt{\frac{1,5 \cdot 195000}{\frac{0,07 \cdot 1395}{4 \cdot 10^3}}} = 3,5 \cdot 10^3 \text{ mm}$$

Question 13.10

The **midspan** cross-section of the beam is now analyzed at **ULS** at $t = \infty$. Bending moment capacity needs to be calculated. Sketch the strain distribution over the height of the **midspan** cross-section and show the equilibrium between external and internal forces. Use the bi-linear ULS stress - strain diagram of concrete and mark important points/values.

Answer 13.10



Example 14 - Prestressed concrete

A post-tensioned beam has an $L = 30$ m span, see figure 14.1. The beam has a **rectangular** cross-section; width $b = 0{,}6$ m and height $h = 2{,}0$ m. The beam is prestressed using curved tendons. The fictitious tendon profile expression is:

$y = 4{,}348 \cdot 10^{-3}\, x^2 - 0{,}1338\, x + 0{,}3$

where x and y are in [m]. The origin of the axis system is at support A, at the level of the neutral axis, see figure 14.1. The radius of curvature $R = 115$ m.
Tendon eccentricities at the anchors: $e_L = 0{,}3$ m; $e_R = 0{,}2$ m

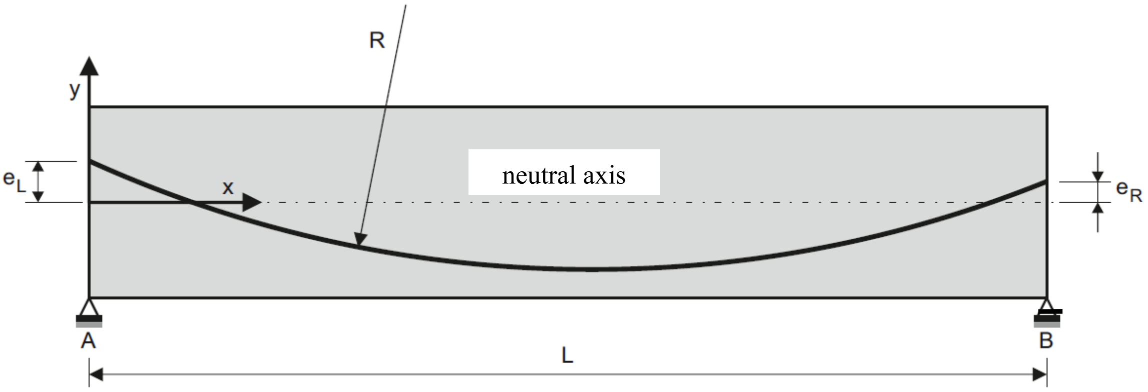


Figure 14.1 Side view of the girder including fictitious tendon profile (not to scale).

Parameters:

Density concrete	: $\rho = 25$ kN/m^3
Variable load	: $q_{Qk} = 20$ kN/m
Strength class of concrete	: C45/55
Initial tensile stress	: $\sigma_{pmo} = 0{,}75 \cdot 1860 = 1395$ N/mm^2
Elastic modulus prestressing steel	: $E_p = 195000$ N/mm^2

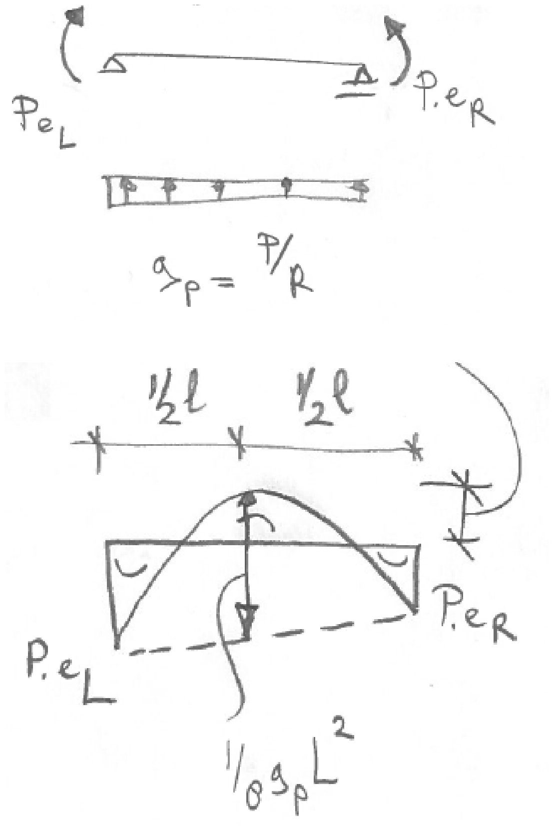
Question 14.1

Calculate and draw the bending moment diagram resulting from a prestressing force P. Assume that there is no friction and no wedge set.

Answer 14.1

Prestressing loads on the beam:
1
Concentrated bending moments from anchor eccentricity at both beam ends:
2
Uniformly distributed load from curvature pressure (upwards)

Total:



midspan cross-section:
from 1: +(0,3 + 0,2)P / 2 = +0,25P

from 2: q_p = P/R where R = 115 m
$q_pL^2/8 = -(P/115)\cdot 30^2/8 = -P/0,978$

Total: -0,728P

Question 14.2

Calculate the **maximum** initial prestressing force P_{m0} [kN] allowed to have no tension in the midspan cross-section when stressing the tendons.

Answer 14.2

G = 0,6·2,0·25 = 30 kN/m
Q = 0 kN/m (prestressing the beam!)

Bending moment from G at midspan: $ql^2 / 8 = 30\cdot 30^2 / 8 = 3375$ kNm

Check top fiber level.

Units [kN, m]:

$$-\frac{P_0}{0,6\cdot 2,0}+\frac{0,728P_0}{\frac{1}{6}\cdot 0,6\cdot 2,0^2}-\frac{3375}{\frac{1}{6}\cdot 0,6\cdot 2,0^2}\leq 0$$

$P_0 \leq 8552$ kN

Question 14.3

Calculate the **minimum** working prestressing force $P_{m\infty}$ [kN] required to have no tension in the midspan cross-section, at maximum SLS load.

Answer 14.3

G = 30 kN/m
Q = 20 kN/m

Bending moment from Q at midspan: $ql^2 / 8 = 20 \cdot 30^2 / 8 = 2250$ kNm

Check bottom fiber level.

Units [kN, m]:

$$-\frac{P_\infty}{0,6 \cdot 2,0} - \frac{0,728 P_\infty}{\frac{1}{6} \cdot 0,6 \cdot 2,0^2} + \frac{3375 + 2250}{\frac{1}{6} \cdot 0,6 \cdot 2,0^2} \leq 0$$
$$P_\infty \geq 5300 \text{ kN}$$

Question 14.4

The tendons are stressed from one end, namely at support A. Sketch the profile of the prestressing force at $t = 0$ over the full length of the beam. Mark important points/values. Assume that the friction coefficient μ = 0,30 and the wobble factor $k = 0,01$ rad/m.

Use the following expression and input:

Prestressing force including frictional loss:

$$P_\mathrm{m}(x) = P_\mathrm{m}(x = 0) \cdot e^{-\mu(\theta + kx)}$$

friction coefficient $\mu = 0,30$
wobble factor $k = 0,01$ rad/m

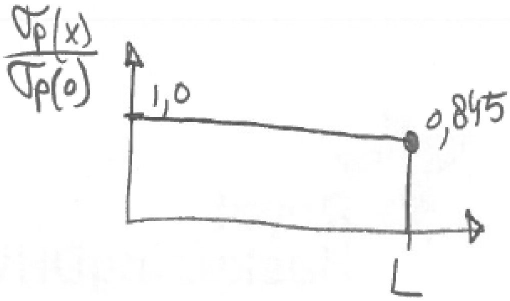
Answer 14.4

Use $\Delta\theta = \left|\frac{\Delta x}{R}\right|$

at 30 m:

$$\Delta\theta = \left|\frac{\Delta x}{R}\right| = \frac{305}{115} \text{rad}$$

$$P_\mathrm{m}(x = 30 \text{ m}) = P_\mathrm{m}(x = 0) \cdot e^{-\mu(\theta + kx)} = P_\mathrm{m}(x = 0) \cdot e^{-0,30 \cdot (30/115 + 30 \cdot 0,01)} = 0,845 P_\mathrm{m}(x = 0)$$



Question 14.5

The tendons are assumed to be stressed to a stress $\sigma_{\mathrm{p,max}} = 1395$ N/mm^2. Wedge set appears to be $w_{\mathrm{set}} = 4$ mm. Calculate the wedge set influence length and sketch the profile of the prestressing steel stress immediately after anchoring, using results from question 14.4.

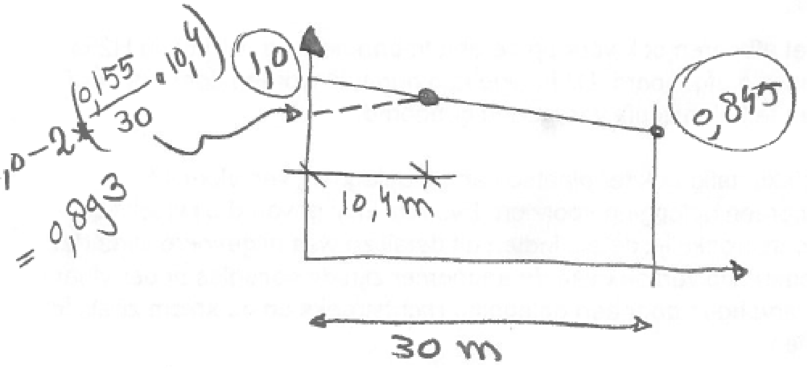
$$l_{\mathrm{set}} = \sqrt{\frac{w_{\mathrm{set}}\, E_{\mathrm{p}}}{\Delta\sigma_{\mathrm{p},\mu} / \Delta x}}$$

Answer 14.5

Units: N & mm:

$$\Delta\sigma_{\mathrm{p},\mu} / \Delta x = \frac{(1-0{,}845)\cdot\sigma_{\mathrm{p,max}}}{30\cdot10^3} = \frac{(1-0{,}845)\cdot1395}{30\cdot10^3} = 7{,}2\cdot10^{-3}\ \mathrm{N/mm^2}$$

$$l_{\mathrm{set}} = \sqrt{\frac{w_{\mathrm{set}} E_{\mathrm{p}}}{\Delta\sigma_{\mathrm{p},\mu} / \Delta x}} = \sqrt{\frac{4\cdot195000}{7{,}2\cdot10^{-3}}} = 10{,}4\cdot10^3\ \mathrm{mm}$$



Question 14.6

The beam is now analyzed at $t = \infty$.

Parameters:
Strength class of concrete : C45/55
Strength class of prestressing steel : Y1860S7

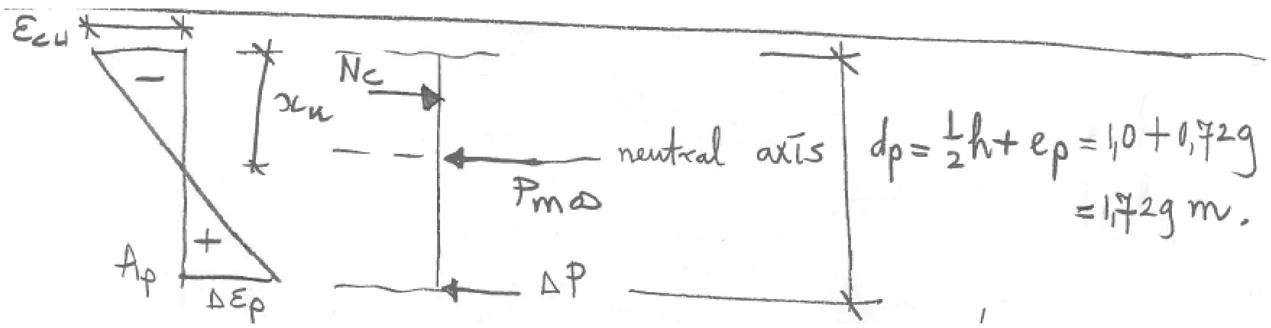
Cross-sectional area of the tendons : $A_{\mathrm{p}} = 4000$ mm^2

Working prestressing stress : $\sigma_{\mathrm{pm}\infty} = 1000$ N/mm^2

Tendons; effective depth of cross-section : d_{p} => follows from the tendon profile

The mid-span cross-section is analyzed in ULS. Bending moment capacity is calculated. Draw the strain distribution over the height of the mid-span cross-section and show the equilibrium between external and internal forces. Use the bi-linear ULS stress - strain diagram of concrete and mark important points/values.

Answer 14.6



Question 14.7

Determine the height of the concrete compression zone.

Answer 14.7

Assume that a prestressing steel stress $0{,}95 f_{\mathrm{pk}}/\gamma_{\mathrm{s}}$ is reached.

$0{,}95 f_{\mathrm{pk}}/\gamma_{\mathrm{s}} = 0{,}95 \cdot 1860/1{,}1 = 1606$ N/mm^2

$$x_{\mathrm{u}} = \frac{N_{\mathrm{cu}}}{\alpha b f_{\mathrm{cd}}} = \frac{A_{\mathrm{p}} \sigma_{\mathrm{p,ULS}}}{\alpha b f_{\mathrm{cd}}} = \frac{4000 \cdot 1606}{0,75 \cdot 600 \cdot \frac{45}{1,5}} = 476 \text{ mm}$$

Question 14.8

Check whether the actual prestressing steel stress equals the assumed stress.

Answer 14.8

Increase of prestressing steel strain:

$$\Delta\varepsilon_{\mathrm{p}} = \left(\frac{d_{\mathrm{p}} - x_{\mathrm{u}}}{x_{\mathrm{u}}}\right) \cdot 3,5 \cdot 10^{-3} = \left(\frac{1729 - 476}{476}\right) \cdot 3,5 \cdot 10^{-3} = 9,2 \cdot 10^{-3}$$

$$\varepsilon_{\mathrm{p}\infty} = \frac{1000}{195000} = 5,1 \cdot 10^{-3}$$

total strain: 14,3 ‰

$7,8\cdot 10^{-3}+\left(\dfrac{35\cdot 10^{-3}-7,8\cdot 10^{-3}}{2}\right)=21,4\cdot 10^{-3}$ required to arrive at $0,95f_{pk}/\gamma_s$. The strain is too small; actual prestressing steel stress will be smaller.

Question 14.9

Calculate the bending moment resistance M_{Rd} at mid-span.

Answer 14.9

Horizontal force equilibrium:

$N_P = N_c \approx 4000\cdot 1606$ N

Bending moment resistance (relative to centroidal axis level):

$$M_{\text{Rd}}=\left(\frac{1}{2}h-\beta x_{\text{u}}\right)N_{\text{C}}+\left(d_{\text{p}}-\frac{1}{2}h\right)\cdot\Delta N_{\text{P}}$$

$$=\left(\frac{1}{2}\cdot 2000-0,39\cdot 476\right)\cdot 6424\cdot 10^{3}+\left(1729-\frac{1}{2}\cdot 2000\right)\cdot\left(1606-1000\right)\cdot 4000$$

$$=6999\cdot 10^{6}\text{ Nmm}$$

Question 14.10

Check whether there is sufficient rotational capacity at the mid-span cross-section.

Rotational capacity:

$$\frac{x_{\text{u}}}{d}\leq\frac{500}{500+f}$$

where

$$f=\frac{\left(\dfrac{f_{\text{pk}}}{\gamma_{\text{s}}}-\sigma_{\text{pm},\infty}\right)A_{\text{p}}+f_{\text{yd}}A_{\text{s}}}{A_{\text{p}}+A_{\text{s}}}$$

Answer 14.10

$A_s = 0$

$$f=\frac{\left(\dfrac{f_{\text{pk}}}{\gamma_{\text{s}}}-\sigma_{\text{pm},\infty}\right)A_{\text{p}}+f_{\text{yd}}A_{\text{s}}}{A_{\text{p}}+A_{\text{s}}}=\frac{\left(\dfrac{1860}{1,1}-1000\right)\cdot 4000+0}{4000+0}=691\text{ N/mm}^{2}$$

$$\frac{476}{1729}?\leq ?\frac{500}{500+691}$$

$$0,28!\leq !0,42$$

OK

Example 15 - Prestressed concrete

A post-tensioned beam has an $\ell = 16$ m span, see figure 15.1. The beam has a **T-shaped** cross-section, see figure 15.2. The beam is prestressed using bonded tendons. The fictitious tendon profile is modeled using two kinks.

Beam span: $\ell = 16$ m ($\ell_1 = 8$ m, $\ell_2 = 4$ m).
Tendon eccentricities at the anchors: $e_{span} = e_{p1} = 0{,}5$ m; $e_{support} = e_{p2} = 0{,}2$ m.

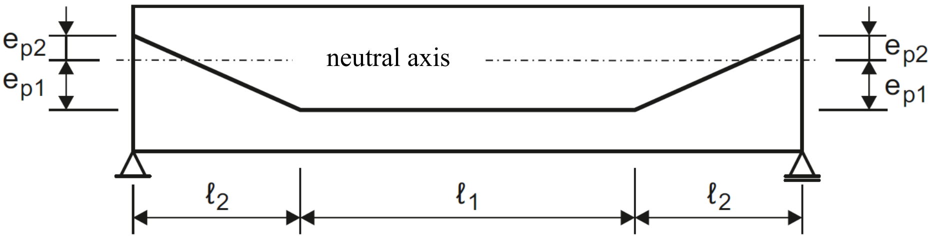


Figure 15.1 Side view of the girder including fictitious tendon profile (not to scale).

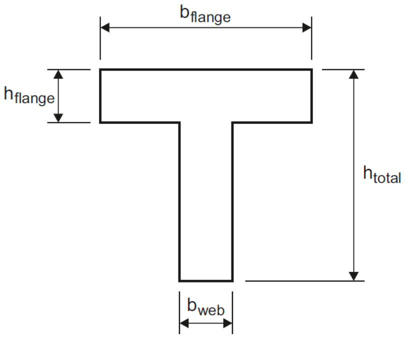


Figure 15.2 Cross-section of the girder.

Parameters:

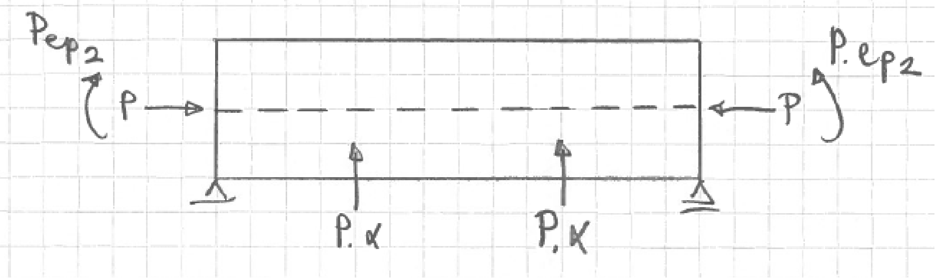
h_{total}	: 1,20 m
h_{flange}	: 0,25 m
b_{flange}	: 0,70 m
b_{web}	: 0,30 m
Cross-sectional area girder	: $A_c = 0{,}46$ m^2
Moment of inertia	: $I_c = 61{,}4 \cdot 10^{-3}$ m^4
Distance from neutral axis to top fibre	: $z_t = 0{,}497$ m
Distance from neutral axis to bottom fibre	: $z_b = 0{,}703$ m
Density concrete	: $\rho = 25$ kN/m^3
Variable load	: $q_{Qk} = 20$ kN/m
Concrete modulus of elasticity	: 30000 N/mm^2
Creep coefficient	: $\varphi = 1{,}8$
Deformation caused by shrinkage	: $\varepsilon_{cs} = 0{,}20$ ‰
Strength class of prestressing steel	: Y1860
Prestressing steel modulus of elasticity	: 195000 N/mm^2

Question 15.1

Show in a figure all the loads on the beam from a prestressing force *P*. Assume that there is no friction and no wedge set.

Answer 15.1

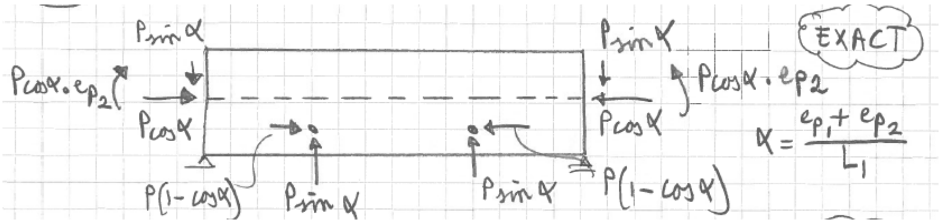
Engineering model:



where:

$$\alpha = \frac{e_{\mathrm{p2}} + e_{\mathrm{p1}}}{l_2}$$

Additional information (exact model)



Question 15.2

Calculate the bending moment from selfweight (*G*) and variable load (*Q*) at midspan (i.e. in the midspan cross-section), at maximum SLS load.

Answer 15.2

G + Q = (25·0,46 + 20) = 11,5 + 20 = 31,5 kN/m
$M_{\mathrm{G+Q}} = (G+Q)l^2/8 = 31{,}5 \cdot 16^2/8 = 1008$ kNm

Question 15.3

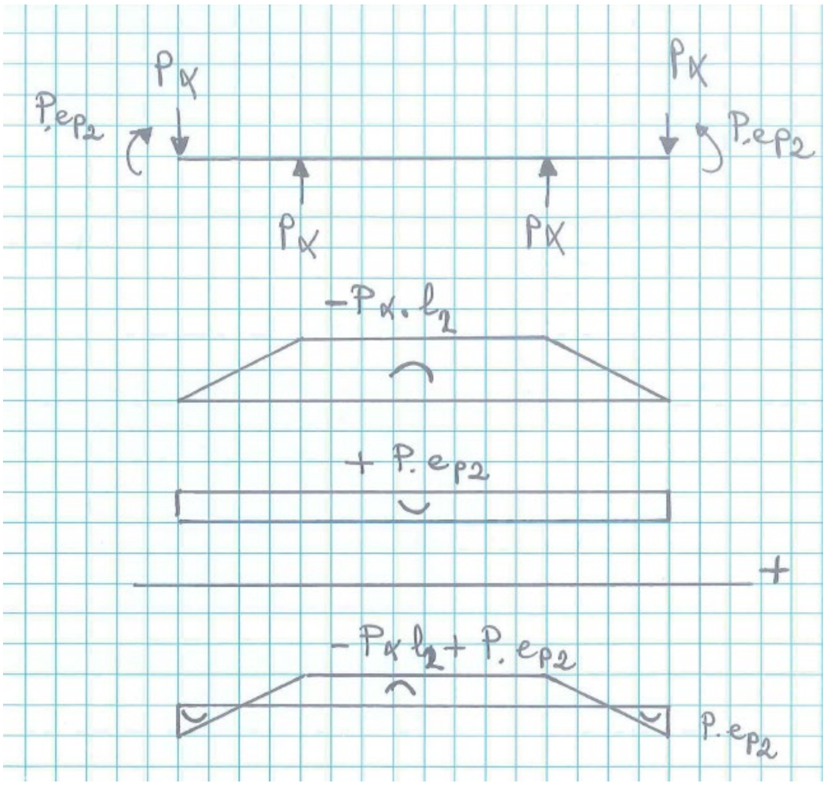
Calculate the **minimum** working prestressing force $P_{\mathrm{m}\infty}$ [kN] required to have no tension at midspan (i.e. in the midspan cross-section) at maximum SLS load.

Answer 15.3

Bending moment caused by prestressing:

Two components:
1
Anchor eccentricities.
2

Point loads at kinks (upwards).



Bending moment at midspan position:

$$-P\alpha l_2 + Pe_{\mathrm{p2}} = -P\left(\frac{e_{\mathrm{p2}} + e_{\mathrm{p1}}}{l_2}\right)l_2 + Pe_{\mathrm{p2}} = -Pe_{\mathrm{p1}}$$

Check bottom fiber level.

Units [kN, m]:

$$-\frac{P}{0,46} - \frac{0,5P\cdot 0,703}{61,4\cdot 10^{-3}} + \frac{1008\cdot 0,703}{61,4\cdot 10^{-3}} \le 0$$

$$P \ge 1461\,\mathrm{kN}$$

Question 15.4

A structural engineer decides to apply a prestressing force P_{m0} = 1746 kN. Calculate the concrete stress at prestressing steel level at midspan (i.e. in the midspan cross-section), at maximum SLS load. Assume that there is no friction and no wedge set.

Answer 15.4

Concrete stress at prestressing steel level:

Bending moment from G and Q: 1008 kNm

Units [kN, m]:

$$\sigma_c = -\frac{1746}{0,46} - \frac{(0,5\cdot 1746)\cdot 0,5}{61,4\cdot 10^{-3}} + \frac{1008\cdot 0,5}{61,4\cdot 10^{-3}} = -3796 + 1099 = -2697 \text{ kN/m}^2 = -2,7 \text{ N/mm}^2$$

Question 15.5

A prestressing force P_{m0} = 1746 kN is used. Calculate the time-dependent prestress losses due to shrinkage, creep and relaxation in N/mm^2 at $t = \infty$ at midspan (i.e. in the midspan cross-section). The tendons are assumed to be stressed to an initial stress σ_{pi} = 1200 N/mm^2.

The equation for the calculation of relaxation is:

$$\frac{\Delta\sigma_{pr}}{\sigma_{pi}} = 0,66\,\rho_{1000}\,e^{(9,1\,\mu)}\left(\frac{t}{1000}\right)^{(0,75\,(1-\mu))}\cdot 10^{-5}$$

where:
$\Delta\sigma_{pr}$: is absolute value of the relaxation losses of the prestressing steel
σ_{pi} : is the initial stress in the prestressing steel
t : is the time after tensioning ($t = \infty$ = 500000 hours)
μ : = σ_{pi} / f_{pk}, where f_{pk} is the characteristic value of the tensile strength of the prestressing steel
ρ_{1000} : is the value of relaxation loss (in %), at 1000 hours after tensioning and at a mean temperature of 20°C
ρ_{1000} : = 2,5%

The influence of shrinkage and creep reduces the relaxation. When taking into account shrinkage and creep losses, relaxation loss is: $\Delta\sigma_{pr(incl,c+s)} = 0,8\,\Delta\sigma_{pr(excl,c+s)}$

Answer 15.5

Concrete strain at prestressing steel level:

ε_c = -2,7/30000

Creep:

$$1{,}8\frac{-2{,}7}{30000}E_p = -32 \text{ N/mm}^2$$

Shrinkage:

$$-200\cdot 10^{-6}\cdot E_p = -39 \text{ N/mm}^2$$

Relaxation:

$$\sigma_{pi} = 1200 \text{ N/mm}^2$$

$$\mu = \frac{1200}{1860} = 0,645$$

$$\frac{\Delta\sigma_{pr}}{\sigma_{pi}} = 0,66\cdot 2,5\cdot e^{9,1\cdot 0,645}\cdot 500^{0,75\cdot(1-0,645)}\cdot 10^{-5} = 0,03$$

$$\Delta\sigma_{pr} = 0,03\cdot 1200 = 36 \text{ N/mm}^2$$

Total loss: 32 + 39 + 0,8·36 = 100 N/mm^2

Example 16 - Prestressed concrete

A post-tensioned beam has an ℓ = 14 m span, see Figure 16.1. The beam has a rectangular cross-section; beam height is 0,78 m and beam width is 0,35 m. The beam is prestressed using tendons. The fictitious tendon profile is modeled using a parabola (radius of curvature R = 25 m) over ℓ_1 = 4 m at midspan and two straight parts of ℓ_2 = 5 m each.

Beam span: ℓ = 14 m (ℓ_1 = 4 m, ℓ_2 = 5 m)

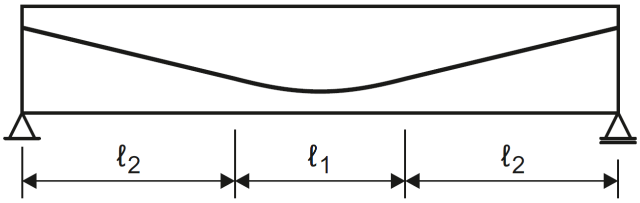


Figure 16.1 Side view of the girder including fictitious tendon profile (not to scale).

Question 16.1

Calculate the drape of the parabola part of the tendon profile.

Answer 16.1

parabola expression: $f = \frac{l^2}{8R} = \frac{(4000)^2}{8 \cdot 25000} = 80 \text{ mm}$

Question 16.2

Calculate the position/height of the anchors, given that (1) the bottom of the parabola part of the tendon profile is at 80 mm from the cross-section's bottom fibre and (2) there are no kinks in the tendon profile.

Answer 16.2

$$y = \frac{x^2}{2R}$$

parabola: $\frac{dy}{dx} = \frac{x}{R}$

$$x = 2 \text{ m}: \frac{dy}{dx} = \frac{2}{25}$$

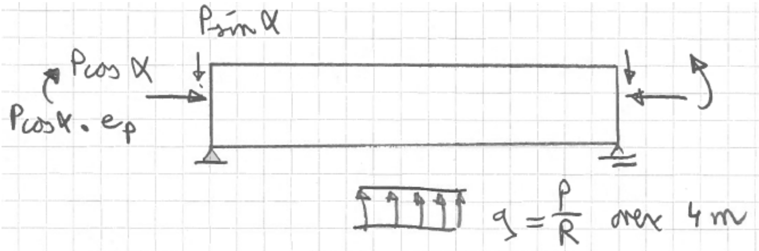
Vertical distance over l_2: (2/25)·5 = 0,4 m

Distance relative to bottom fiber: 0,08 + 0,4 = 0,48 m => 0,09 m above centroidal axis level.

Question 16.3

Show in a figure all the loads on the beam from a prestressing force P. Assume that there is no friction and no wedge set.

Answer 16.3



where

e_P = 0,09 m and α = 2/25 = 0,4/5 = 0,08 rad.

Question 16.4

The tendons are stressed from **one** end, namely the left support. Sketch the profile of the prestressing force at t = 0 over the full length of the beam. Mark important points/values. Use the following expression and input:

Prestressing force including frictional loss:
$P_m(x) = P_m(x = 0) \cdot e^{-\mu(\theta + kx)}$

friction coefficient $\mu = 0{,}2$
wobble factor $k = 0{,}01$ rad/m

Answer 16.4

Use $\Delta\theta = \left|\frac{\Delta x}{R}\right|$

x = 0: $P_m(0)$

x = 5 m:

$\Delta\theta = 0$ (no curvature)

$$P_m(5\text{ m}) = P_m(0) \cdot e^{-\mu(\theta + kx)} = P_m(0) \cdot e^{-0{,}2\cdot(0+5\cdot 0{,}01)} = 0{,}99 P_m(0)$$

x = 9 m:

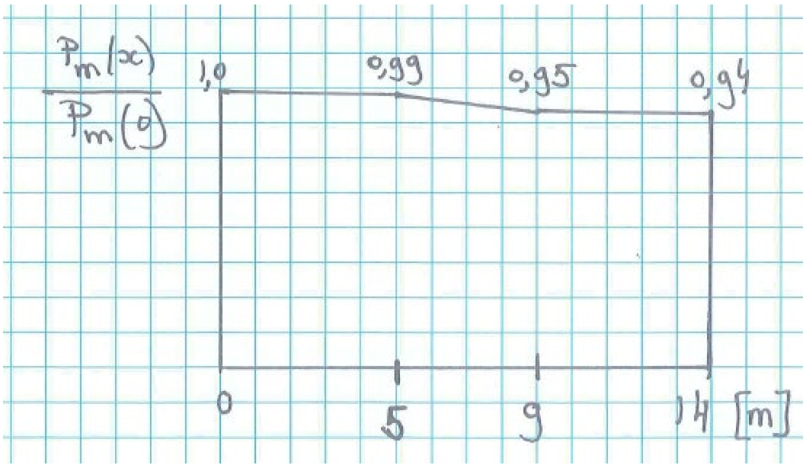
$$\Delta\theta = 0 + \frac{4}{25} \text{ (total)}$$

$$P_m(9\text{ m}) = P_m(0) \cdot e^{-\mu(\theta + kx)} = P_m(0) \cdot e^{-0{,}2\cdot(4/25+9\cdot 0{,}01)} = 0{,}95 P_m(0)$$

x = 14 m:

$$\Delta\theta = 0 + \frac{4}{25} + 0 \text{ (total)}$$

$$P_{\mathrm{m}}\left(14\ \mathrm{m}\right)=P_{\mathrm{m}}\left(0\right)\cdot e^{-\mu\left(\theta+kx\right)}=P_{\mathrm{m}}\left(0\right)\cdot e^{-0,2\cdot\left(4/25+14\cdot0,01\right)}=0,94P_{\mathrm{m}}\left(0\right)$$



Example 17 - Prestressed concrete

A pre-tensioned double T girder with a span of 20 m (see figure 17.1) is fully prestressed by 24 linear tendons. The fictitious tendon is positioned 150 mm from the bottom as shown in figure 17.1. The production and the construction phase are considered, both without a top layer.

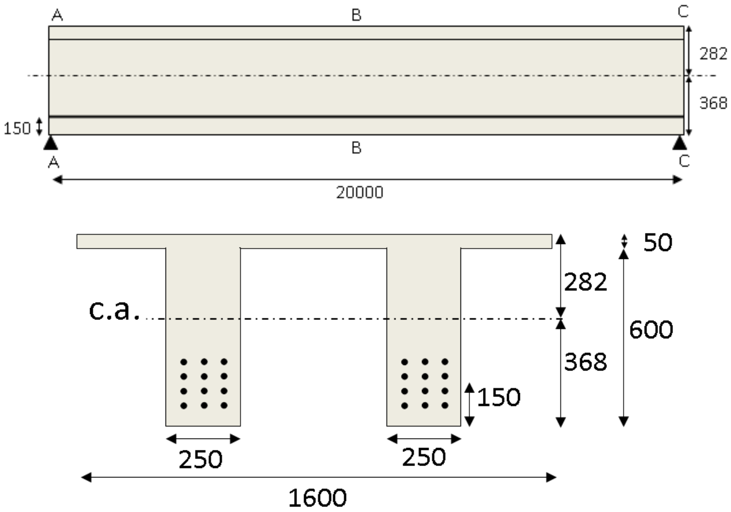


Figure 17.1 Overview of the girder including fictitious tendon profile and cross section B (midspan)(tendons are indicative) (dimensions [mm])

Parameters:

Density concrete : $\rho = 25$ kN/m^3

Variable load : $q_{Qk} = 10$ kN/m

Dimensions girder : see figure 17.1

Cross-sectional area girder : $A_c = 0{,}38$ m^2

Moment of inertia : $I_c = 0{,}0157$ m^4

Distance from centroidal axis to top fibre : $z_t = 0{,}282$ m

Distance from centroidal axis to bottom fibre : $z_b = 0{,}368$ m

Section modulus (top fibre) : $W_t = 0{,}0557$ m^3

Section modulus (bottom fibre) : $W_b = 0{,}0426$ m^3

Strength class of concrete : C45/55

Elastic modulus concrete : 36000 N/mm^2

Strength class of prestressing steel : Y1860S7

Cross-section of one strand (12.7 mm) : 98,71 mm^2

Initial tensile stress : $\sigma_{pmo} = 0.75 \cdot 1860 = 1395$ N/mm^2

Elastic modulus prestressing steel : 195000 N/mm^2

Partial load factors : $\gamma_G = 1{,}2$: $\gamma_Q = 1{,}5$

Creep coefficient : $\varphi = 2{,}0$

Deformation caused by shrinkage : $\varepsilon_{cs} = 0{,}24$ ‰

Question 17.1

Calculate the maximum initial prestress force P_{m0} [kN] required in cross-section B to fully prestress the girder. Governing situations to be calculated are (at tendon release): 1) no tensile stresses at t = 0 and 2) maximum concrete compressive stress at t = 0 (allowed: $0{,}6f_{ck}$).

Answer 17.1

load: $G = 0{,}38 \cdot 25 = 9$ kN/m

Selfweight and prestressing only.

Tendon eccentricity:
368 - 150 = 218 mm

Prestressing loads:
1
Axial compressive force P.
2
Concentrated bending moment at both anchors; $P \cdot e_p$ where $e_p = 218$ mm

Bending moment is constant over beam length; $P \cdot e_p$

Bending moment G: $ql^2/8 = 9 \cdot 20^2/8 = 450$ kNm

Check top fiber level.

Units [kN, m]

$$-\frac{P_{\text{m0}}}{0,38}+\frac{0,218P_{\text{m0}}}{0,0557}-\frac{450}{0,0557}\leq 0$$
$$P_{\text{m0}}\leq 6301\,\text{kN}$$

Check bottom fiber level.

Units [kN, m]

$$-\frac{P_{\text{m0}}}{0,38}-\frac{0,218P_{\text{m0}}}{0,0426}+\frac{450}{0,0426}\geq -0,6f_{\text{ck}}=-0,6\cdot 45000$$
$$7,75P_{\text{m0}}\geq -37563\,\text{kN}$$
$$P_{\text{m0}}\leq 4847\,\text{kN}$$

Check bottom fiber level.

Units [kN, m]

$$-\frac{P_{\text{m0}}}{0,38}-\frac{0,218P_{\text{m0}}}{0,0426}+\frac{450}{0,0426}\leq 0$$
$$7,75P_{\text{m0}}\leq -10563$$
$$P_{\text{m0}}\geq 1363\,\text{kN}$$

Question 17.2

Calculate the distance over which prestress losses occur at t = 0 and sketch the profile of the prestressing stress, in N/mm^2, in the linear tendons at t = 0 over the full length of the beam. Mark important points/values.

Equations for anchorage of pre-tensioned steel

$$l_{pt} = \alpha_1 \; \alpha_2 \; \phi \frac{\sigma_{pm0}}{f_{bpt}}$$

α_1 1,25 for sudden release

α_2 0,19 for 3 and 7-wire strands

ϕ is the nominal diameter of the strand

σ_{pm0} is the strand stress just after release

$f_{bpt} = 3{,}9 \text{ N/mm}^2$

$$l_{disp} = \sqrt{l_{pt}^2 + d^2}$$

$$l_{pt1} = 0{,}8\, l_{pt} \text{ or } l_{pt2} = 1{,}2\, l_{pt}$$

$$l_{bpd} = l_{pt2} + \frac{\alpha_2 \; \phi \; (\sigma_{pd} - \sigma_{pm\infty})}{f_{bpd}}$$

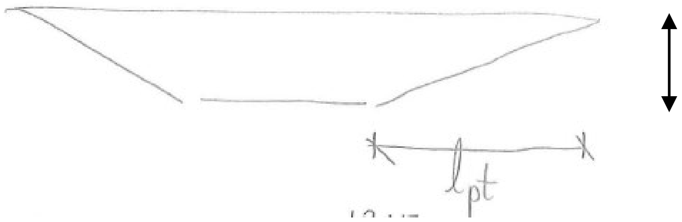
σ_{pd} is the tendon stress corresponding to the force in the cracked cross-section

$\sigma_{pm\infty}$ is the stress after taking into account all losses

$f_{bpd} = 2.2 \text{ N/mm}^2$

Answer 17.2

$$l_{pt} = \alpha_1 \; \alpha_2 \; \phi \frac{\sigma_{pm0}}{f_{bpt}} = 1{,}25 \cdot 0{,}19 \cdot 12{,}7 \cdot \frac{1395}{3{,}9} = 1079 \text{ mm}$$



1395 N/mm²; losses due to elastic shortening of the concrete not taken into account.

Question 17.3

The elastic losses in the prestressing steel are caused by an average concrete compressive stress (at the fictitious tendon level) of -5,0 N/mm².

Calculate the average stress loss in the prestressing steel [N/mm²] due to elastic deformation. Explain briefly whether or not this loss can be compensated for; and if so, how this should be done.

Answer 17.3

Concrete strain:
$\varepsilon_{c0} = -5/36000$

$$(-5/36000) \cdot E_p = -27 \text{ N/mm}^2$$

Stress loss can be compensated for by overstressing the tendons (initial stress level > 1395 N/mm²)

Question 17.4

Calculate the time dependent prestress losses due to shrinkage, creep and relaxation in N/mm^2 at t = ∞ in cross-section B.

The equation for the calculation of relaxation is:

$$\frac{\Delta\sigma_{pr}}{\sigma_{pi}} = 0,66\ \rho_{1000}\ e^{(9,1\,\mu)} \left(\frac{t}{1000}\right)^{(0,75\,(1-\mu))} \cdot 10^{-5}$$

where: $\Delta\sigma_{pr}$: is absolute value of the relaxation losses of the prestressing steel

σ_{pi} : is the initial stress in the prestressing steel

t : is the time after tensioning ($t = \infty = 500000$ hours)

μ : $= \sigma_{pi} / f_{pk}$, where f_{pk} is the characteristic value of the tensile strength of the prestressing steel

ρ_{1000} : is the value of relaxation loss (in %), at 1000 hours after tensioning and at a mean temperature of 20°C

ρ_{1000} : = 2,5%

The influence of shrinkage and creep reduces the relaxation. When taking into account shrinkage and creep losses, relaxation loss is: $\Delta\sigma_{\text{pr(incl,c+s)}} = 0,8\ \Delta\sigma_{\text{pr(excl,c+s)}}$

Answer 17.4

Prestressing steel stress directly after tendon release (tendon shortening due to concrete shortening): 1395 - 27 = 1368 N/mm^2.

Elastic concrete strain at prestressing steel level:

$\varepsilon_c = -5/36000$

Creep:

$$2,0 \cdot \left(\frac{-5}{36000}\right) \cdot E_p = -54 \text{ N/mm}^2$$

Shrinkage:

$$-240 \cdot 10^{-6} \cdot E_p = -47 \text{ N/mm}^2$$

Relaxation:

$$\sigma_{pi} = 1368 \text{ N/mm}^2$$

$$\mu = \frac{1368}{1860} = 0,74$$

$$\frac{\Delta\sigma_{pr}}{\sigma_{pi}} = 0,66 \cdot 2,5 \cdot e^{9,1 \cdot 0,74} \cdot 500^{0,75 \cdot (1-0,74)} \cdot 10^{-5} = 0,047$$

$$\Delta\sigma_{pr} = 0,047 \cdot 1368 = 64 \text{ N/mm}^2$$

Total loss: 54 + 47 + 0,8·64 = 152 N/mm^2

Question 17.5

Explain briefly whether casting a concrete top layer at t = 50 days will have a positive, negative or no effect on each of these losses (question 17.5) and why.

Answer 17.5

Shrinkage: No effect
Creep: Concrete compressive stress at prestressing steel level increases (> -5 N/mm^2) => positive effect.
Relaxation: No effect.

Question 17.6

The double T girder (see figure 17.2) is now considered at t = ∞. At t = 50 days a structural concrete top layer has been cast. Top layer thickness is *x* mm. The fictitious tendon is at 150 mm from the bottom fiber level.

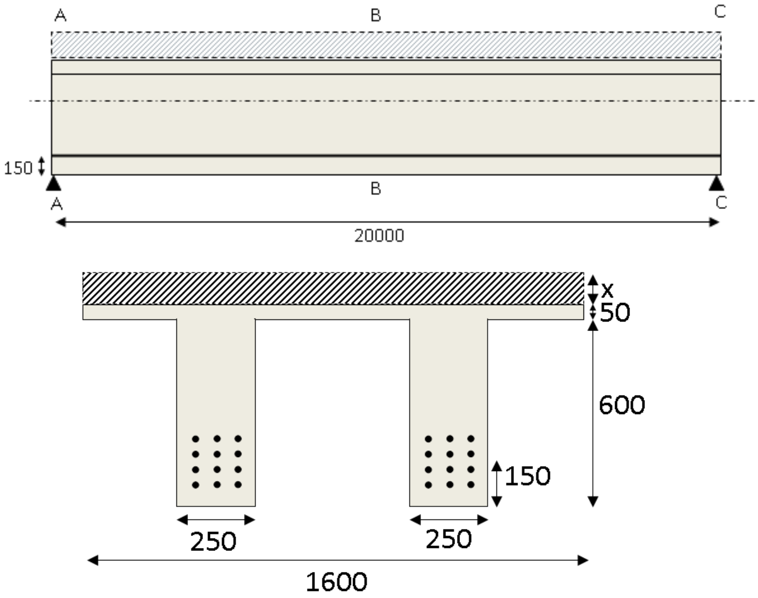


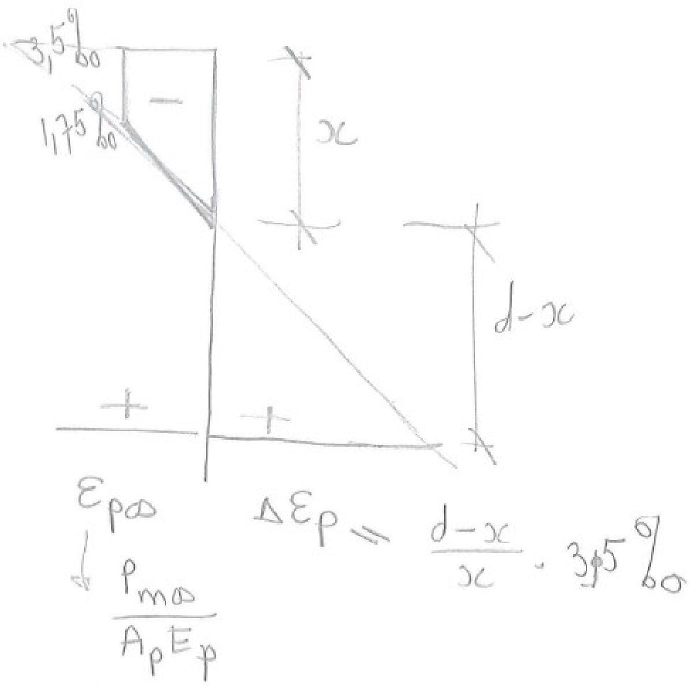
Figure 17.2 Overview of the girder including fictitious tendon profile and cross section B (tendons are indicative).

Parameters:
Density concrete : $\rho = 25$ kN/m^3
Variable load : $q_{Qk} = 10$ kN/m
Dimensions girder : see figure 17.2
Strength class of concrete : C45/55
Elastic modulus concrete : 36000 N/mm^2
Strength class of prestressing steel : Y1860S7
Number of strands : 24
Center of gravity of fictitious tendon : 150 mm from bottom fibre
Cross-section of one strand (12.7 mm) : 98,71 mm^2
Initial tensile stress : $\sigma_{pmo} = 0{,}75 \cdot 1860 = 1395$ N/mm^2
Elastic modulus prestressing steel : 195000 N/mm^2
Partial load factors : $\gamma_G = 1{,}2$: $\gamma_Q = 1{,}5$

Twenty-four tendons are applied ($A_p = 2369$ mm^2). At t = ∞ the prestressing force $P_{m,\infty}$ is assumed to be 2930 kN.

Draw the ULS strain distribution over the height of cross section B in combination with the equilibrium between external and internal forces in cross section B. Use the bi-linear concrete stress strain relationship and mark important points/values.

Answer 17.6



Question 17.7

Determine, from the force equilibrium in cross-section B, the minimum required height of the top layer [mm] to be cast on the double T girder to assure that the concrete compressive zone is in the flange. Assume that $\sigma_{pu} = 0.95 f_{pk}/\gamma_s$.

Answer 17.7

$0{,}95 f_{pk}/\gamma_s = 0{,}95 \cdot 1860/1{,}1 = 1606$ N/mm^2

$$x_u = \frac{N_{cu}}{\alpha b f_{cd}} = \frac{A_p \sigma_{p,ULS}}{\alpha b f_{cd}} = \frac{2369 \cdot 1606}{0{,}75 \cdot 1600 \cdot \frac{45}{1{,}5}} = 106 \text{ mm}$$ => 106 - 50 = min. 56 mm top layer required;

100 mm is OK.

Question 17.8

A 100 mm top layer has been cast on the double T girder. This results in the following properties:

Cross-sectional area girder : $A_c = 0{,}54$ m^2
Moment of inertia : $I_c = 0{,}0282$ m^4
Distance from centroidal axis to top fibre : $z_t = 0{,}283$ m
Distance from centroidal axis to bottom fibre : $z_b = 0{,}467$ m
Section modulus (top fibre) : $W_t = 0{,}0995$ m^3
Section modulus (bottom fibre) : $W_b = 0{,}0604$ m^3

Check whether $M_{\mathrm{Rd}} \geq M_{\mathrm{Ed}}$ at midspan B.

Answer 17.8

Bending moment resistance (relative to centroidal axis level):

$$\begin{aligned}
M_{\mathrm{Rd}} &= \left(z_{\mathrm{t}} - \beta x_{\mathrm{u}}\right) N_{\mathrm{C}} + \left(e_{\mathrm{p}}\right) \cdot \Delta N_{\mathrm{P}} \\
&= \left(283 - 0{,}39 \cdot 106\right) \cdot 2369 \cdot 1606 \cdot 10^{3} + \left(467 - 150\right) \cdot \left(1606 - 1395\right) \cdot 2369 \\
&= (919 + 158) \cdot 10^{6} = 1077 \cdot 10^{6} \text{ Nmm} = 1077 \text{ kNm}
\end{aligned}$$

$$\begin{aligned}
M_{\mathrm{Ed}} &= \frac{1}{8}\left(1{,}2 \cdot 0{,}54 \cdot 25 + 1{,}5 \cdot 10\right) \cdot 20^{2} - e_{\mathrm{p}} P_{\mathrm{m}\infty} \\
&= 1560 - 2369 \cdot 1395 \cdot \left(467 - 150\right) \cdot 10^{-6} \\
&= 1560 - 1047 = 513 \text{ kNm}
\end{aligned}$$

Result: $M_{\mathrm{Rd}} \geq M_{\mathrm{Ed}}$; OK.

Question 17.9

Check the assumption of question 17.8. Hint: Use the strain distribution over the height at cross-section B.

Answer 17.9

Increase of prestressing steel strain:

$$\Delta \varepsilon_{\mathrm{p}} = \left(\frac{d_{\mathrm{p}} - x_{\mathrm{u}}}{x_{\mathrm{u}}} \right) \cdot 3{,}5 \cdot 10^{-3} = \left(\frac{600 - 106}{106} \right) \cdot 3{,}5 \cdot 10^{-3} = 16{,}3 \cdot 10^{-3}$$

$$\varepsilon_{\mathrm{p}\infty} = \frac{2930 \cdot 10^{3}}{2369 \cdot 195000} = 6{,}3 \cdot 10^{-3}$$

total strain: 22,6 ‰

$7{,}8 \cdot 10^{-3} + \left(\frac{35 \cdot 10^{-3} - 7{,}8 \cdot 10^{-3}}{2} \right) = 21{,}4 \cdot 10^{-3}$ is required to arrive at $0{,}95 f_{\mathrm{pk}}/\gamma_{\mathrm{s}}$. The strain is a bit greater; estimated prestressing steel stress is OK.

Example 18 – Reservoir and crack width

A water tank comprises a foundation slab and on top a circular reinforced concrete tank wall with a thickness of 150 mm. The inner diameter of the wall is 18 meters and the height is *x* meters (to be determined). The tank wall is reinforced with two layers of bars with diameter 12 mm and a center to center distance of 75 mm. There is a sliding connection without friction between the tank wall and the foundation, see Figure 18.1.

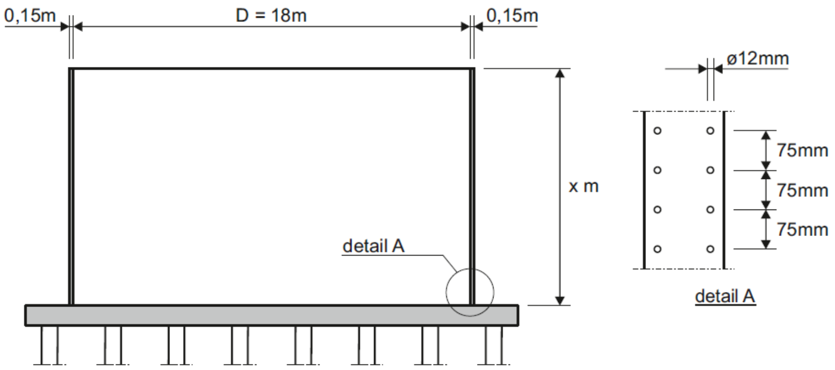


Figure 18.1 Cross-section of the water tank

Concrete strength class : C25/30
Density concrete : $\rho = 25$ kN/m^3
Reinforcement class B500B : $f_{yd} = 435$ N/mm^2
Modulus of elasticity : $E_s = 200000$ N/mm^2
Concrete wall
  thickness : 150 mm
  concrete cover : 25 mm
  allowable crack width : 0,20 mm
: $h_{c.eff} = 2{,}5\,(h - d)$
  mean concrete tensile strength: $f_{ctm} = 2{,}6$ N/mm^2
  bond strength : $\tau_{bm} = 2\,f_{ctm}$
Concrete modulus of elasticity : $E_{cm} = 31000$ N/mm^2
Partial load factors : $\gamma_G = 1{,}2$
: $\gamma_Q = 1{,}5$

Question 18.1

Calculate the maximum tensile force resistance per meter (N_{Rd}) of the tank wall in the ultimate limit state when the wall is reinforced with 2 layers of 12 mm bars at a center to center spacing of 75 mm.

Answer 18.1

Amount of reinforcement:

$$a_s = 2 \cdot \frac{1}{4}\pi \cdot 12^2 \cdot \frac{1000}{75} = 3013\ \text{mm}^2/\text{m}$$

Resistance (ULS):

$$N_{\text{Rd}} = a_s f_{\text{yd}} = 3013 \cdot 435 = 1311 \cdot 10^3\ \text{N/m} = 1311\ \text{kN/m}$$

Question 18.2

Calculate the maximum height of the tank wall in the ultimate limit state if the tank is completely filled with water. Hint: Express the tensile force over 1 meter height in the tank wall as a function of the water height x, keeping the maximum tensile force constant over 1 meter and assume $N_{Rd} = N_{Ed}$.

Answer 18.2

Hoop force (ring force) from the hydrostatic water pressure: $N = qR$.
Maximum design hoop force is at the bottom: $N_{Ed,max} = \gamma_Q \, h \, \rho_{water} \, R$.

$N_{Ed,max} = 1{,}5 \cdot h \cdot 10 \cdot 9 = 135h$ units kN &m

$N_{Ed,max} = N_{Rd} \Rightarrow 135h = 1311 \Rightarrow h = 9{,}7$ m

Question 18.3

Calculate the crack width w_{max} and check if the crack width is smaller than 0,2 mm.

Crack width control:

$$w_{max} = \frac{1}{2} \frac{f_{ctm}}{\tau_{bm}} \frac{Ø}{\rho_{s,eff}} \frac{1}{E_s} \left(\sigma_s - \alpha \, \sigma_{sr} \right)$$

$$\sigma_{sr} = \frac{N_{crack}}{A_s}$$

Table 15,III Values for τ_{bm}, α and β from eq. (15.15) for various conditions. The values for α between brackets are the recalibrated values as applied in the Eurocode by means of the coefficient k_t (EN 1992-1-1 eq. (7.9))

	crack formation stage	stabilized cracking stage
Short term loading	α = 0,5 (0,6) β = 0 $\tau_{bm} = 2{,}0 \, f_{ctm}$	α = 0,5(0,6) β = 0 $\tau_{bm} = 2{,}0 \, f_{ctm}$
long term or dynamic loading	α = 0,5 (0,6) β = 0 $\tau_{bm} = 1{,}6 \, f_{ctm}$	α = 0,3 (0,4) β = 1 $\tau_{bm} = 2{,}0 \, f_{ctm}$

Answer 18.3

Concrete cover = 25 mm
$h - d = 25 + Ø/2 = 25 + 12/2 = 31$ mm
$h_{c.eff} = 2{,}5 \, (h - d) = 78$ mm

$2h_{c.eff}$ = 156 mm > wall thickness = 150 mm => $h_{c.eff}$ zones overlap => effective tensile member width = $h/2$ = 75 mm

Bars: Ø12-75

$$\rho_{\mathrm{s,eff}} = \frac{\frac{1}{4}\pi \cdot 12^2}{75 \cdot 75} = 0,02$$

$$\rho = \frac{\frac{1}{4}\pi \cdot 12^2}{75 \cdot 75} = 0,02$$

Hoop force (ring force) from the hydrostatic water pressure: $N = qR$.
SLS check: Maximum hoop force is at the bottom: $N_{\mathrm{E,max}} = h\, \rho_{\mathrm{water}}\, R$.

$N_{\mathrm{E,max}} = 9{,}7 \cdot 10 \cdot 9 = 873$ kN/m

Steel stress in SLS: $\sigma_{\mathrm{s,max}} = \frac{N_{\mathrm{E,max}}}{a_{\mathrm{s}}} = \frac{873 \cdot 10^3}{3013} = 290\ \mathrm{N/mm^2}$

Note: This result is as expected. It's SLS, so no load factor applied. ULS steel stress 435 N/mm^2 now is 435/1,5 = 290 N/mm^2.

Steel stress directly after cracking:

$$\sigma_{\mathrm{sr}} = f_{\mathrm{ctm}}\left(1 + \frac{E_{\mathrm{s}}}{E_{\mathrm{c}}}\rho\right)\frac{1}{\rho} = 2,6 \cdot \left(1 + \frac{200}{31} \cdot 0,02\right) \cdot \frac{1}{0,02} = 147\ \mathrm{N/mm^2}$$

Maximum crack width:

Assume long term loading:

$$w_{\max} = \frac{1}{2}\frac{f_{\mathrm{ctm}}}{\tau_{\mathrm{bm}}}\frac{Ø}{\rho_{\mathrm{s,eff}}}\frac{1}{E_{\mathrm{s}}}\left(\sigma_{\mathrm{s}} - \alpha\,\sigma_{\mathrm{sr}}\right) = \frac{1}{2} \cdot \frac{2,6}{2 \cdot 2,6} \cdot \frac{12}{0,02} \cdot \frac{1}{200 \cdot 10^3} \cdot \left(290 - 0,3 \cdot 147\right) = 0,18\ \mathrm{mm}$$

Smaller than 0,2 mm; OK.

Question 18.4

Is the calculated crack width in the phase "fully developed crack pattern" or not?

Answer 18.4

$N_{\mathrm{E,max}} = 873$ kN/m

Concrete tensile stress assuming that the concrete is uncracked in SLS:

$$\sigma_{\text{c,max}} = \frac{N_{\text{E,max}}}{a_c} = \frac{873 \cdot 10^3}{1000 \cdot 150} = 5,8 \text{ N/mm}^2 >> \sigma_{\text{cr}}$$

Fully developed crack pattern (or: stabilized cracking stage) since $N_E > N_{crack}$.

Question 18.5

Calculate the crack width when the bars are replaced by 16 mm bars without changing the cross-sectional area of steel per meter tank wall (A_s/m is kept constant).

Answer 18.5

Ø12 mm bars (from Answer 18.4):

$$w_{\max} = \frac{1}{2}\frac{f_{\text{ctm}}}{\tau_{\text{bm}}}\frac{\text{Ø}}{\rho_{\text{s,eff}}}\frac{1}{E_{\text{s}}}\left(\sigma_{\text{s}} - \alpha\,\sigma_{\text{sr}}\right) = \frac{1}{2} \cdot \frac{2,6}{2 \cdot 2,6} \cdot \frac{12}{0,02} \cdot \frac{1}{200 \cdot 10^3} \cdot \left(290 - 0,3 \cdot 147\right) = 0,18 \text{ mm}$$

Now apply Ø16 mm bars; same amount of reinforcement.

The only variable that changes is the bar diameter Ø:

$$w_{\max} = \frac{16}{12} \cdot 0,18 = 0,24 \text{ mm}$$

Question 18.6

Is the crack spacing (crack distance) larger or smaller in case 16 mm bars are used instead of 12 mm bars, given that A_s/m is constant?

Answer 18.6

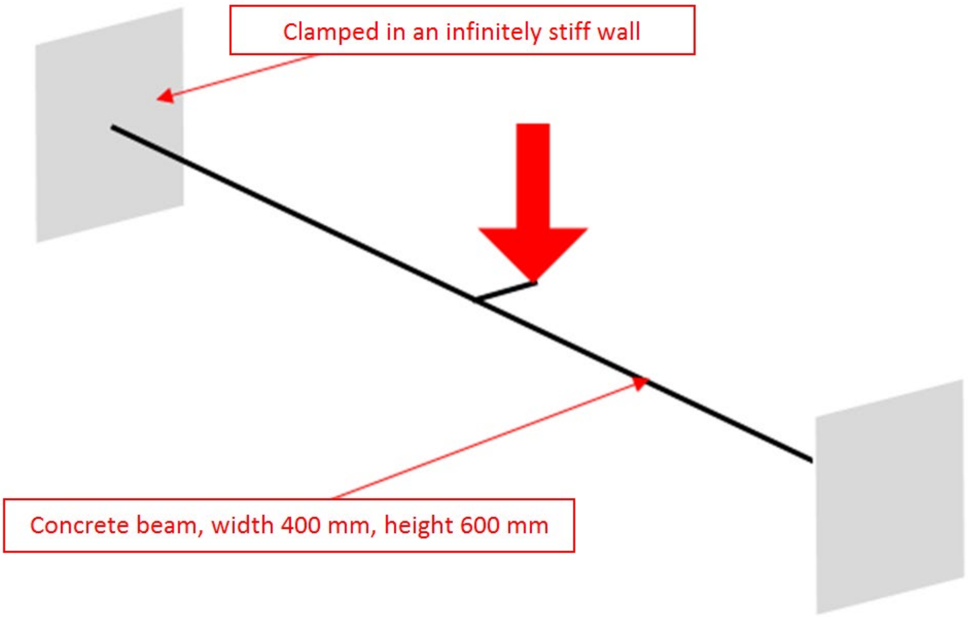
Crack width expression:

$$w_{\max} = \frac{1}{2}\frac{f_{\text{ctm}}}{\tau_{\text{bm}}}\frac{\text{Ø}}{\rho_{\text{s,eff}}}\frac{1}{E_{\text{s}}}\left(\sigma_{\text{s}} - \alpha\,\sigma_{\text{sr}}\right)$$

The crack spacing is the first part of the expression; the mean steel strain is the second part.
=> An increase of Ø results in an increase of the crack spacing.

Example 19 - Torsion

A rectangular concrete beam (see figure) is clamped on both sides in infinitely rigid walls. The span of the beam is 7.5 meters. The width of the beam is 400mm. The beam height is 600 mm. The beam is loaded by an evenly distributed load q_{Ed} = 7.2 kN/m due its own weight, and by a point load F_{Ed} = 600 kN in the middle of the span. Because this point load is introduced with an eccentricity of 200 mm, a twisting (torsion) moment (T_{Ed} = 120 kNm) is exerted on the beam.



General parameters:

Concrete strength class : C35/45
weight density of concrete : $\gamma_c = 25$ kN/m^3

Reinforcement class B500B : $f_{yd} = 435$ N/mm^2
modulus of elasticity : $E_s = 200000$ N/mm^2
estimated diameter of the main longitudinal reinf. : Ø20
diameter of the stirrups : Ø10

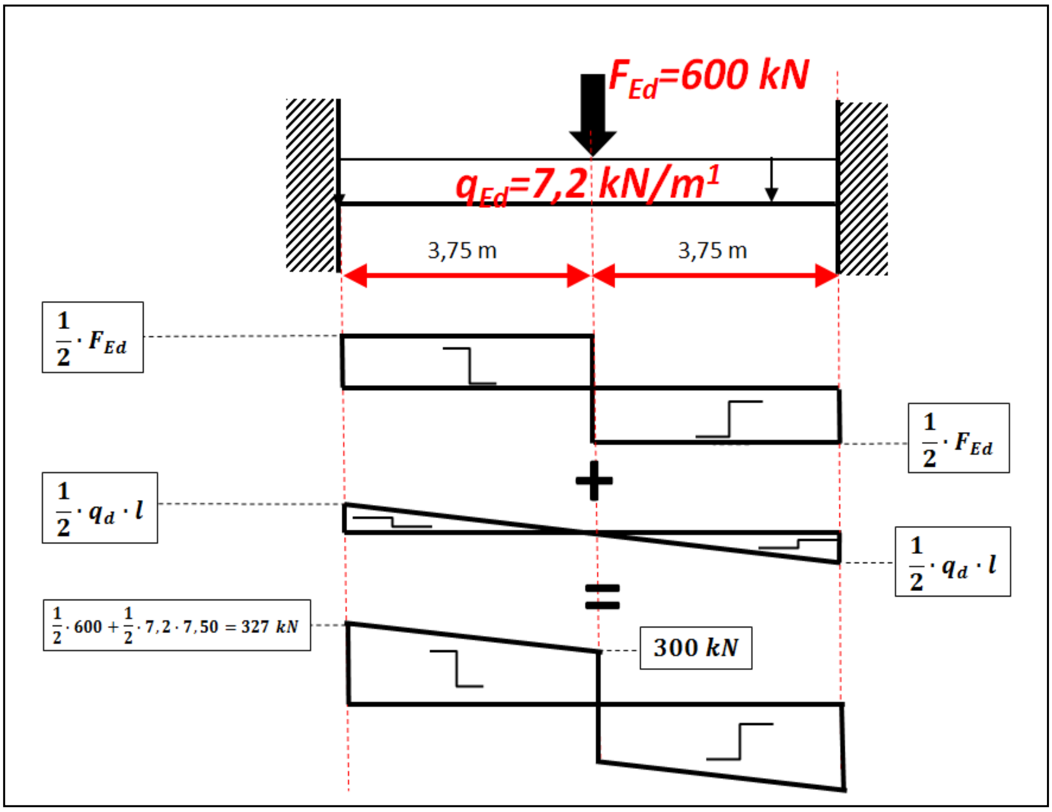
Concrete cover : c=40 mm
(from outer reinforcement, stirrups)

Question 19.1

Determine the distribution of the shear force and draw the shear distribution diagram (in kN).

Answer 19.1

Shear force in the beam is caused by the uniformly distributed load q_{Ed} = 7.2 kN/m and by the point load in the middle of the span F_{Ed} = 600 kN. Shear force diagram can be drawn as follows:

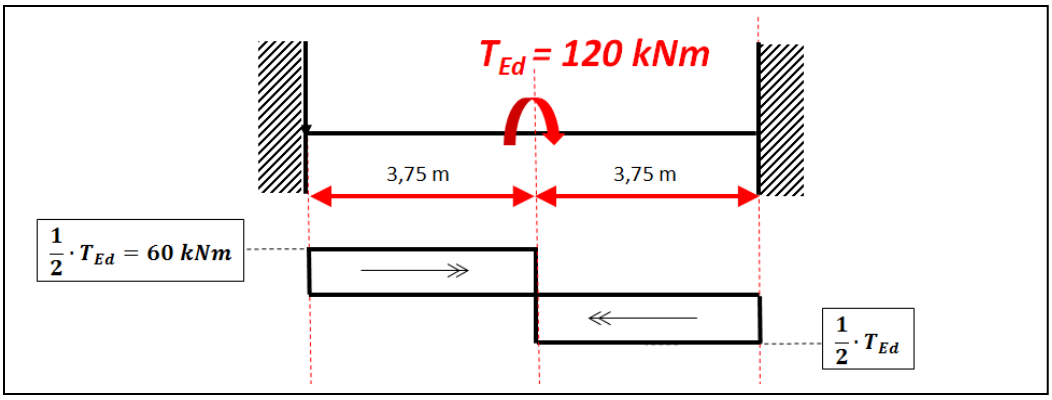


Question 19.2

Determine the distribution of the torsion moment and draw the corresponding diagram (in kNm).

Answer 19.2

The torsion moment in the beam is caused by the eccentric point load F_{Ed} = 600 kN (with an eccentricity of 0.2 m) in the middle of the span: $M = F_{Ed} \times e = 600 \times 0.2 = 120kNm$.
The torsion moment diagram can be drawn as follows:



Question 19.3

Which cross-section is governing for the control of the combination of shear and torsion, and which combination of forces must be included there?

Answer 19.3

Governing cross-section is the cross-section immediately adjacent to the supports;
This cross-section must be checked for:

- shear force of 327 kN ($V_{Ed,1} = 327$ kN)

Additional information

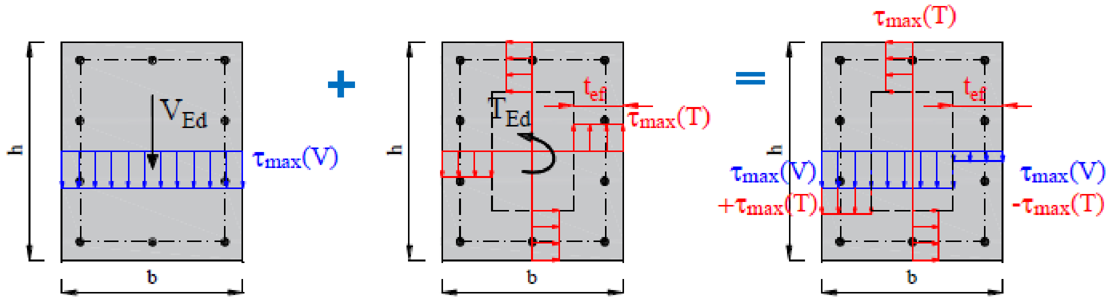
In theory, in order to check if the governing combination can be secured without shear and torsion reinforcement, and for the calculation of the required reinforcement (stirrups), the shear force at the support can be reduced by $d \times q_{Ed} \cong 0.54 \times 7.2 = 3.9\text{kN}$; when checking the maximum compressive stress this reduction should not be applied.

Because the difference in this specific case is small, in this example it is calculated with the "not reduced" shear force of 327 kN.

- torsion moment of 60 kNm ($T_{Ed,1} = 60$ kNm)

Additional information

Shear stresses are caused by a shear force and a torsion moment. Therefore, the maximum shear stresses in the cross-section can be determined by adding the shear stresses due to the shear force and the shear stresses due to the torsion moment, in the parts of the cross-section where the shear stresses have the same direction:



Question 19.4

Check whether the governing combination of forces (as calculated in question 10.3) can be resisted without additional reinforcement (that is: with minimum reinforcement only).

Answer 19.4

The reinforcement is not needed if the following is satisfied:

$$\frac{V_{Ed}}{V_{Rd,c}} + \frac{T_{Ed}}{T_{Rd,c}} > 1$$

The shear capacity $V_{Rd,c}$ (without shear reinforcement) can be determined as follows:

$$V_{Rd,c} = v_{Rd,c} \times b \times d$$

where $v_{Rd,c}$ is the shear resistance stress:

$$v_{Rd,c} = 0.12 \times k \times (100 \rho f_{ck})^{\frac{1}{3}} + k_1 \sigma_{cp} \text{ with a minimum of } v_{\mathrm{Rd,c}} = v_{\min} = 0.035 \times k^{3/2} \sqrt{f_{\mathrm{ck}}} + k_1 \sigma_{cp}$$

where σ_{cp} is the concrete compressive stress in cross-section due to axial loading and/or prestressing. In this case σ_{cp}=0.

and $d = h - c - Ø_{stirrup} - \frac{1}{2} \times Ø = 600 - 40 - 10 - \frac{1}{2} \times 20 = 540$ mm

Since the amount of longitudinal reinforcement is not known (ρ), the value of $v_{Rd,c}$:

$v_{Rd,c} = 0.12 \times k \times (100 \rho f_{ck})^{\frac{1}{3}}$ cannot be determined.

Therefore, the shear capacity can be determined only based on the minimum shear resistance stress: $v_{\min} = 0.035 \times k^{3/2} \sqrt{f_{\mathrm{ck}}} + k_1 \sigma_{cp}$ where

$$k = \min\left(1 + \sqrt{\frac{200}{d}}; 2\right) = \min\left(1 + \sqrt{\frac{200}{540}}; 2\right) = 1{,}61 \text{ and } f_{\mathrm{ck}}(\mathrm{C35/45}) = 35 \text{ MPa}$$

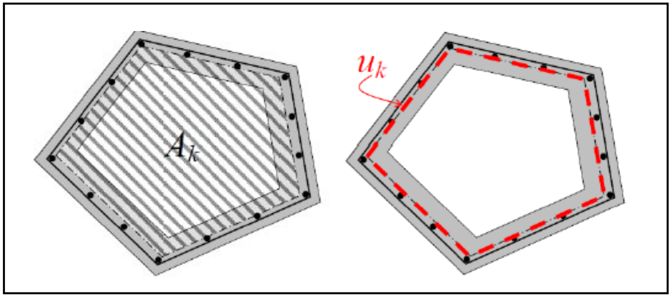
$$v_{\min} = 0{,}035 \times k^{3/2} \sqrt{f_{\mathrm{ck}}} = 0.035 \times 1.61^{3/2} \times \sqrt{35} = 0.42 \mathrm{MPa}$$

$$V_{Rd,c} = v_{\min} \times b \times d = 0.42 \times 400 \times 540 = 90720 \text{ N=90 kN}$$

The torsion moment capacity without torsion reinforcement, or the torsional cracking moment, can be determined as follows:

$$T_{Rd,c} = 2 \times A_k \times f_{ctd} \times t_{ef,i}$$

where A_k is the area enclosed by the centre-lines of the connecting walls, including inner hollow areas. For the rectangular cross-section:



$$A_k = \left(b - 2 \times \frac{1}{2} t_{ef,i}\right) \times \left(h - 2 \times \frac{1}{2} t_{ef,i}\right)$$

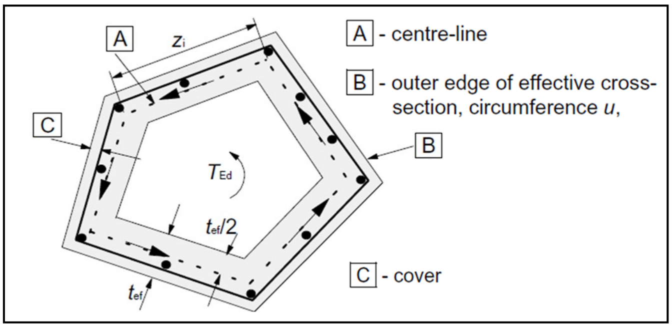
and $t_{ef,i}$ is the effective wall thickness:

$$t_{ef,i} = \max(\frac{A}{u}; 2 \times f_i)$$

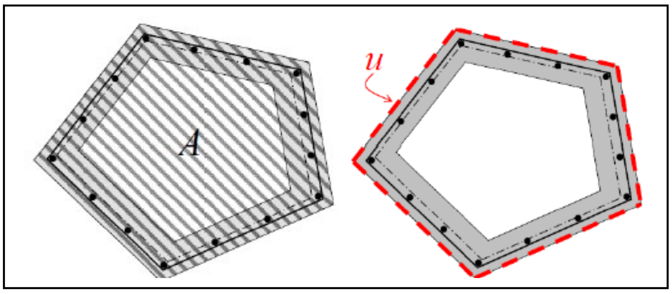
where f_i is the distance between the edge and the centre of the longitudinal reinforcement:

$$f_i = c + Ø_{stirrup} + \frac{1}{2} \times Ø$$

$$f_i = 40 + 10 + \frac{1}{2} \times 20 = 60mm$$



A is the total area of the cross-section within the outer circumference u, including inner hollow areas:



$$A = b \times h = 400 \times 600 = 240000mm^2$$

$$u = 2 \times (b+h) = 2 \times (400+600) = 2000mm$$

$$t_{ef,i} = \max(\frac{A}{u};2 \times f)$$

$$t_{ef,i} = \max\left(\frac{240000}{2000};2 \times 60\right) = 120mm$$

$$A_k = b_1 \times h_1 = \left(b - 2 \times \frac{1}{2}t_{ef,i}\right) \times \left(h - 2 \times \frac{1}{2}t_{ef,i}\right) = \left(400 - 2 \times \frac{1}{2}120\right) \times \left(600 - 2 \times \frac{1}{2}120\right) = 134400mm^2$$

$$f_{ctd} = f_{ctk,0,05} / \gamma_c = (0.7 \times 0.3 \times f_{ck}{}^{2/3}) / \gamma_c = (0.7 \times 0.3 \times 35^{2/3}) / 1.5 = 1.50MPa$$

The torsion moment capacity without torsion reinforcement is:

$$T_{Rd,c} = 2 \times A_k \times f_{ctd} \times t_{ef,i} = 2 \times 134400 \times 1.5 \times 120 = 48.4kNm$$

$$\frac{V_{Ed}}{V_{Rd,c}} + \frac{T_{Ed}}{T_{Rd,c}} > \frac{327}{91} + \frac{60}{48.4} = 3.59 + 1.24 = 4.84 >> 1.0$$

Conclusion: Additional reinforcement is required to resist this combination of shear and torsion forces.

Question 19.5

Check if the governing combination of forces (as calculated in question 19.3) can be taken by the concrete strut in compression, and at which slope of the strut (and thus also the crack angle) is the maximum capacity found?

Answer 19.5

The maximum resistance of a member subjected to torsion and shear is limited by the capacity of the concrete struts. In order not to exceed this resistance the following should be satisfied:

$$\frac{V_{Ed}}{V_{Rd,max}} + \frac{T_{Ed}}{T_{Rd,max}} \leq 1,0$$

where the maximum design shear resistance is:

$$V_{Rd,max} = \frac{\alpha_{cw} \times b \times z \times v_1 \times f_{cd}}{\cot\theta + \tan\theta} = \alpha_{cw} \times b \times z \times v_1 \times f_{cd} \times \sin\theta \times \cos\theta$$

For non-prestressed structures $\alpha_{cw} = 1$ and v_1 is a strength reduction factor for concrete cracked in shear:

$$v_1 = 0.6 \times \left[1 - \frac{f_{ck}}{250}\right] = 0.6 \times \left[1 - \frac{35}{250}\right] = 0.516$$

The maximum combination of shear and torsion can be withstood with the largest crack inclination angle θ, θ = 45°.

$$V_{Rd,max} = \frac{\alpha_{cw} \times b \times z \times v_1 \times f_{cd}}{\cot\theta + \tan\theta}$$

$$V_{Rd,max} = \alpha_{cw} \times b \times z \times v_1 \times f_{cd} \times \sin\theta \times \cos\theta = 1 \times 400 \times 0.9 \times 540 \times 0.516 \times \frac{35}{1.5} \times (\frac{\sqrt{2}}{2})^2 = 1170kN$$

The design torsional resistance moment is:

$$T_{Rd,max} = 2 \times \nu \times \alpha_{cw} \times f_{cd} \times A_k \times t_{ef,i} \times \sin\theta \times \cos\theta \text{ where } \nu = \nu_1 \text{ and } \alpha_{cw} = 1$$

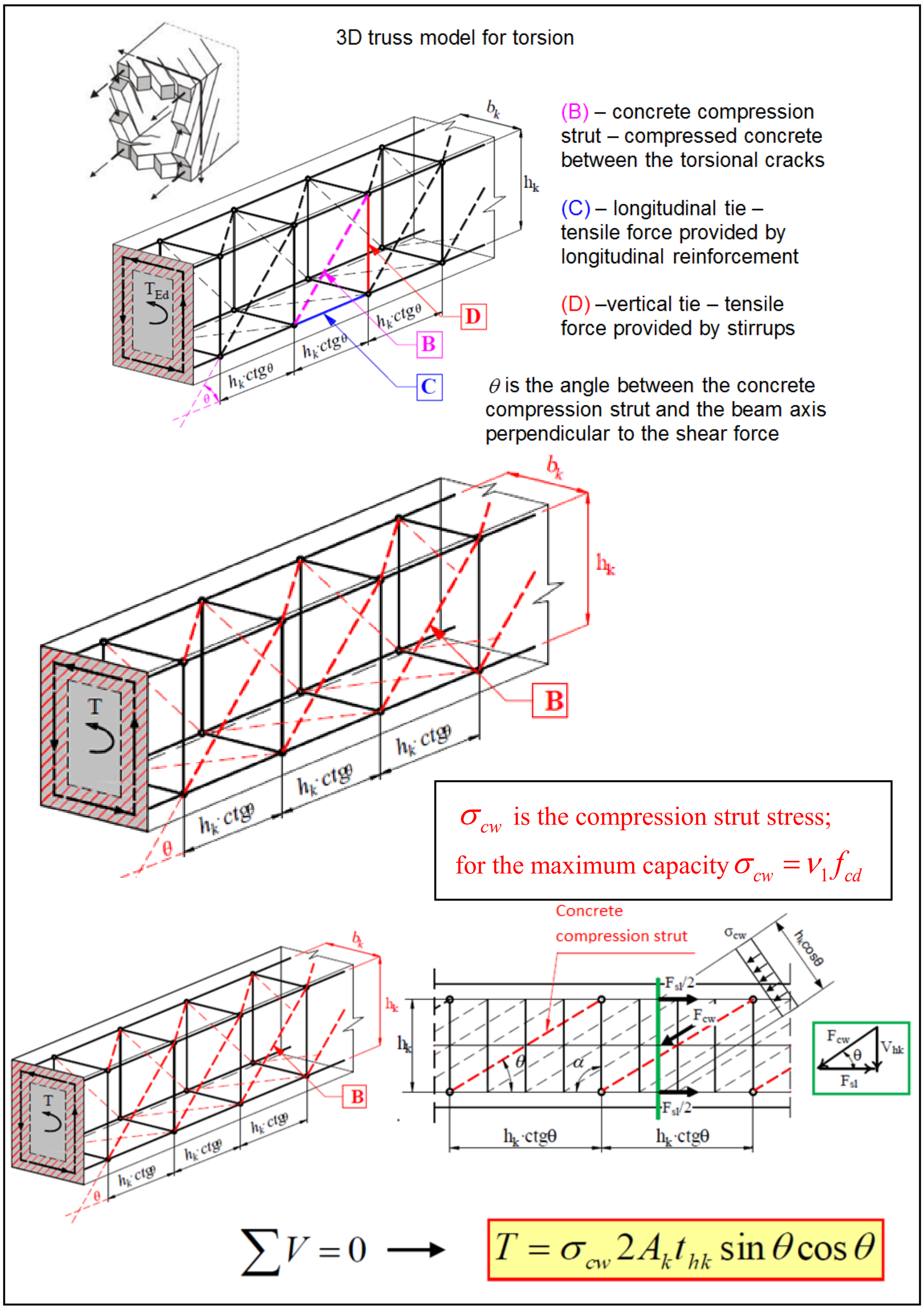
$$T_{Rd,max} = 2 \times 0.516 \times 1 \times \frac{35}{1.5} \times 134400 \times 120 \times \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} = 194kNm$$

$$\frac{V_{Ed}}{V_{Rd,max}} + \frac{T_{Ed}}{T_{Rd,max}} =$$

$$= \frac{327}{1170} + \frac{60}{194} =$$

$$= 0.28 + 0.31 = 0.59 < 1$$

Conclusion: Concrete struts can withstand the acting combination of forces if the crack inclination angle is 45°.



(B) – concrete compression strut – compressed concrete between the torsional cracks

(C) – longitudinal tie – tensile force provided by longitudinal reinforcement

(D) –vertical tie – tensile force provided by stirrups

θ is the angle between the concrete compression strut and the beam axis perpendicular to the shear force

σ_{cw} is the compression strut stress;

for the maximum capacity $\sigma_{cw} = \nu_1 f_{cd}$

$$\sum V = 0 \longrightarrow \boxed{T = \sigma_{cw} 2 A_k t_{hk} \sin\theta \cos\theta}$$

Question 19.6

Calculate the required stirrups (in mm^2 / m per side of the beam) and longitudinal reinforcement (in mm^2) in order to be able to resist the occurring combination of forces (calculated in question 19.3), assuming the crack inclination angle of 45°.

Answer 19.6

Required stirrups for the shear force:

$$\frac{A_{sw}}{s_w} = \frac{V_{Ed}}{z \times \cot\theta \times f_{ywd}} = \frac{327000}{0.9 \times 540 \times 1 \times 435} = 1.55 mm^2 / mm$$

Required stirrups for the torsion moment (see the figure below):

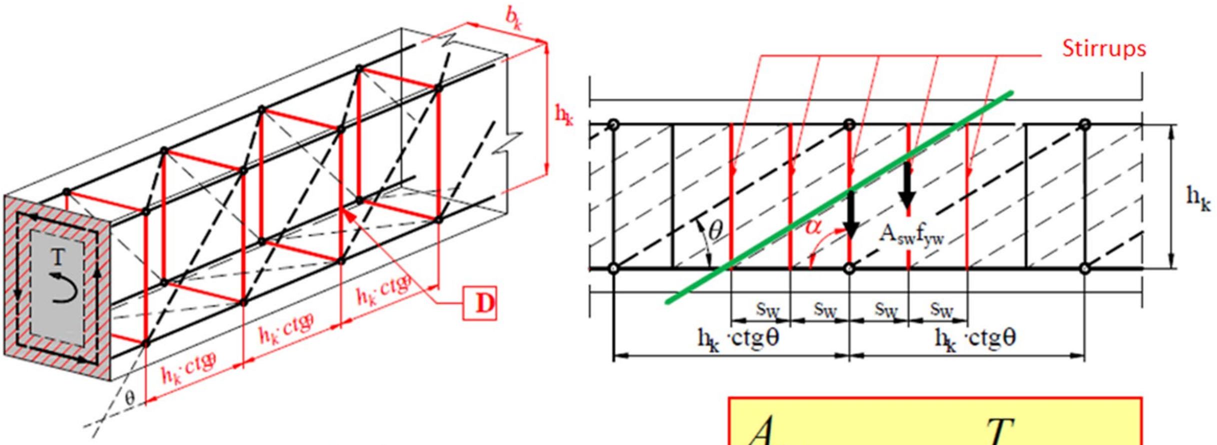
$$\frac{A_{sw}}{s_w} = \frac{T_{Ed}}{2 \times A_k \times \cot\theta \times f_{ywd}} = \frac{60 \times 10^6}{2 \times 134400 \times 1 \times 435} = 0.51 mm^2 / mm$$

Additional information

Shear force due to torsion in the rectangular cross section (longer side) is $V_{Ed,i} = \frac{T_{Ed}}{2A_k} \times h_1$

Shear reinforcement for the torsion can be determined analogous to the shear reinforcement needed for the shear force: $\frac{A_{sw}}{s_{w,\max}} = \frac{V_{Ed,i}}{h_1 \times \cot\theta \times f_{yd}}$, from which the following equation for the required reinforcement for the torsion moment can be derived: $\frac{A_{sw}}{s_w} = \frac{T_{Ed}}{2 \times A_k \times \cot\theta \times f_{ywd}}$

Calculation of the required stirrups



$$\sum V = 0 \longrightarrow \boxed{\frac{A_{sw}}{s_w} = \frac{T}{2A_k f_{yw} \cot\theta}}$$

Required stirrup reinforcement for the shear force can be distributed over the two legs of the stirrups. The required stirrup reinforcement for torsion applies only to a single bar (leg) cross-section. So the required stirrup reinforcement per side of the beam is:

$$\frac{A_{sw}}{s_w}=\frac{1}{2}\times 1.55+0.51=1.29\,mm^2/mm=1.29mm^2/m$$

By keeping the Ø$_{stirrup}$ =10 mm, the spacing between the stirrups is 60 mm.
This is a small spacing. It is more practical to opt for a larger stirrup diameter, e.g. for Ø12 or even Ø16.

Longitudinal reinforcement required for the torsion moment (see the figure below):

$$\sum A_{s,langs}=\frac{T_{Ed}\times\cot\theta\times u_k}{2\times A_k\times f_{yd}}=\frac{60\times 10^6\times 1\times(2\times(480+280))}{2\times 134400\times 435}=780\,mm^2$$

Longitudinal reinforcement can be also determined per side, as follows:

- for the long side:

$$A_{s,l}=\frac{T_{Ed}\times\cot\theta}{f_{yd}\times 2\times b_1}=\frac{60\times 10^6\times 1}{435\times 2\times 280}=246.3\,mm^2$$

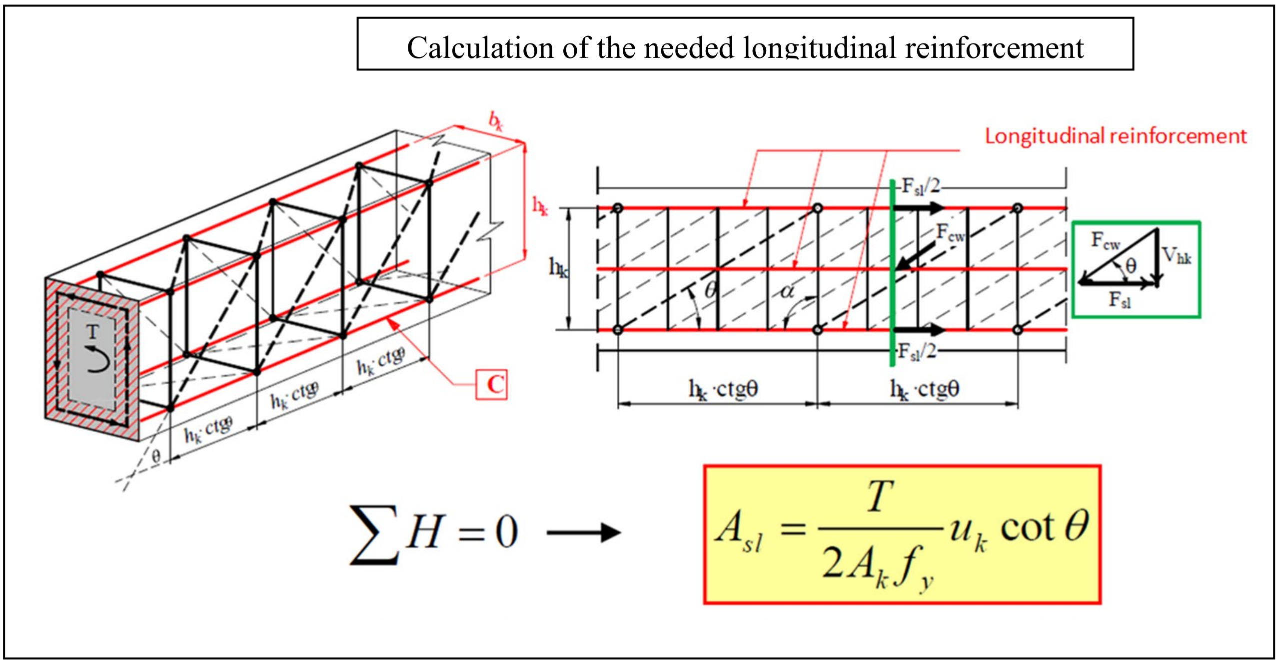
- for the short side:

$$A_{s,l}=\frac{T_{Ed}\times\cot\theta}{f_{yd}\times 2\times h_1}=\frac{60\times 10^6\times 1}{435\times 2\times 480}=143.7\,mm^2$$

The total amount of longitudinal reinforcement can then be calculated as:

$$\sum A_{s,langs}=\frac{T_{Ed}\times\cot\theta\times u_k}{2\times A_k\times f_{yd}}=2\times 246.3+2\times 143.7=780mm^2$$

Calculation of the needed longitudinal reinforcement



$$\sum H=0\longrightarrow \boxed{A_{sl}=\frac{T}{2A_kf_y}u_k\cot\theta}$$

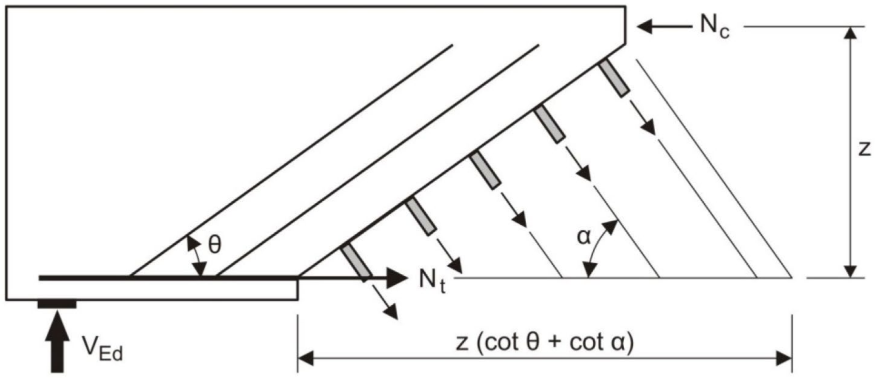
Note that in this problem there will be also longitudinal reinforcement due to the shear force. In general, this additional longitudinal reinforcement is taken into account by shifting the bending moment line.

The longitudinal reinforcement coming from the bending moment (note that in this example, there are also bending moments present in the beam) is not calculated here as it seemed too much for this example.

The horizontal force resulting from the shear force (that needs to be taken by longitudinal reinforcement) will be:

$$0,5V_{Ed}\,(\cot\theta - \cot\alpha)$$

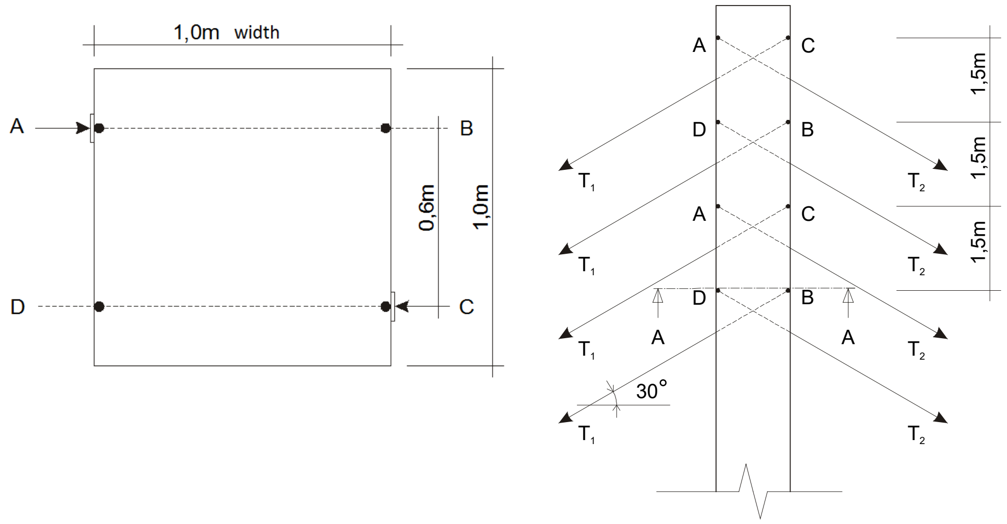
Note that this horizontal component is both dependent on the angle of the crack, θ, as well as the angle of shear reinforcement, α.



Try to think where this additional reinforcement will be placed. In top or/and bottom zone?

Example 20 - Torsion

The cables of a cable-stayed bridge are loading the top of the pylon according to the figure below. The anchor anchors are placed diagonally in A and C or in D and B, see the cross-section. In Figure 20.1, the cross-section A-A just above the next cable closure (D / B) is given.



Cross-section A-A

Side view pylon

Figure 20.1: The pylon of the cable-stayed bridge.

General parameters:

Concrete strength class : C35/45
weight density of concrete : $\gamma c = 25$ kN/m^3

Reinforcement class B500B : $f_{yd} = 435$ N/mm^2
modulus of elasticity : $E_s = 200000$ N/mm^2
minimal diameter of the stirrups : Ø16

Concrete cover : $c = 50$ mm
(from outer reinforcement, stirrups)

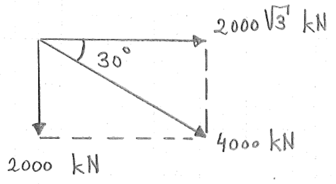
Force in the cable : $T_1 = T_2 = 4000\ kN$
cable at an angle of 30º

Question 20.1

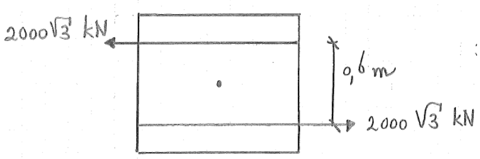
Determine all the forces and moment distributions in the top 4.5 m of the pylon.

Answer 20.1

The forces that act on the pylon can be decomposed as follows:



Side view



Top view

The horizontal component of a cable force is: $4000 \times \cos 30^{\circ} = 2000\sqrt{3} = 3464\ kN$

The vertical component is: $4000 \times \sin 30^{\circ} = 4000 \times 0.5 = 2000\ kN$

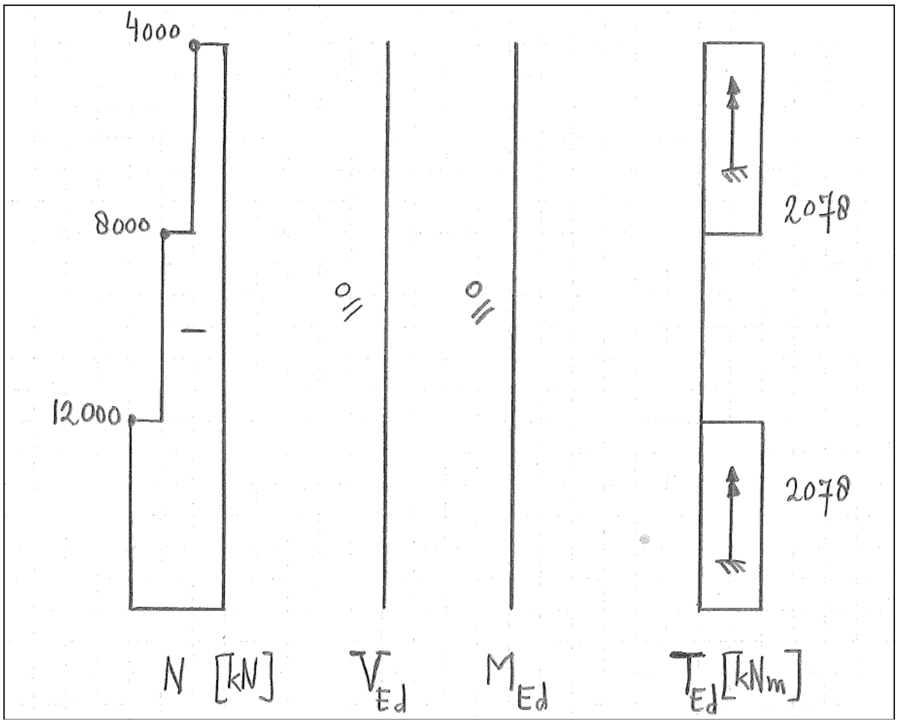
The normal force in the column increases per engagement level of the cables by:

$$2 \times 2000 = 4000 kN\,.$$

With regard to the transverse force, the horizontal components cancel each other.
However, they provide a torsion moment equal to the horizontal component times the arm =

$$3464 kN \times 0.6 m = 2078 kNm$$

For each level of engagement of the cables, the normal forces of the cables increase the already present normal force in the pylon; in relation to the torsional moments these turn in the opposite direction and eliminate each other.



Question 20.2

Calculate the stirrup reinforcement in the cross-section A-A.

Answer 20.2

The design value of the torsion moment is T_{Ed} = 2078 kNm.

The effective wall thickness is:

$$t_{ef} = \frac{A}{u} = \frac{1000 \times 1000}{4 \times 1000} = 250\, mm$$

This should not be smaller than $2 \times f$ where f is the distance from the edge to the centre of the longitudinal reinforcement:

$$f = c + Ø_{stirrup} + \frac{1}{2} \times Ø$$

Assuming that the dimeter of the bending reinforcement is $Ø = 20mm$

$$f = 50 + 16 + \frac{1}{2} \times 20 = 76mm \rightarrow \ 2 \times f = 152mm << 250mm \rightarrow t_{ef} = 250mm$$

The area enclosed by the centre-lines of the connecting walls, including inner hollow areas is:

$$A_k = (1000 - 250) \times (1000 - 250) = 562.5 \times 10^3\ mm^2$$

Torsional shear stress in the wall is:

$$\tau_{T,i} = \frac{T_{Ed}}{2 \times A_k \times t_{ef}} = \frac{2078 \times 10^6}{2 \times 562.5 \times 10^3 \times 250} = 7.39\ MPa$$

Checking if the maximum torsional resistance moment is not exceeded:

$$T_{Rd,\max} = 2 \times \nu \times f_{cd} \times A_k \times t_{ef} \times \sin\theta \times \cos\theta$$

$$\nu = 0.6 \times \left[1 - \frac{f_{ck}}{250}\right] = 0.6 \times \left[1 - \frac{35}{250}\right] = 0.52$$

- First check the inclination angle of the compression strut which leads to the smallest number of stirrups, θ = 21.8º:

$$T_{Rd,\max} = 2 \times 0.52 \times \frac{35}{1.5} \times 562.5 \times 10^3 \times 250 \times \sin 21.8^o \times \cos 21.8^o = 1176.7 \times 10^6\, Nmm$$

This is smaller than the applied design torsion moment T_{Ed} = 2078 kNm.

- Then check the maximum allowed inclination angle of the compression strut θ = 45º:

$$T_{Rd,\max} = 2 \times 0.52 \times \frac{35}{1.5} \times 562.5 \times 10^3 \times 250 \times \sin 45^o \times \cos 45^o = 1706.3 \times 10^6\, Nmm$$

This is also smaller than the applied design torsion moment T_{Ed} = 2078 kNm.

Conclusion: The resistance of the compression strut is exceeded; the cross-sectional dimensions and / or the concrete strength class must be adjusted (increased).

Calculation of the required amount of stirrups for the given cross-section:

Shear force in the effective wall thickness, which (due to the symmetry of the square cross-section) is equal at all the four sides, is:

$$V_{Ed} = \tau_T \times t_{ef} \times h_1$$

With the shear stress of 7.39 MPa, the effective wall thickness of 250 mm and the length of the wall $h_1 = 1000 - t_{ef} = 750\ mm$:

$$V_{Ed} = 7.39 \times 250 \times 750 = 1.39 \times 10^6\ N$$

Additional information

Shear force due to torsion in the rectangular cross-section can be also calculated as follows: $V_{Ed} = \frac{T_{Ed}}{2A_k} \times h_1$

For the square cross-section: $V_{Ed} = \frac{T_{Ed}}{2A_k} \times h_1 = \frac{T_{Ed}}{2h_1} = \frac{2078 \times 10^6}{2 \times 750} = 1.39 \times 10^6 N$

This shear force has to be taken by stirrups. With the inclination of the compression strut of 45°:

$$\frac{A_{sw}}{s_w} = \frac{V_{Ed}}{h_1 \times f_{ywd} \times \cot 45^{\circ}} = \frac{1.39 \times 10^6}{750 \times 435 \times 1} = 4.26\ mm^2 / mm$$

Use the cross-sectional area of one stirrup leg. For a stirrup Ø 16 mm, 201 mm^2 of reinforcement is present. This results in a stirrup spacing $s_w = \frac{201}{4.26} = 47\ mm$, which is too small for practical application.

Question 20.3

Is the amount of longitudinal reinforcement affected by this specific loading case? If so, determine the quantity (mm^2).

Answer 20.3

Yes, due to the torsional moment, additional longitudinal reinforcement is required:

$$\Sigma A_{sl} = \frac{T_{Ed}}{2A_k} \times \frac{u_k}{f_{yd}} \times \cot\theta = \frac{2078 \times 10^6}{2 \times 562.5 \times 10^3} \times \frac{4 \times 750}{435} \times \cot 45^{\circ} = 12.7 \times 10^3\ mm^2$$

This is approximately $\frac{12.7 \times 10^3}{4} = 3175 mm^2$ per side of the cross-section. The area of a reinforcement bar Ø 25 mm is 491 mm^2. This results in 7 Ø 25 bars per side.

Note: Of course, the favourable influence of the normal force may still be included when the final required amount of reinforcement is calculated: On the compression side, the longitudinal reinforcement may be reduced in relation to the available compressive force. In the upper part of the pylon the calculation value of the pressure force is 2000 kN. This implies that the total reduction of the required longitudinal reinforcement over all 4 sides is $\frac{4000 \times 10^3}{435} = 9195\ mm^2$.

Question 20.4

As a result of an asymmetrical load the cable forces T_1 and T_2 change. Increase the cross-section to 1.5 m x 1.5 m. The position of the gripping points of the lines changes; they slide along, each over 250 mm because the cross-section becomes wider. But, in the direction perpendicular to it, their location does not change; they remain there 0.6 m apart.

T_1 = 4500 kN and T_2 = 3500 kN (calculation value).
Answer questions 20.1 and 20.2 again.

Answer 20.4

The two cable forces can be composed as follows:

T_1:

- The horizontal component of a cable force is: $4500 \times \cos 30^\circ = 3897\ kN$
- The vertical component is: $4500 \times \sin 30^\circ = 2250\ kN$

T_1:

- The horizontal component of a cable force is: $3500 \times \cos 30^\circ = 3031\ kN$
- The vertical component is: $3500 \times \sin 30^\circ = 1750\ kN$

For each engagement level of two cables there is now:

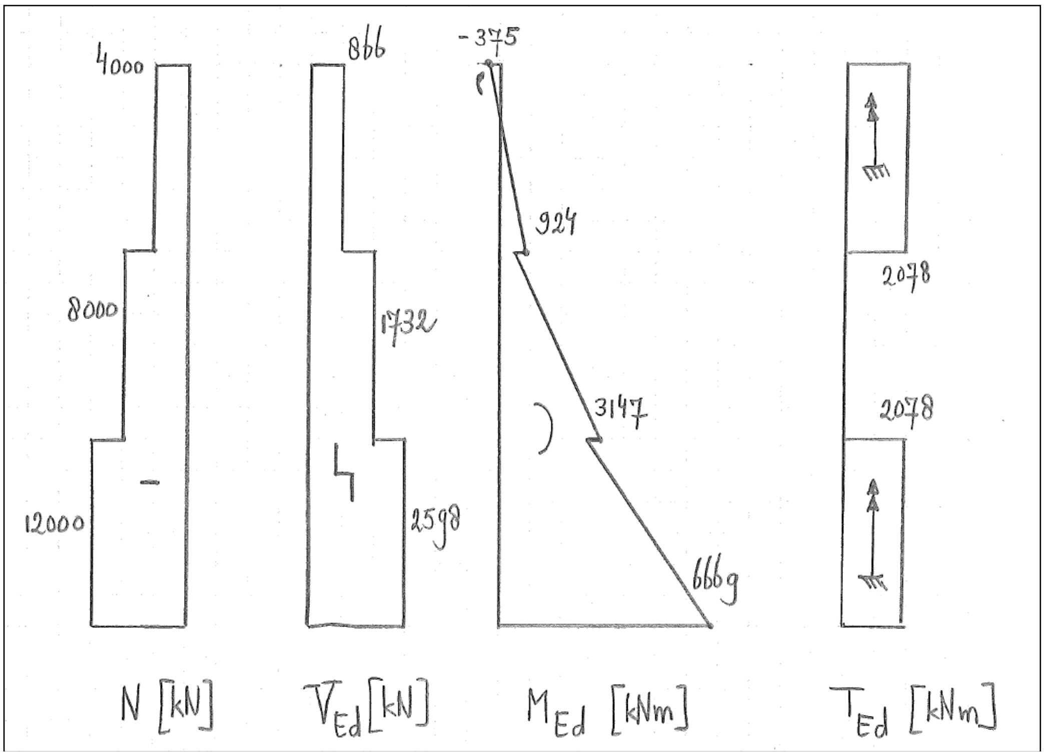
- Shear force: $3897 - 3031 = 866 kN$
- Torsion moment from the horizontal forces in the pylon, each with an arm of 0.3 m from the centre line of the pylon: $0.3 \times 3897 + 0.3 \times 3031 = 2078 kNm$
- Vertical compression force: $2250 + 1750 = 4000 kN$
- Bending moment:
  The differences between the vertical and horizontal components of the cable forces give bending moments. With respect to an axis perpendicular to the plane of the given cross section, the moment consists of two components:
  - As a consequence of vertical cable forces $2250 \times 0.75 - 1750 \times 0.75 = -375\ kNm$
    This is a local, concentrated moment.
  - As a consequence of horizontal cable forces
    These forces only contribute to the bending moment in the lower cross-sections. So, in a cross-section 1.5 m lower, precisely where the new set of cables is engaged, the bending moment is: $3897 \times 1.50 - 3031 \times 1.50 = 1299\ kNm$

    This moment increases linearly between the two engagement levels. The number of cables is also increasing in the pylon downwards. In section A-A there are ultimately three upper engagement levels with cables. This provides a total moment of:

    $$3 \times (-375) + 3 \times 1299 + 2 \times 1299 + 1 \times 1299 = 6669\ kNm$$

    A layer of cables higher: $2 \times (-375) + 2 \times 1299 + 1 \times 1299 = 3147\ kNm$

    A layer of cables higher: $1 \times (-375) + 1 \times 1299 = 924\ kNm$



The design values of the normal force and torsion moment did not change.

The effective wall thickness is:

$$t_{ef} = \frac{A}{u} = \frac{1500 \times 1500}{4 \times 1500} = 375\, mm$$

This is, of course, again smaller than $2 \times f = 152mm$

The area enclosed by the centre-lines of the connecting walls, including inner hollow areas is:
$A_k = (1500 - 375) \times (1500 - 375) = 1265.6 \times 10^3\, mm^2$

Torsional shear stress in the wall is:
$$\tau_{T,i} = \frac{T_{Ed}}{2 \times A_k \times t_{ef}} = \frac{2078 \times 10^6}{2 \times 1265.6 \times 10^3 \times 375} = 2.19\, MPa$$

Check if the maximum torsional resistance moment is not exceeded:

$$T_{Rd,\max} = 2 \times \nu \times f_{cd} \times A_k \times t_{ef} \times \sin\theta \times \cos\theta$$
$$\nu = 0.6 \times \left[1 - \frac{f_{ck}}{250}\right] = 0.6 \times \left[1 - \frac{35}{250}\right] = 0.52$$

- First checking for the inclination angle of the compression strut which leads to the smallest number of stirrups, θ = 21.8°:

$$T_{Rd,\max} = 2 \times 0.52 \times \frac{35}{1.5} \times 1265.6 \times 10^3 \times 375 \times \sin 21.8^o \times \cos 21.8^o = 3971.2 \times 10^6\, Nmm$$

This is larger than the applied design torsion moment T_{Ed} = 2078 kNm.

In the cross-section is also there is also a shear force, since the horizontal components in the two cables are not equal. In the considered cross section A-A this is 2598 kN.
The shear resistance of the compression strut is:

$$V_{Rd,\max} = b_w \times h_1 \times v_1 \times f_{cd} \times \sin\theta \times \cos\theta$$

$$v_1 = v = 0,6 \cdot \left(1 - \frac{f_{ck}}{250}\right) = 0,6 \cdot \left(1 - \frac{35}{250}\right) = 0,52$$

For the inclination angle of the compression strut θ = 21.8º:

$$V_{Rd,\max} = 1500 \times (1500 - 375) \times 0.52 \times \frac{35}{1.5} \times \sin 21.8^\circ \times \cos 21.8^\circ = 7060.0 \cdot 10^3 N$$

Unity check for the most heavily loaded side:

$$\frac{T_{Ed}}{T_{Rd,\max}} + \frac{V_{Ed}}{V_{Rd,\max}} = \frac{2078}{3971} + \frac{2598}{7060} = 0.52 + 0.37 = 0.89 < 1.0$$

Therefore, the concrete strut can withstand the acting combination of forces, also with the most unfavourable inclination of the compression strut.

When calculating the required amount of stirrups for the given cross-section, the shear force in one wall, due to torsion moment. should be considered:

$$V_{Ed} = \tau_T \times t_{ef} \times h_1$$

Due to the symmetry, this shear force is equal in all four sides of the square cross section. With the shear stress of 2.19 MPa, effective wall thickness of 375 mm and the length of the wall $h_1 = 1500 - t_{ef} = 1125\ mm$:

$$V_{Ed} = 2.19 \times 375 \times 1125 = 0.92 \times 10^6\ N$$

The shear force in the cross-section is 2598 kN. This will be taken with two sided stirrups, meaning that 1299 kN ($\approx 1.3 \times 10^6$kN) should be taken per one stirrup leg. With the inclination angle of the compressive strut of 21.8º, the amount of required reinforcement can be calculated as follows:

$$\frac{A_{sw}}{s_w} = \frac{V_{Ed}}{h_1 \times f_{ywd} \times \cot 21.8^\circ} = \frac{(0,92 + 1.30) \times 10^6}{1125 \times 435 \times 2.5} = 1.81\ mm^2 / mm$$

For one Ø 16 mm stirrup (201 mm^2 per one side of the stirrup) the spacing between stirrups is s_w = 201 / 1,81 = 111 mm.

Question 20.5

Can the bending moment be taken by a 1.5 x 1.5 m^2 cross section? (Show this with a very global calculation).

Answer 20.5

The design value of the bending moment is 6669 kNm and of the normal, compressive force is 12000 kNm. The reinforcement can be calculated based on:

$$M_{Rd} = z \times A_s \times f_{yd}$$

With $d = 1500 - 50 - 16 - 20/2 = 1424$ mm and $z = 0.9 \times d = 1282\ mm$, the required amount of reinforcement is:

$A_s = \dfrac{6669 \cdot 10^6}{1282 \cdot 435} = 12{,}0 \cdot 10^3\ mm^2$, which corresponds to 38 Ø 20 mm; 24 Ø 25 mm or 15 Ø 32 mm.

If this reinforcement cannot be placed in one line, the effective depth of the cross-section, *d*, and consequently the lever arm of internal forces *z*, have to be calculated again.

In this calculation the favourable effect of the normal pressure force is not considered. This can be done by splitting the pressure force into two forces of equal size. One force is shifted to the centre of the reinforcing steel; the other force is shifted over the same distance in the direction of the concrete compression zone. The latter force is assumed to be taken up immediately by an additional concrete compression force acting at the same place. The compressive force at the location of the reinforcing bar results in a reduction of the required quantity of reinforcing steel:

$$A_{s,reduction} = \frac{0.5 N_{Ed}}{f_{yd}} = \frac{0.5 \times 12000 \times 10^3}{435} = 13800\ mm^2$$

This approach is only possible if the concrete compression zone can develop. With an internal lever arm of 1282 mm for the bending moment, the resulting concrete pressure is:

$$N_c = \frac{6669 \times 10^6}{1282} = 5.2 \times 10^6\ N$$

With an additional pressure force of $0.5 \times 12000 = 6000 kN$ as a result of the design value of the additional external normal force, the total concrete pressure force becomes 11200 kN. For the 1500 mm wide pressure zone in strength class C35 / 45 (full-density stress-strain relationship under pressure $\alpha = 0.75$):

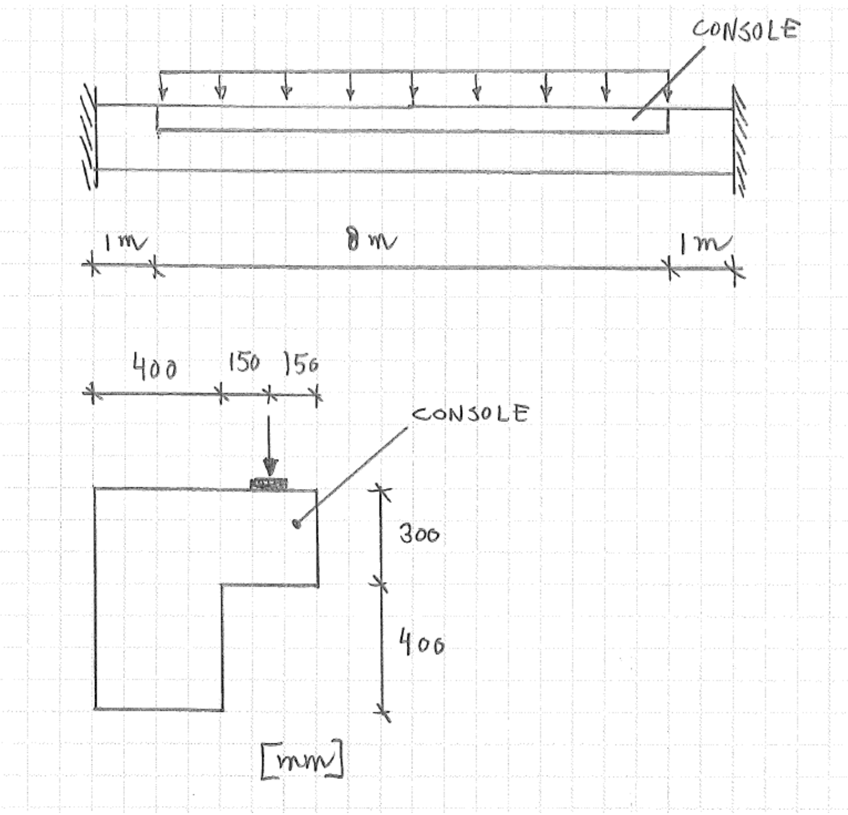
$$x_u = \frac{11200 \times 10^3}{0.75 \times 1500 \times \dfrac{35}{1{,}5}} = 427\ mm$$

The centre of the concrete compressed zone is ${}^{7}\!/_{18} \times 427 \approx 0.39 \times 427 = 166$ mm from the edge.

This is more inward than where the centre of the reinforcing bar is, namely approximately $50 + 16 + 20/2 = 76$ mm from the edge. The assumption has therefore not been correct and a more accurate calculation must show how much reinforcement is really needed.

Example 21 - Torsion

A concrete beam (length 10 m) is fully clamped at both ends in a concrete wall. The beam is has a console over a distance of 8 m. A uniformly distributed line load acts on the console.



General parameters:

Dimensions of the concrete beam : b = 400 mm;
h = 700 mm
Dimensions of the console : b = h = 300 mm

Concrete strength class : C35/45
weight density of concrete : $\gamma c = 24\ kN/m^3$

Reinforcement class B500B : $f_{yd} = 435\ N/mm^2$
modulus of elasticity : $E_s = 200000\ N/mm^2$
diameter of the stirrups : Ø10
diameter of longitudinal reinforcement : Ø20

Concrete cover : $c = 30$ mm
(from outer reinforcement, stirrups)

Characteristic value of the distributed load on console : $q_{Q,k} = 50\ kN/m$

Question 21.1

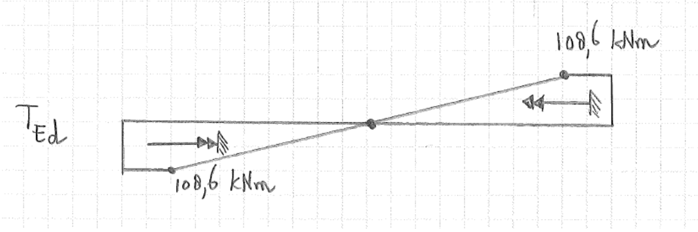
Calculate the design value of the total torsion moment that acts on the girder (taking the selfweight of the console into account) and draw the torsion moment line for the girder.

Answer 21.1

The total distributed load on the console, taking into account the self-weight, can be calculated as follows: $q_{tot} = (1.2 \times 0.3 \times 0.3 \times 24 \times 1.2) + (1.5 \times 50) = 77.59\ kN/m$

Total torsion moment is: $T_{Ed} = q_{tot} \times e \times l = 77.59 \times \left(\frac{0.4}{2} + 0.15 \right) \times 8 = 217.2\ kNm$

This torsional moment is composed out of two equal parts, both ending at the support:



Question 21.2

It is now assumed that stirrups are required. Calculate the amount of stirrups needed to resist the torsion moment.

Answer 21.2

$$t_{\text{ef}} = \frac{A}{u} = \frac{400 \times 700}{2 \times (400 + 700)} = 127.3\ \text{mm}$$

$$b_1 = b - t_{ef,i} = 400 - 127.3 = 272.7\ mm$$

$$h_1 = h - t_{ef,i} = 700 - 127.3 = 572.7\ mm$$

Condition $t_{ef,i} > 2 \times \left(30 + 10 + 20/2\right) = 100\ mm$ is satisfied.

$$A_k = (400 - t_{ef}) \times (700 - t_{ef}) = 156.2 \cdot 10^3\ mm^2$$

Shear stress due to the torsion: $\tau_T = \frac{T_{Ed}}{2 \times A_k \times t_{ef}} \rightarrow \tau_T \times t_{ef} = \frac{T_{Ed}}{2 \times A_k}$

The reinforcement can be calculated as follows:

$\frac{A_{sw}}{s_w} = \frac{V_{Ed}}{h_1 \times f_{ywd} \times \cot\theta}$ where $V_{Ed} = \tau_T \times t_{ef} \times h_1 = \frac{T_{Ed}}{2 \times A_k} \times h_1$

$$\frac{A_{sw}}{s_w} = \frac{\frac{T_{Ed}}{2 \times A_k} \times h_1}{h_1 \times f_{ywd} \times \cot\theta} = \frac{\frac{108.6 \times 10^6}{2 \times 156.2 \times 10^3}}{435 \times 1.0} = \frac{348}{435} = 0{,}80\ mm^2/mm$$

Note: The inclination angle of the compression strut is assumed to be 45°. Furthermore, as h_1, the length of a shear panel, is not in the equation, the required reinforcement is equal for each shear panel. For stirrup reinforcement Ø10 mm, the spacing is $s_w = 98$ mm.

Question 21.2

Calculate the total amount of stirrups that is needed to take the torsion moment and the shear force and calculate the spacing between the stirrups for a stirrup diameter of Ø10 mm.

Answer 21.2

In order to calculate the total amount of reinforcement, first the shear force acting on the beam has to be calculated.

Distributed load from the beam itself (acting over the length of 10 m) is:
$q_{d1} = 1.2 \times 0.4 \times 0.7 \times 24 = 8.06\, kN/m$

Distributed load from the console (acting over the length of 8 m), as previously calculated is:
$q_{d2} = 77.59\, kN/m$

Resulting shear force at the support is:

$$V_{Ed} = \frac{10}{2} \times 8.06 + \frac{8}{2} \times 77.59 = 356.7\, kN$$

When calculating the amount of stirrups needed to take the shear force, again the inclination angle of the compression strut (the same as with torsion) is considered to be 45º:

$$\frac{A_{sw}}{s_w} = \frac{V_{Ed}}{z \times f_{ywd} \times \cot\theta} = \frac{350.7 \times 10^3}{0.9 \times (700 - 30 - 10 - 20/2) \times 435 \times 1.0} = 1.38\, mm^2/mm$$

In order to calculate the total reinforcement for the shear stresses coming both from the shear force and the torsion moment, the calculated torsion and shear reinforcement must be combined:

$$\frac{A_{sw}}{s_w} = 0.8 + \frac{1.38}{2} = 1.49 mm^2/mm$$

The required spacing of the reinforcement Ø10 mm is $\frac{79}{s_w} = 1.49 mm^2/mm \rightarrow s_w = 53mm$.

Question 21.3

Calculate the total required amount of longitudinal reinforcement for torsion and calculate how much reinforcement is needed in each of the four side faces of the beam.

Answer 21.3

The total longitudinal reinforcement can be calculated as follows:

$$\sum A_{sl} = \frac{T_{Ed} \times \cot\theta}{2 \times A_k} \cdot \frac{u_k}{f_{yd}} = \frac{108.6 \times 10^6 \times 1.0}{2 \times 156 \times 10^3} \cdot \frac{2 \cdot (272.7 + 572.7)}{435} = 1353\, mm^2$$

This reinforcement has to be equally distributed over the circumference of the beam:

Per side: $\frac{572.7}{2 \cdot (272.7 + 572.7)} \cdot 1353 = 459\, mm^2 \quad \rightarrow 6\, Ø\, 10 \quad (471 mm^2)$

At top and bottom: $\frac{272.7}{2\cdot(272.7+572.7)}\cdot 1353 = 218\ mm^2 \quad \rightarrow 3\ \text{Ø}\ 10 \quad (236mm^2)$

Note: When determining the definitive reinforcement, the reinforcement needed for the bending moment must also be taken into account.