14 SLABS

14.1 Introduction

A research program carried out by the Dutch organization CUR shows in which parts of buildings most concrete is applied. Table 14. 1 contains the results of this survey.

Table 14. 1: Volume of concrete (in %) applied in the various parts of concrete structures for high rise buildings and housing.

Foundations	22 %
Load bearing walls	4 %
Columns	5 %
Slabs	59 %
Other	10 %

The results show that the major part of the concrete is used for constructing slabs. The choice of an efficient type of slab is therefore very important with regard to the building costs.

Reinforced concrete slabs are mostly used as floors, roofs and as decks of bridges. The slab is a structural element loaded perpendicular to its plane, having a thickness that is significantly smaller than the other two dimensions (Fig. 14. 1). Slabs primarily transfer the loads to the supports in one or two directions by bending. Due to the relatively small thickness (relatively great span/height ratio) of a reinforced concrete slab, the shear deformation is considered to be insignificant; deformation from bending is governing. Contrary to membranes and shells, boundary conditions and load cases for concrete slabs are in most cases such that no in-plane forces occur.

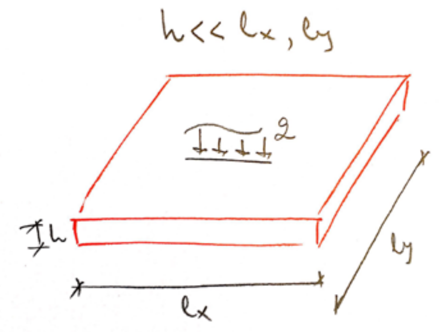


Fig. 14. 1: Definition of the slab element

By the type of execution, three main types of reinforced concrete slabs can be defined:

- Cast in situ
- Prefabricated
- Partially prefabricated and partially cast in situ systems.

More detailed information about different systems and their main advantages and disadvantages can be found in CIE3340/CIE4281 reader Concrete Building Structures.

A structure (e.g. building) usually consists of slabs with different spans and support conditions. The slabs can be supported by beams which further carry the load to columns and these transfer the load to the foundations. These beams can be present at all slab sides, or slabs can have 1, 2 or 3 free edges (Fig. 14. 2). Instead of by beams and columns, slabs can also be supported by masonry or reinforced concrete walls, by structural steel members, directly by columns (i.e. flat slab) or continuously by ground (e.g. warehouse floors on a bedding). As a result, depending on the type of the supports, their size and arrangement, different load transfer mechanisms occur in the structure.

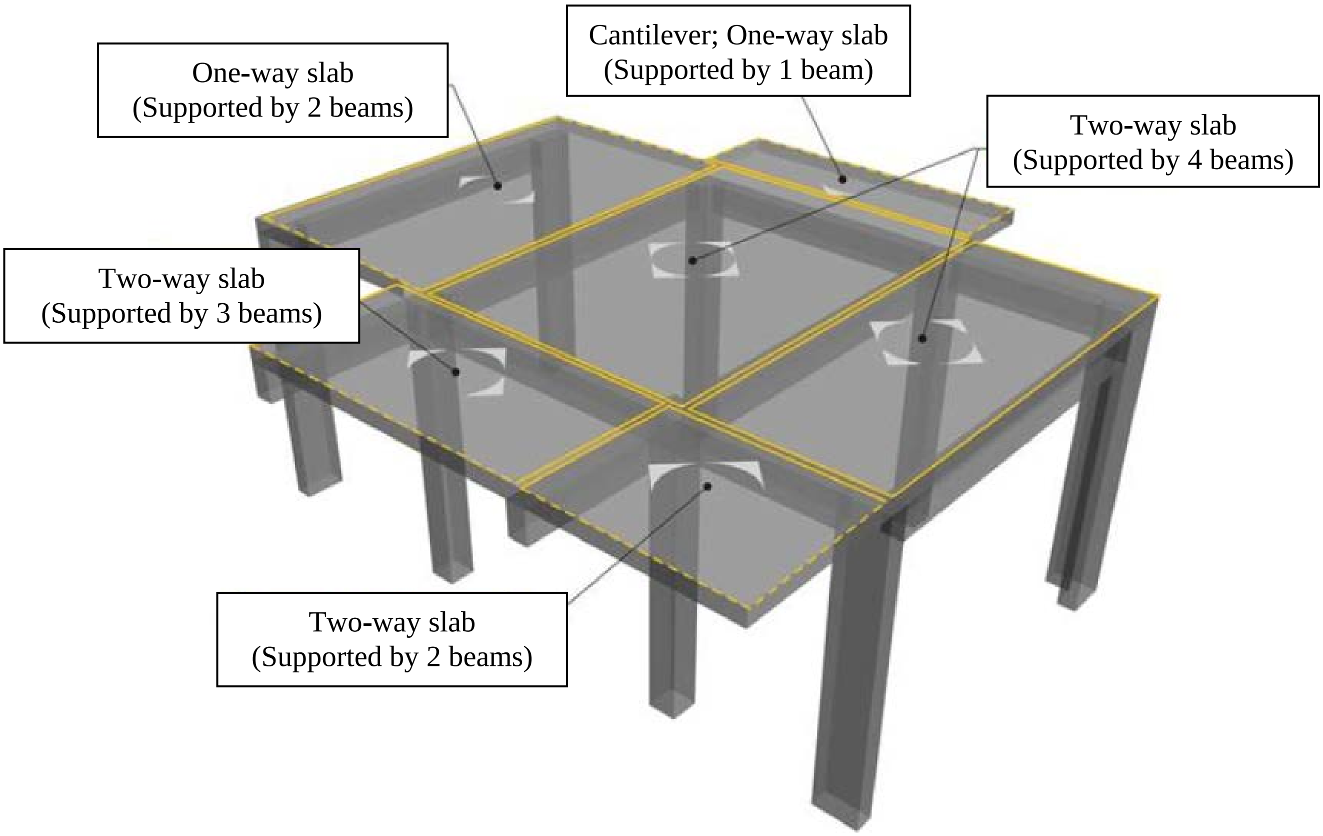


Fig. 14. 2: Realistic floor plan with different types of slabs (solid lines: stiff line support provided by stiff beam; dashed lines: no supports, free edges)

Depending on the way the load on the slab is transferred, a general distinction can be made by defining the following structural types of slabs (Fig. 14. 3):

1) One-way slabs with line supports
2) Two-way slabs with line supports
3) Flat slabs (or flat plates) with and without drop panels and/or column heads

Examples of one-way slabs and two-way slabs with line supports are given in Fig. 14. 3a and Fig. 14. 3b, respectively. These are slabs *with supporting beams*, meaning that the load is first transferred from the slab to the beams, which further transfer the load to the columns. The formwork is complex and applying the reinforcement is labour-intensive. This is the reason why this type of slab is only applied in case of high slab loads. Instead of beams and columns, the supporting structures of one-way and two-way slabs can also be walls. The slabs now first transfer the load to the walls and the walls transfer the load directly to the foundation.

Flat slabs are slabs in which the supporting structure consists of columns only (Fig. 14. 3c). This is a popular floor type thanks to its ease of execution and the corresponding reduction of the labour costs. Moreover, installations can easily be attached to the floor.

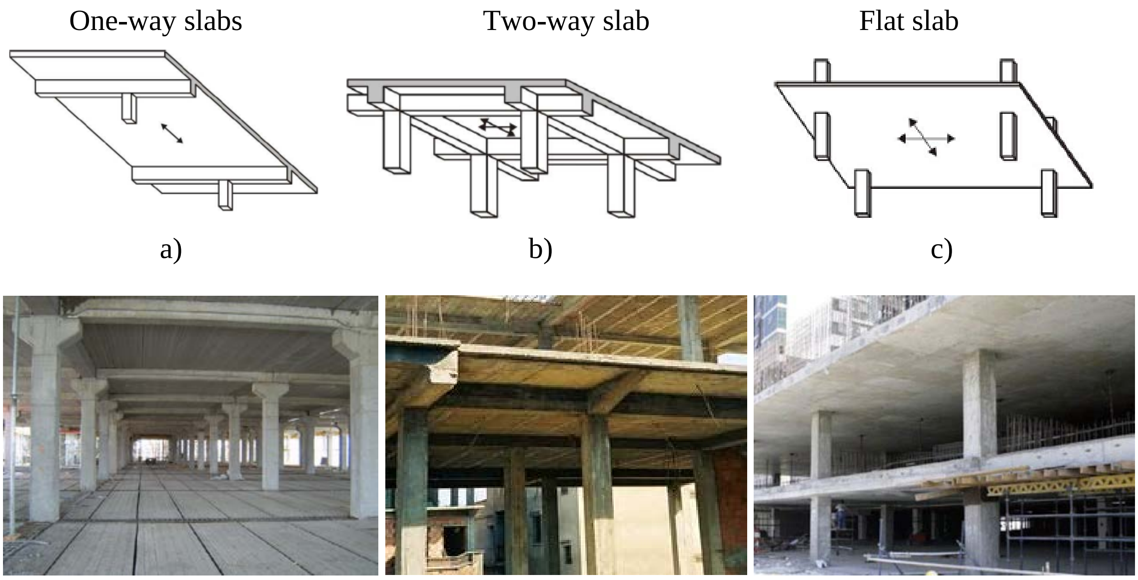


Fig. 14. 3: Structural types of slabs a) one-way slab with line supports, b) two-way slab with line supports and c) flat slab

A flat slab can be constructed with and without drop panels and column heads Fig. 14. 4. These structural elements are commonly applied nowadays. The drop panel or a column head increases the bearing capacity of the slab at the column perimeter, where the highest shear stresses occur.

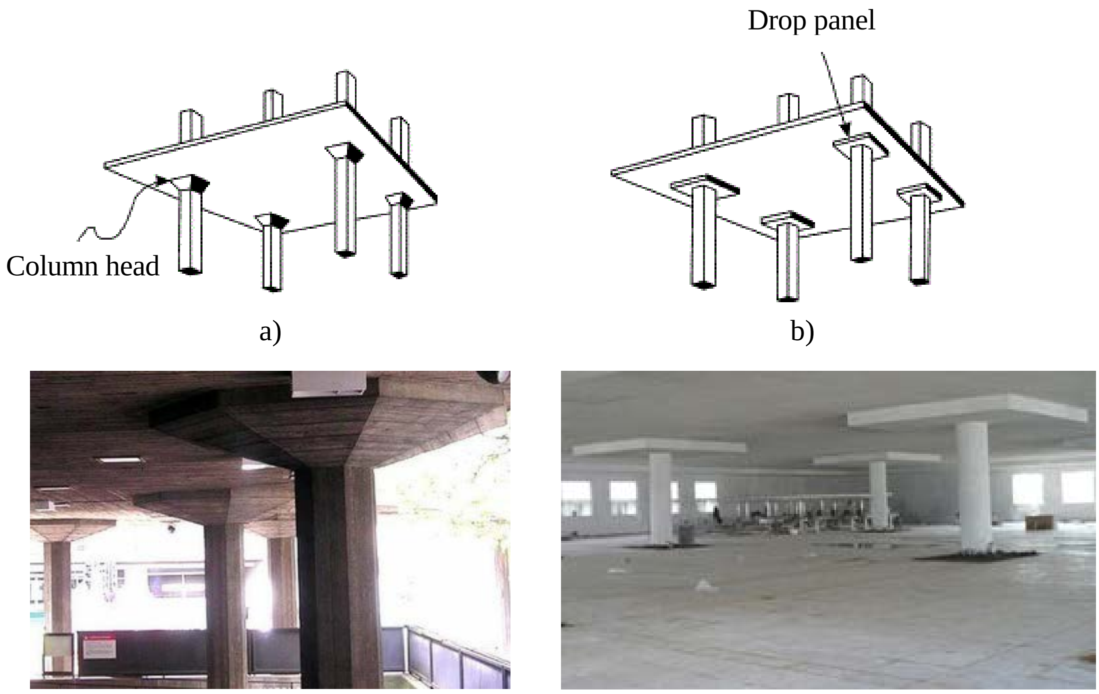


Fig. 14. 4: Flat slabs with a) column heads and b) drop panels

More specific examples of one-way and two-ways slabs, that will not be discussed here in detail, *are ribbed slabs* and *waffle slabs* (Fig. 14. 5). In ribbed slabs, the deck transfers the load in one principal direction to the ribs. The ribs then transfer the loads to the beams which carry the load in one principal direction to the columns. In the waffle slabs, the deck transfers the load in two principal direction to the supporting beams directed in two perpendicular directions. These two types of slabs are not applied frequently as they are labour intensive to construct, i.e. the formwork is expensive and installation of ducts and pipes below the slab is complicated.

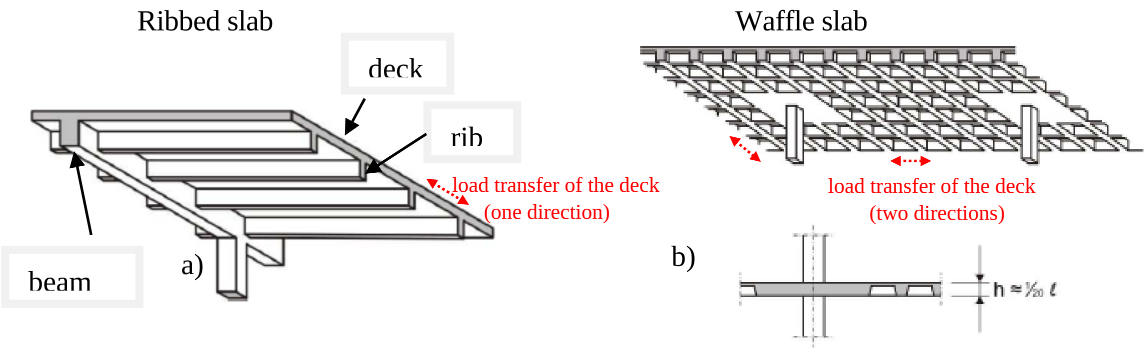


Fig. 14. 5: a) Ribbed and b) waffle slabs

The three main structural types of slabs, being the one-way, the two-way and the flat slab, are separately discussed in the following sections. Strict distinctions will be made regarding the following three main aspects:

- load transfer in the system
- designing slab to fulfil safety (ULS) and functionality requirements (SLS)
- reinforcement detailing

In engineering practice a structural engineer has to deal with several, often project specific, additional design aspects, such as required openings in slabs, concentrated loads, irregular loads, etc. Some of these aspects are dealt with in section 14.5.

14.2 Load transfer

14.2.1 One-way slab

Fig. 14. 6 sketches a system in which a slab is supported by two vertically rigid hinged line supports (rigid beams or walls). In this system the load is transferred by the slab in *y*-direction only.

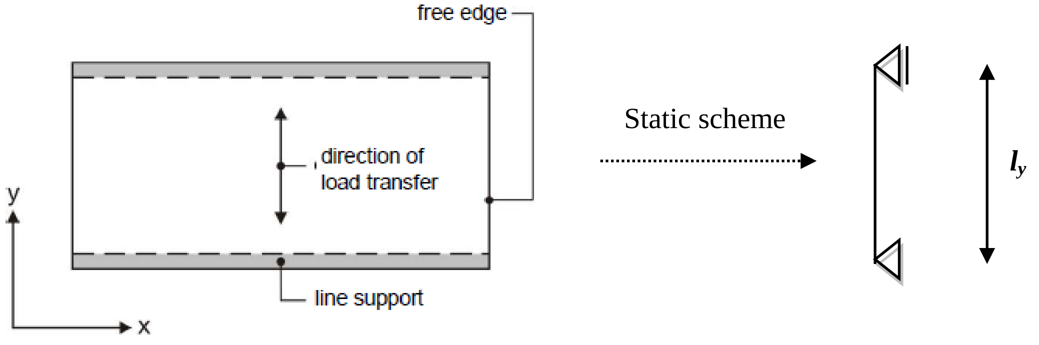


Fig. 14. 6: Slab with two vertically rigid, hinged line supports; slab spanning in one direction

The structural action of this one-way slab may be visualized by the deformed shape of the loaded surface. If a uniformly distributed load is applied to the slab surface, the deformed surface is approximately cylindrical (Fig. 14. 7). The curvature, and consequently the bending moments in *y*-direction, are almost the same in all strips, whereas there is no curvature in *x*-direction. Note that this, however, does not mean that there are no bending moments in *x*-direction, as will be explained later.

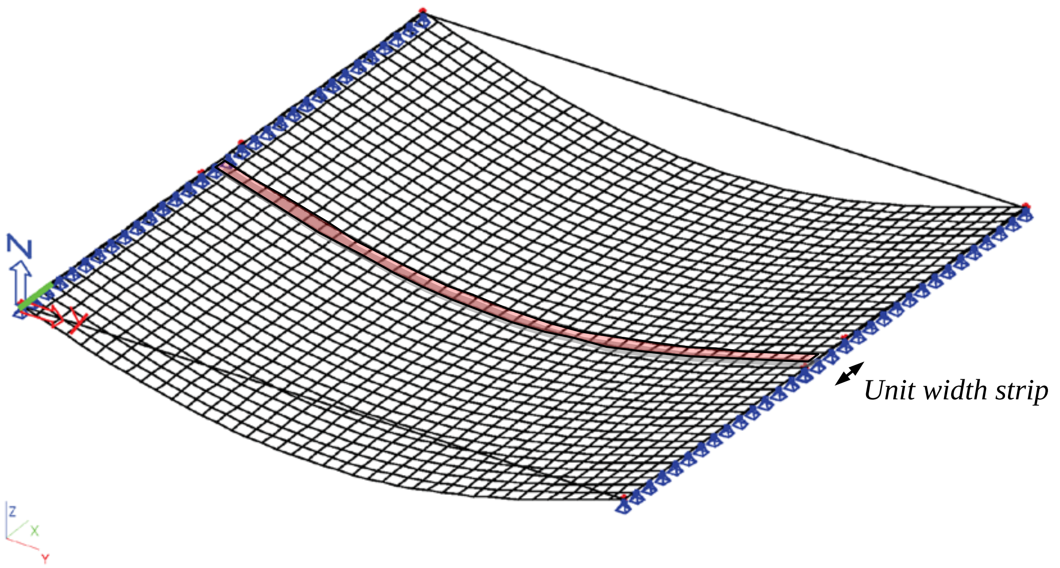


Fig. 14. 7: Deformed shape of a uniformly loaded one-way slab (two rigid hinged line supports)

Examples of one-way slabs with different boundary conditions of the supports are given in Fig. 14. 8. The first example on the left is the previously discussed slab with two rigid line supports. The second example is a continuous three span slab. The four beams supporting the slab are assumed to be rigid in vertical direction (i.e. infinitely stiff) and are therefore modelled as rigid supports. The third example shows a cantilever slab. The beam connected to the slab is assumed to provide a rigid and stiff support. In the static scheme, this is modelled as a fixity. The last example shows a slab that has rigid line supports along all four edges. Strictly speaking, this is not a one-way slab. However, there is a link: in case the ratio of the longer dimension of the slab to the shorter dimension is greater than or equal to 2, the mid part of the slab behaves as a one-way slab. This is because the load on this part of the slab is mainly transferred in the direction of the shorter span (in this case: l_y). Therefore, although the slab is supported by beams in both longitudinal and transversal direction, part of it behaves as if it was a one-way slab. A detailed derivation and explanation why this is the case can be found in Section 14.2.2 (Two-way slabs).

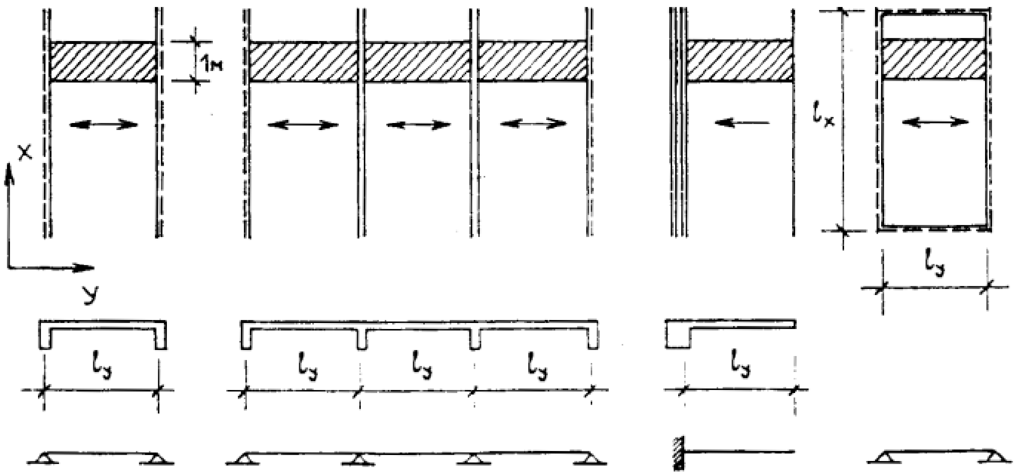


Fig. 14. 8: Examples of one-way slabs together with their corresponding static systems

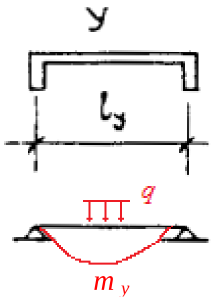
For the one-way slab, the main method of analysis is according to the theory of elasticity. The behaviour of the slab spanning in one direction can be compared with the behaviour of a beam.

Bending moments

In this example, the slab is assumed to be loaded by a uniformly distributed load q. In the calculation this slab is modelled as being split up into parallel strips having a unit width (e.g. *1 m*). Each strip may be considered as a rectangular beam with a height equal to the thickness of the slab and a span equal to the distance between the supported edges. Strips transfer the load in one direction, denoted here as the *y*-direction. Therefore, for the one-way slab given in Fig. 14. 9 the situation is analogous to the case of a simply supported beam. The maximum bending moment for a strip having a width of *1 m* occurs in the cross-section at the midspan and can be calculated as follows:

$$m_{\mathrm{y,max}} = \tfrac{1}{8} \cdot q \cdot l_y^2 \qquad [kNm/m] \qquad (14.1)$$

Note the units of this bending moment (*m*, not *M*); it is a bending moment per unit of width.
Also note that similarly as with a beam, it is assumed that a bending moment is positive when it causes tensile stresses at the bottom of the slab.



Additional information

If the strip with the width of b_x was chosen, then the moment for this strip would be $M_{y,max} = \frac{1}{8} q \cdot l_y^2 \cdot b_x \ [kNm]$.

In reality, instead of span length, l , the effective span, l_{eff}, should be used, taking into account the support lengths. More information about the l_{eff} can be found in section 14.5.1.

Fig. 14. 9: Bending moment in the strip

Whereas a beam is free and can deform in transversal direction, this is not the case with the individual slab strips. Take a closer look at the cross section of a strip in *y*-direction, having a width b_x (Fig. 14. 10a). At the strip top fibre, due to Poison's ratio, flexural compression in *y*-direction results in expansion in *x*-direction (Fig. 14. 10b). This expansion will occur in all the adjacent slab strips which also tend to expand at the top fibre level (Fig. 14. 10c). The opposite occurs at strip bottom fiber level. The bottom fibers are in tension in *y*-direction and, as a result, contract (tend to shrink/shorten) in *x*-direction. Compatibility requirements between adjacent strips makes that the free transversal deformations are restrained ($\varepsilon_x = 0$). As a result, there is also a bending moment in the opposite (transversal) direction which develops in the slab (M_x).

To determine the magnitude of this bending moment, take a closer look at a single strip in *y*-direction, having a width b_x (Fig. 14. 10a). As already indicated, the top fibre of this strip in *y*-direction is in compression while the bottom fibre is in tension. Strain at the bottom of the slab in *y*-direction is:

$$\varepsilon_{yb} = \frac{\sigma_{cy}}{E_c} = \frac{M_y}{E_c W_y} \tag{14.2}$$

where σ_{cy} is the maximum tensile stress in the load bearing strip, at the bottom of the strip (see Fig. 14. 10b). Similarly, the strain at the top of the strip is:

$$\varepsilon_{yt} = \frac{\sigma_{cy}}{E_c} = \frac{-M_y}{E_c W_y} \tag{14.3}$$

If the strip can deform freely in transverse direction (*x*-direction), the strain in this direction would be:

$$\varepsilon_x = -\nu \, \varepsilon_y \tag{14.4}$$

So for the outer tensile fiber (bottom of strip):

$$\varepsilon_{xb} = -\nu \, \varepsilon_{yb} = -\nu \frac{M_y}{E_c W_y} \tag{14.5}$$

This strain can occur only in a strip that is free to deform in *x*-direction (e.g. a beam), but not in a strip that is an integral part of a slab. Free deformation would imply introducing an opening at bottom fiber level between adjacent beams. At top fiber level, the opposite would occur, namely transverse expansion (ε_{xt}). As a result each strip tends to bend in the direction perpendicular to its longitudinal axis (Fig. 14. 10c). From compatibility requirements, the side faces of the strip should stay in a vertical position (the slab remains plane in *x*-direction). Compatibility is achieved by introducing bending moments in the transverse direction, which bring the strain in *x*-direction back to zero:

$$M_x = E_c\,(-\varepsilon_{xb})\,W_x = \nu\,M_y\,\frac{W_x}{W_y} \tag{14.6}$$

The total strain at the bottom fibre from M_x and M_y now is zero. The same result is obtained for the top fibre of the strip.

In a solid, uncracked isotropic slab $W_x = W_y$. Considering that Poisson's ratio for uncracked concrete is about 0,2, the bending moment in transverse direction results in:

$$M_x = \nu\,M_y \approx 0{,}20\,M_y \tag{14.7}$$

Therefore, for the bearing mode in transverse direction account should be taken of the restrained lateral curvature which causes secondary bending moments in transverse direction. Due to this effect transverse reinforcement is needed. This reinforcement will also enable, if required, the distribution of concentrated loads, or will take up stresses due to shrinkage. In general, 20% of the maximum reinforcement in the major load transfer direction is applied in the transverse direction (EN 1992-1-1 cl. 9.3.1.1).

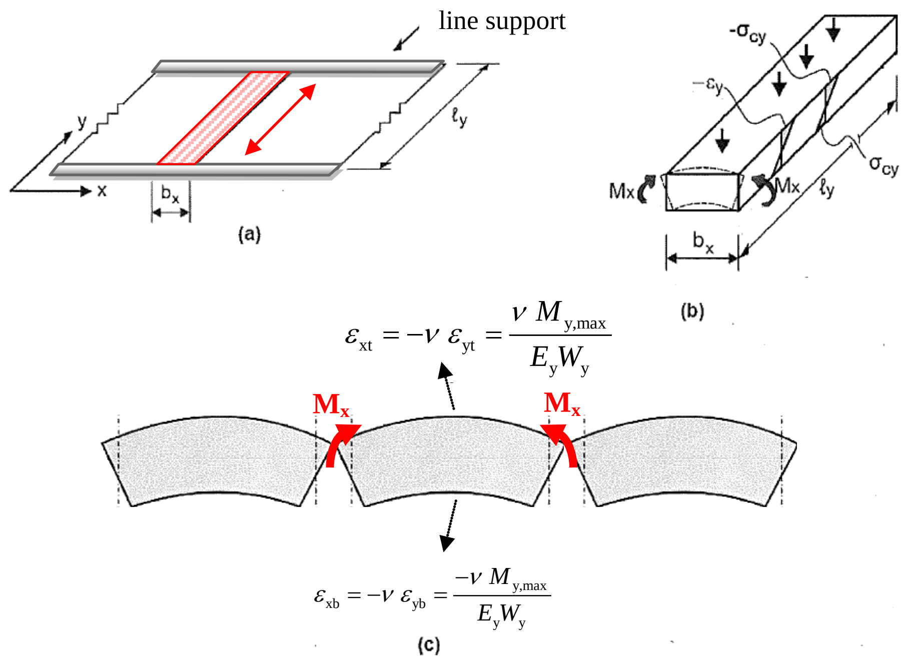


Fig. 14. 10: A strip from a slab transferring load in *y*-direction (bending moment M_y) and also loaded by a bending moment M_x in *x*-direction

Shear stresses and support reactions

The slab transfers its loads to the line supports. These supports can be walls or rigid beams. To design the supports, information on their loads is required. Therefore, the support reactions of the slab must be known. Furthermore, the slab has to be designed to resist the shear forces that result from load transfer to the line supports. This requires insight into the flow of forces in the slab. Maximum shear forces occur along the supported edges of a slab.

According to Eurocode, the shear stress at a section in a solid slab is calculated as:

$$v_{\mathrm{Ed}} = \frac{V_{\mathrm{Ed}}}{bd} \tag{14.8}$$

where V_{Ed} is the shear force due to the ultimate load, d is the effective depth of the slab and b is the width of the section considered. As shown previously for a bending moment, the calculation is usually based on a *1 m* wide strip from a slab.

In a case of a one-way slab with rigid hinged line supports and a uniformly distributed load q (Fig. 14. 11), all the load is transferred equally to the two opposite supports. The effective span is l. Therefore, the two reaction forces at the line supports (per unit length) are:

$$R_{\max} = \tfrac{1}{2} ql \qquad [kN/m] \tag{14.9}$$

The maximum shear stress per unit length in the slab occurs at the supports:

$$v_{\mathrm{Ed,\,max}} = \frac{\tfrac{1}{2} ql}{d} \qquad [kN/m^2] \tag{14.10}$$

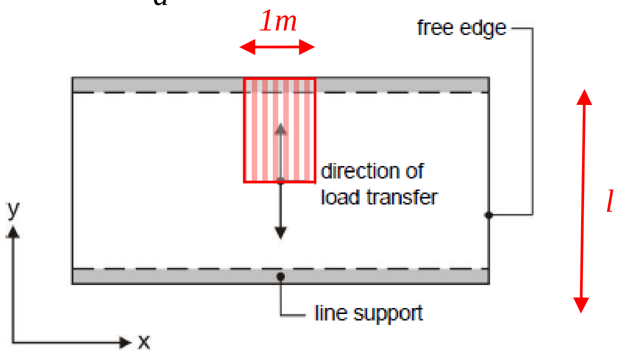


Fig. 14. 11: The load distribution for each support

Note that the maximum shear stress, calculated in this way, is overestimated. In fact, the maximum shear stress in the slab, similarly as with beam analysis, should be calculated not directly above the theoretical supports, but at the critical shear section, which is at a distance d from the face (inner edge) of the support. Note that d is the effective depth of the slab. This approach is used because the distributed load acting between the critical cross section and the face of the support is directly taken by the support. This part of the load is transferred by concrete compressive struts and, as a result, do not introduce shear stresses as known from the slender beam theory. As a result, the shear force acting on a critical cross section is smaller than the reaction force. Therefore, the presented method of calculation results in an overestimation of the shear stress, but is on the safe side and can be used for a quick check.

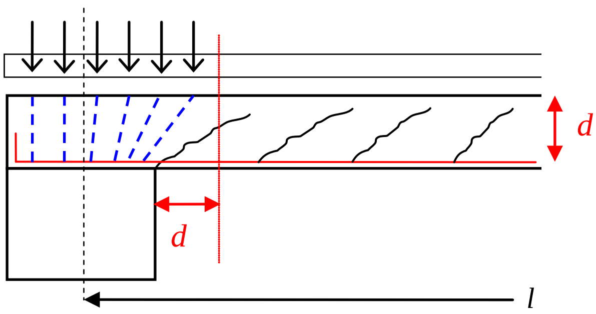


Fig. 14. 12: Critical shear cross section (note that d is the effective depth of the slab)

Stiffness of supports

If the supporting beam or wall not only provides vertical but also rotational constraint, the boundary conditions change from hinged to partially of fully clamped supports. A similar situation might occur when a masonry wall is constructed above the supporting masonry wall (Fig. 14. 13). This change of boundary conditions of course changes the occurring bending moments, shear forces and support reactions. They should be calculated according to the corresponding static scheme (Fig. 14. 14).

In reality, full fixity can be obtained if the slab extends sufficiently into a rigid concrete wall of sufficient thickness or is cast monolithically with the concrete wall. Then the connection can be modelled as providing full or flexible/partial fixity, and corresponding effects, i.e. bending moment and shear force distribution in each strip and in the slab can be calculated according to the corresponding static scheme.

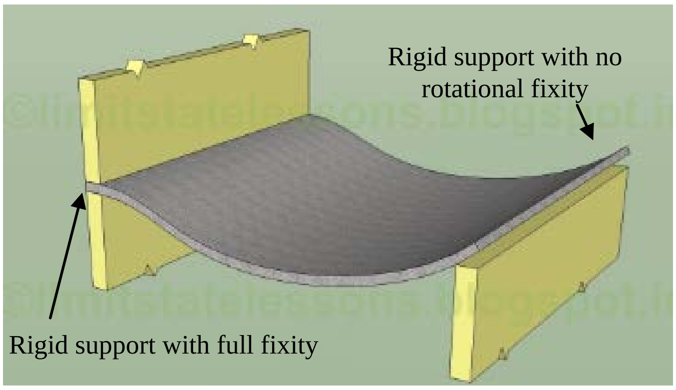


Fig. 14. 13: Partial fixity due to a wall constructed above the supporting wall.

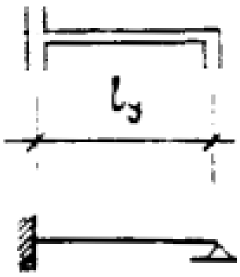


Fig. 14. 14: Corresponding static scheme for the system given in Fig. 14. 13.

It can also be that the slab is supported by a rigid beam, providing full fixity only from one side. A basic example when such system might be present is shown in Fig. 14. 15. In the figure, a balcony slab is connected to a beam. It is now assumed that the rotational stiffness and torsional strength of the beam are such, that a full fixity is provided. Note that the beam is now loaded in shear, torsion and bending in its longitudinal direction (in case it is not supported by a rigid wall).

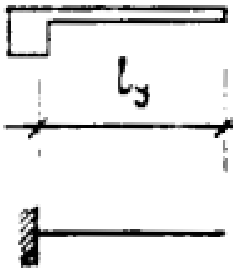


Fig. 14. 15: Example of the cantilever slab

In these examples the assumption is made that the supporting beams or walls act as vertically stiff supports. If this is not the case, and a supports become flexible, this will also influence the load distribution. For example, consider a one-way slab with two line supports that have a certain stiffness in vertical direction. The beams are supported by columns. At the position of the columns, the beams provide a stiff support. However, at positions in between the columns the beams provide a flexible support. The load transfer in case of flexible line supports is hard to determine by hand calculations since it depends significantly on the ratios between the stiffnesses of the various components (beams, slab, spans, column spacing, local boundary conditions). Nowadays numerical programs are used (e.g. the Finite Element Method, FEM) to calculate the actual behaviour. This aspect is not further elaborated here. Only the slabs with vertically rigid line supports are further discussed.

Example – comparison FEM and simplified model

In practice, a slab is often supported by beams or walls with limited and different vertical translational and rotational stiffness. The actual boundary conditions in the slab therefore usually corresponds to partial fixity. The Finite Element Method (FEM) can be used to depict this behaviour. To give an impression of FEM results, two simulations have been performed (Fig. 14. 16). In the first simulation, the slab is supported by very thin walls at both sides. In the second case, the slab is supported by thick walls. Both slabs have a span of 10 m and are loaded by a uniformly distributed load of 10 kN/m^2.

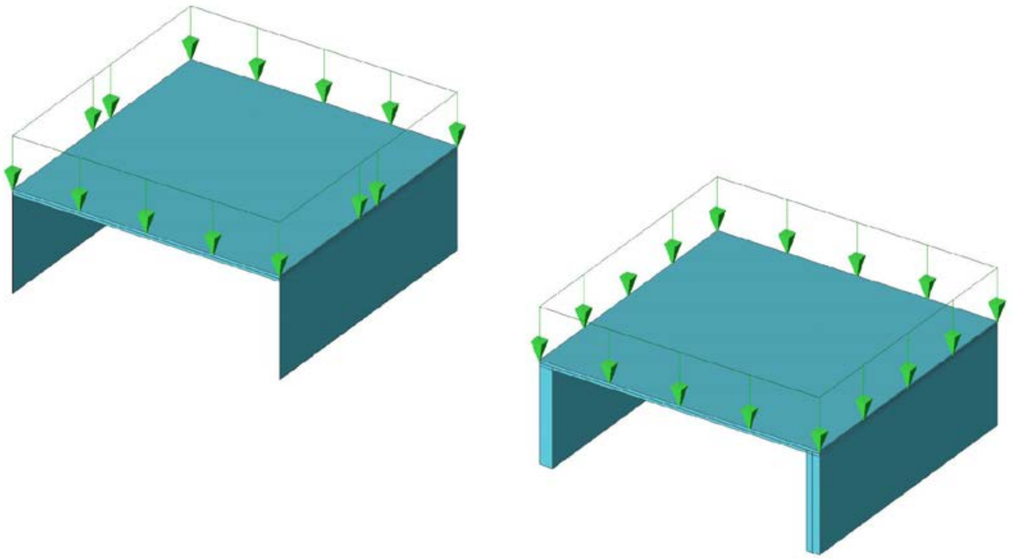


Fig. 14. 16: Two simulated cases, thin supporting walls (left) and thick supporting walls (right)

Parallel to this, a hand calculation is performed in order to check the results of the numerical simulations (Fig. 14. 16). Three types of slabs are calculated. The first slab is supported by hinged supports and the second one by fixed supports. The third slab is a combination of these two. The same as in simulations, the slabs have a span of 10 m and are loaded by a uniformly distributed load of 10 kN/m^2. The maximum sagging and hogging moments in each of the slabs can be simply calculated by using forget-me-not rules from beam design (see Fig. 14. 20).

The maximum bending moment in the span occurs for the slab with the two hinged line supports (125 kNm/m). In case the support provides full rotational fixity, the maximum bending moment in the slab is at the supports (~83 kNm/m). The third case is given only for comparison and to indicate the situation for which the maximum support moment would occur. Note that the presented bending moment values are extremes, corresponding to infinitely stiff behaviour of the supports.

These calculations are made according to the beam theory. In slabs there are also secondary moments which come from restrained deformations in transversal directions. These transversal moments (m_y) can be calculated as:

$m_{yy} = \nu \cdot m_{xx} = 0.2 m_{xx}$, where ν is Poisson's ratio of concrete.

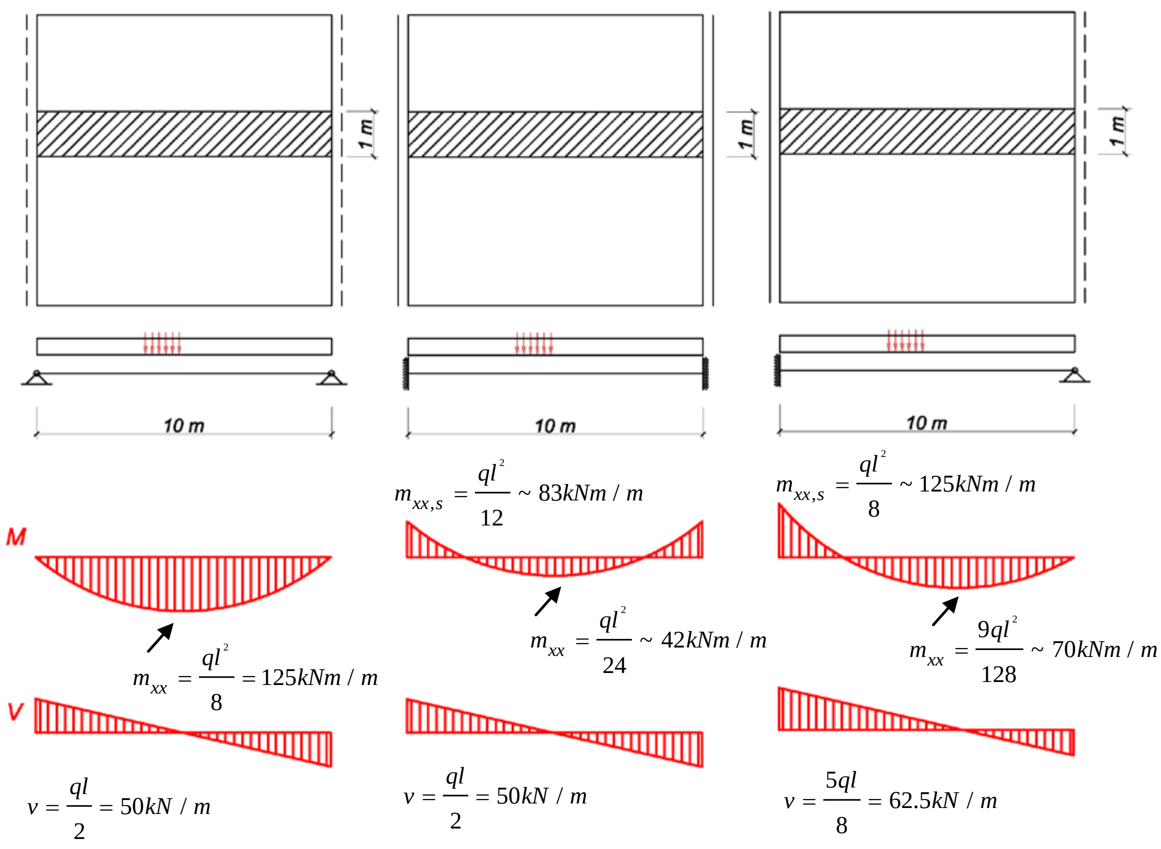


Fig. 14. 17: Different examples of one-way slabs and effects

Consider now the results from FEM analysis (Fig. 14. 18). In the first simulation, the walls are very thin and do not provide any rotational stiffness. Therefore, the connection between the slab and the wall is close to a hinged connection. The maximum sagging moments in the middle of the slab (~123 kNm/m) are almost equal to those obtained by hand calculation, assuming that the supports provide no rotational fixity at all (125 kNm/m).

In the second case, the slab is supported by thick walls. The thick walls provide almost full fixity at the connection (no rotation allowed). As a result, the maximum sagging moment and hogging moments at the supports correspond now to those obtained by hand calculations for a situation with clamped supports.

The secondary moments obtained by FEM (Fig. 14. 19) are in order of magnitude around 15-20% of the main bending moments, as expected. It is interesting to notice that the bending moment isolines (m_x) are not straight lines. This can be attributed to the reduced confining effects towards the free edges of the slab. Transverse bending moments reduce to zero at free sides (which are free to deform), so there are disturbing effects in the longitudinal moments which start at the free edges but gradually diminish towards the middle of the slab. These effects cannot be considered by simplified beam analysis. Note that results obtained by numerical analysis should always be carefully checked by an engineer and by, usually applicable, hand calculations.

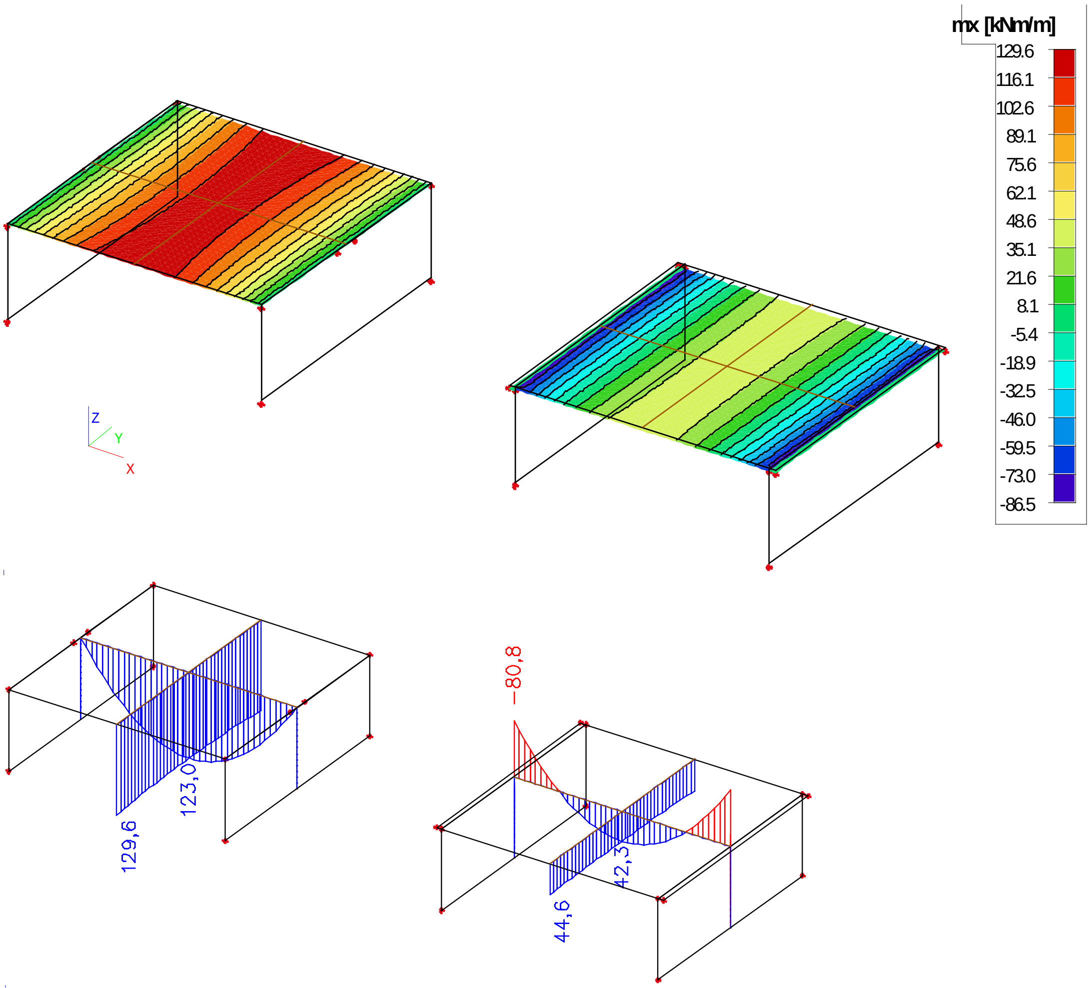


Fig. 14. 18: Bending moments m_x

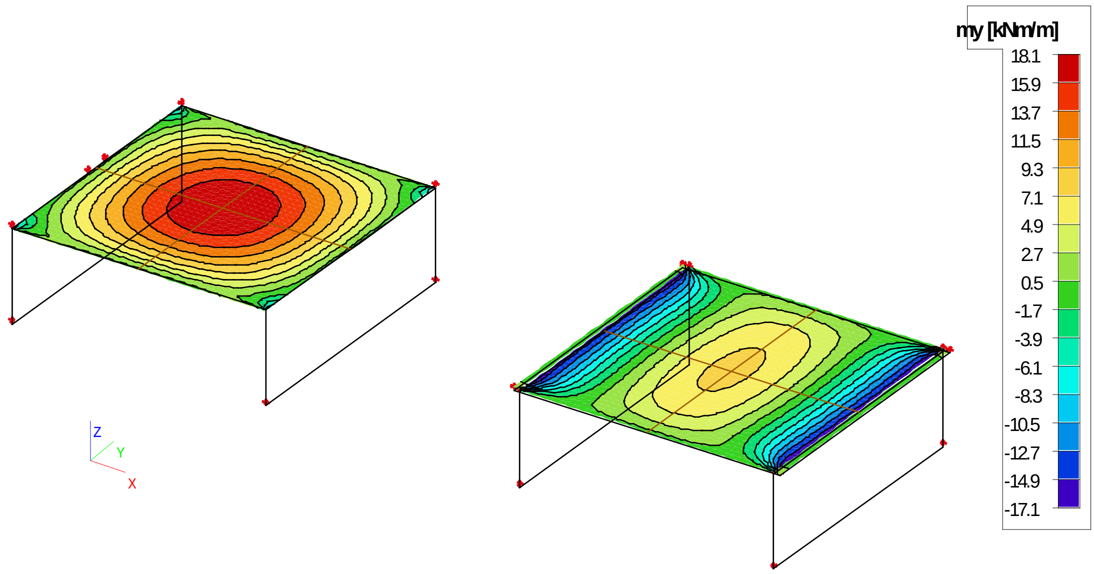


Fig. 14. 19: Bending moments m_y

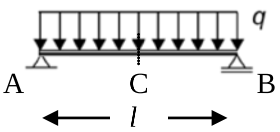
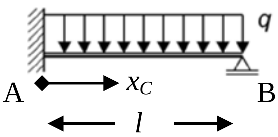
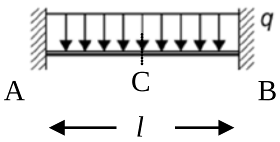
	R	M_{max}	f_{max}
	$A = B = \frac{ql}{2}$	$M_C = \frac{ql^2}{8}$ $x_C = \frac{l}{2}$	$f_C = \frac{5}{384}\frac{ql^4}{EI}$ $x_C = \frac{l}{2}$
	$A = \frac{5ql}{8}$ $B = \frac{3ql}{8}$	$M_A = -\frac{ql^2}{8}$ $M_C = \frac{9}{128}ql^2$ $x_C = \frac{5}{8}l$	$f_C = 0.005416\frac{ql^4}{EI}$ $x_C = \frac{1}{16}(15-\sqrt{33})$
	$A = B = \frac{ql}{2}$	$M_A = M_B = -\frac{ql^2}{12}$ $M_C = \frac{ql^2}{24}, x_C = \frac{l}{2}$	$f_C = \frac{1}{384}\frac{ql^4}{EI}$ $x_C = \frac{l}{2}$

Fig. 14. 20: Forget-me-not formulas for bending moments, reaction forces and deflections

Apart from the dimensions of the elements in the structure, also the reinforcement distribution might influence the bending moment distribution. So far, the linear elastic theory for the analysis of the slabs was discussed. The Eurocode also allows using the linear elastic theory with limited redistribution for the verification of the Ultimate Limit State (ULS). This means that the bending moments at ULS calculated using a linear elastic analysis in statically undetermined structures may be redistributed provided that the resulting distribution of moments remains in equilibrium with the applied loads. In Eurocode 2 (section/clause 5.5.4) the maximum allowed values for the ratio of the redistributed moment to the elastic bending moment are given. In order to enable this redistribution, a plastic hinge with sufficient rotational capacity has to be present. Therefore, the redistribution should not be applied when the rotation capacity is too small or cannot be defined with confidence.

Continuous one-way slabs

Often slabs are continuous and extend over a number of supports. Continuous slabs should in principle be designed to withstand the most unfavourable arrangements of loads, in the same way as beams. Therefore, the governing load combination has to be found.

A continuous slab with *n* spans given in Fig. 14. 21 is now considered. The slab is loaded by a permanent load and a live load, both uniformly distributed loads. Since the permanent load *G* is always present, the most unfavourable arrangements of live loads will be governing for the design. The question now is, how to arrange the live load *Q*.

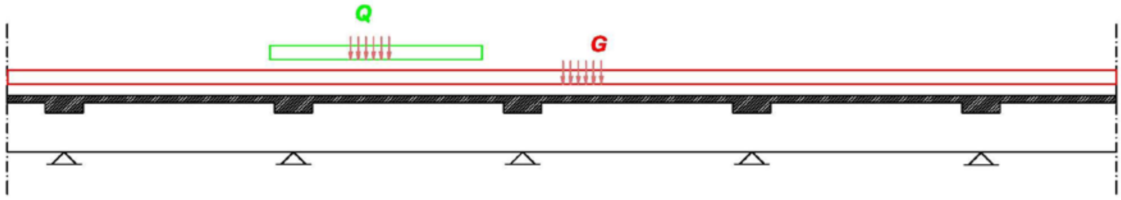


Fig. 14. 21: Continuous one-way slabs

Maximum positive bending moment:

In order to get the largest sagging moment in the slab by hand calculations, the load distribution should be according to Fig. 14. 22. This load distribution (*G*+*Q*) can then be divided in two basic parts: the first one consisting of *G*+*Q*/2 and the second one consisting of *Q*/2. When the analysed area is considered, different boundary conditions arise from the two loading parts:

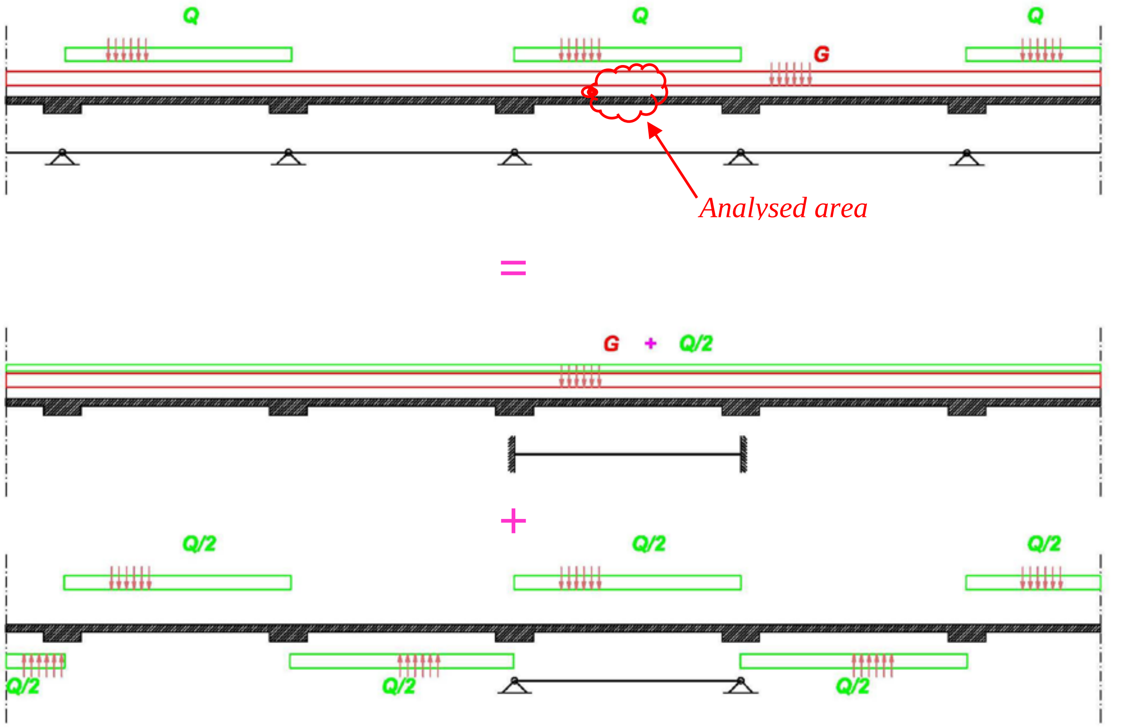


Fig. 14. 22: Load distribution for the maximum sagging moment and the corresponding boundary conditions of the considered area

The maximum sagging moment can then simply be estimated by using forget-me-not rules (Fig. 14. 20) and by superposition of the effects corresponding to these two load combinations

(*G*+*Q*/2 and *Q*/2) and their corresponding boundary conditions. The boundary conditions follow from the deformation of the slab. At the load *G* + *Q*/2 it is clear that, because of symmetry, no rotation is present at supports (Fig. 14. 24). As a result, the static scheme of a span next to the support where the maximum hogging moment occurs has two fixed ends. At the load *Q*/2 however, the alternating load is present at all spans, resulting in slab rotation at supports. Therefore, the static scheme of the span now has two ends where rotation is allowed to occur. Note that this is the static scheme that corresponds to the maximum slab moment, as also indicated in Fig. 14. 22.

Maximum negative bending moment:
In order to get the largest hogging moment in the slab, the load should be acting above the supports, according to the load distribution presented in Fig. 14. 23. Similarly as in case of the maximum sagging moment, the maximum hogging moment can then simply be estimated by the superposition of the effects corresponding to the two load combinations (*G*+*Q*/2 and *Q*/2) and corresponding boundary conditions.

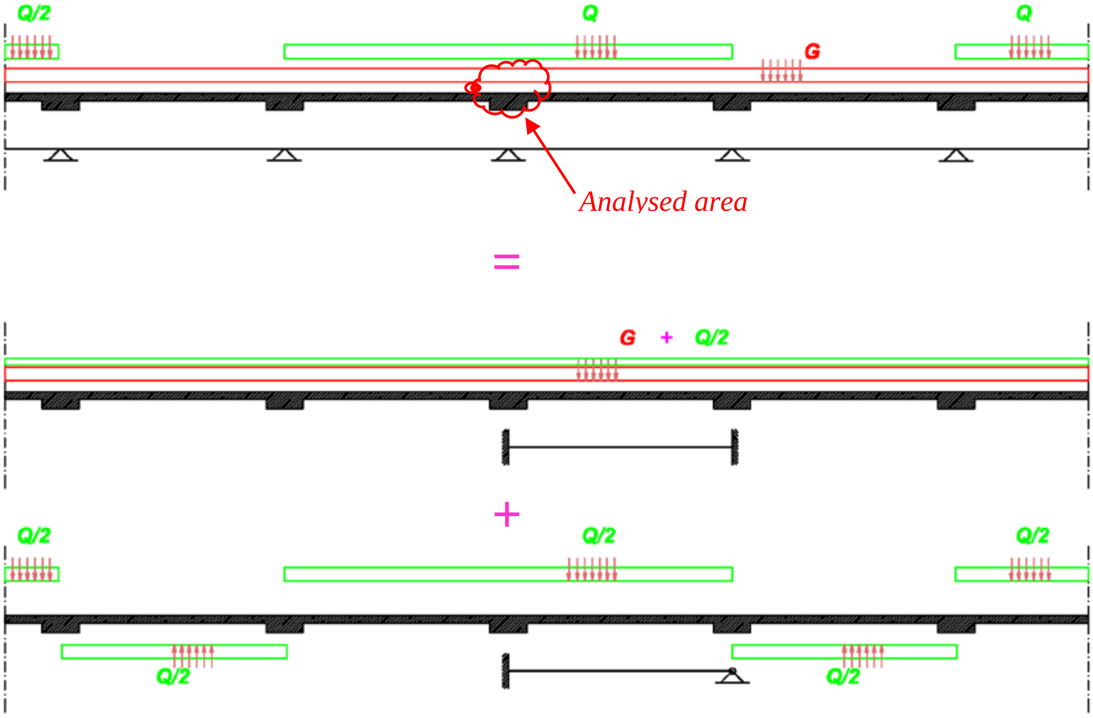


Fig. 14. 23: Load distribution for the maximum hogging moment and corresponding boundary conditions

At the load *Q*/2, at one support there is no rotation (both adjacent spans are loaded), whereas the alternating loads at all the remaining supports make that slab rotation occurs at these supports. The static scheme of the span now has one fixed end and one end where rotation is allowed to occur.

Additional information:

How loading conditions determine the boundary conditions in the system:

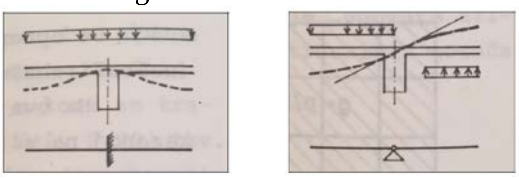


Fig. 14. 24: Fixed and hinged boundary conditions

Similarly, the maximum shear forces, present next to the supports in the solid slab, should be calculated and checked for their governing load combinations too.

Note that in case of multiple spanning one-way slabs, also the tabulated values for continuous two, three spans etc. spanning beams are available and can be used to determine the maximum moments and shear forces in the slab and the support reactions. Be aware, however, that the application of these tables is subjected to some restrictions (e.g. the assumed ratio between live load and self-weight and span ratio in the case of different spans). Also in that case, account should be taken for the fact that the live load does not need to be present at all the slab spans simultaneously. These analyses can be nowadays done in FEM programs where the envelopes of effects are calculated.

Transition from one-way to two-way slabs

One should be careful at locations where the uni-axial load transfer from a one-way spanning slab is disturbed by other support reactions. Fig. 14. 25 shows an example of a slab (line supports 1 & 2) where at one end (side 3) an additional support reaction occurs, for instance by a supporting wall. This extra support results in a local reduction of the load transfer in the *y*-direction, but simultaneously causes a local bending moment (m_{xx}) in *x*-direction which is larger than the transversal moment in one way slab, νm_{yy}. As a result, in a part of the slab the load is not transferred in one but in two directions.

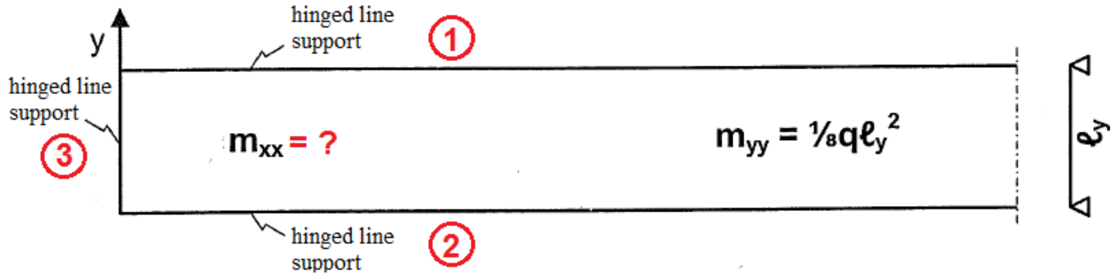


Fig. 14. 25: Top view and longitudinal cross-section of a one-way spanning slab with distortion at the edge

This can be depicted by FEM analysis (Fig. 14. 26). A slab with dimensions 10 m x 50 m and simply supported (vertically rigid, hinged line supports) at three sides is modelled. The slab is loaded with a uniformly distributed load of 10 kN/m^2. At the certain distance from the wall (~ 20 m from the left support) the moments in *y*-direction become uniform, the load is transferred in one direction only (the magnitude of the primary moment reaches the moment of a one-way slab: $ql^2/8 = 125 kNm/m$) and the slab behaves as a one-way slab. However, this is not valid for the first 10 m next to the left support where it appears that the load is transferred in 2 directions. Note that in this 10 m also the transversal moments are larger than $\nu m_{yy} = 0.2 \cdot 125 = 25 kNm/m$.

The theory of elasticity can be applied to find the actual two-dimensional load transfer mechanism and the bending and torsional moments that occur. The reinforcement to be applied should be adjusted to the two-dimensional flow of forces. Section 14.2.2 deals with these aspects of two-way spanning slabs. Therefore, also the question of transition between one-way to two-way slabs is dealt with in detail.

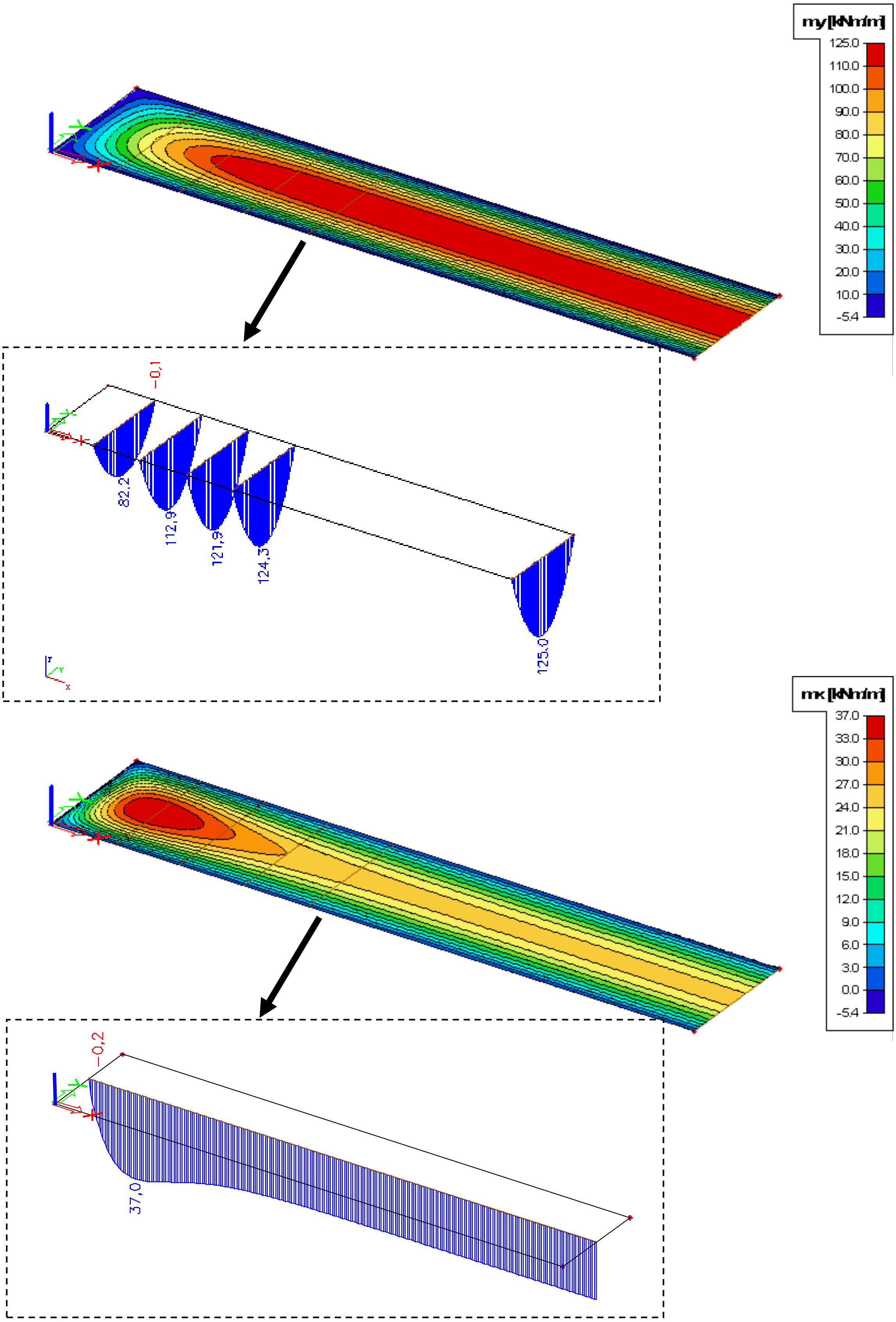


Fig. 14. 26: Results of FEM analysis and cross sectional moments in *x*- and *y*-direction in a slab simply supported from 3 sides

14.2.2 Two-way slab

Fig. 14. 27 sketches the system in which a slab is supported by four rigid line supports (beams or walls). In this system the load is transferred both in *x*- and *y*-direction.

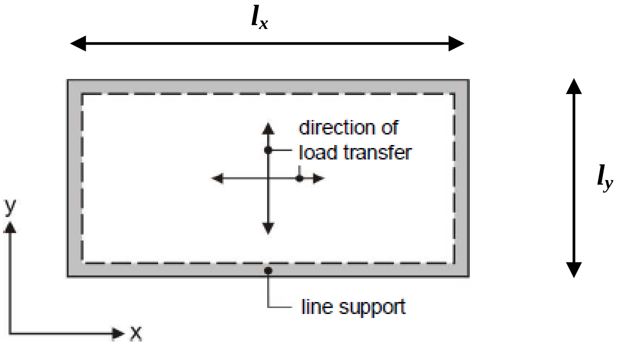


Fig. 14. 27: Slab with line supports spanning in two directions

The structural behaviour of a two-way slab may be visualized by the deformed shape of the loaded surface. Fig. 14. 28 shows a rectangular slab, supported by rigid line supports along its four sides. These supports might be deep, stiff (monolithic) concrete beams, or walls, or stiff steel girders. With a uniformly distributed load at the surface, such a slab bends into a dished surface, rather than a cylindrical one, as it was the case with a one-way slab. This means that at any point the slab is curved in two principal directions. Since bending moments are proportional to curvatures, bending moments also exist in both directions. If the concrete edge beams are flexible, the vertical deformation of the slab along the sides significantly alters the distribution of bending moments in the slab panel. This is valid for flat slabs, which is a specific type of two-way slab and will be separately considered.

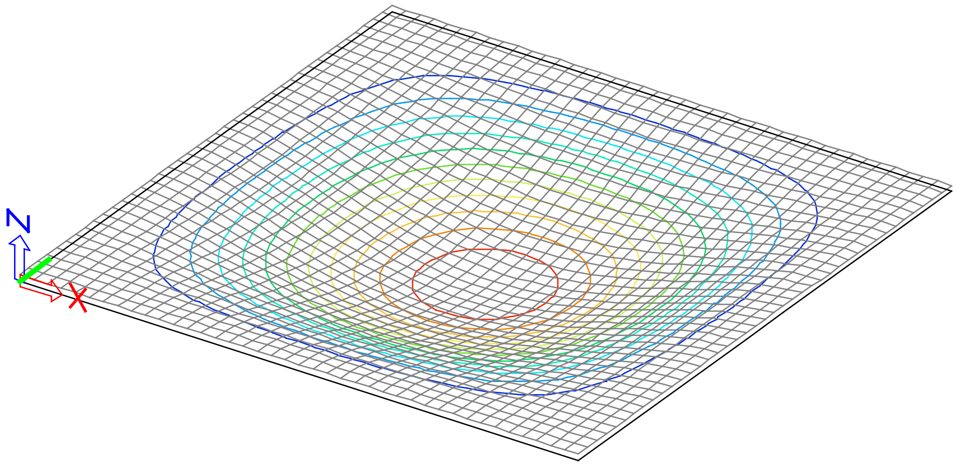


Fig. 14. 28: Deflected shape of uniformly loaded two-way slab with four stiff line supports providing full fixity

For two-way spanning slabs with line supports three methods of analysis apply:

The theory of elasticity

> Under the assumption of a homogeneous (uncracked) and elastic slab material the bending and torsional moments are calculated. The moments obtained are used as a starting point to perform cross-sectional analyses to determine the amount of reinforcement required. This method is on the safe side (lower bound value approach).

The strip method (the equilibrium method)

According to this method the slab is assumed to be composed of strips that transfer the load in two directions. When defining the load transfer mechanism the designer has a certain freedom. The goal is to create a system of equilibrium that results in a practical and economical reinforcement layout, but also is realistic in a way that it is close to the actual behaviour of the slab in the uncracked state (i.e. linear elastic). Similarly as the theory of elasticity, this method is also on the safe side (lower bound value approach). However, compared to the theory of elasticity, this method is less economic as more reinforcement is required. The main advantages are that it is always applicable, it is fast and the designer has insight and decides how the load can be transferred to the supports.

The yield line theory

This method aims at finding the failure mechanism for a slab with given reinforcement detailing. A number of potential failure mechanisms has to be researched to find the one which gives the smallest failure load. Since the designer is never sure to have found the mechanism that results in the lowest bearing capacity, this method is in principle not a safe one (it is an upper bound value approach).

Today, nearly always the theory of elasticity or the strip method is used for the dimensioning and detailing of slab reinforcement.

Analysis with the theory of elasticity

The classical theory of elasticity for plates loaded perpendicular to the plate plane was derived in the 1800s. Take an infinitesimally small plate element loaded in *z*-direction by a uniformly distributed load *q*. Fig. 14. 29 shows the occurring bending moments m_{yy} and m_{xx}, torsion moments m_{xy} and m_{yx} and shear forces v_x and v_y. Note that all these are per unit length.

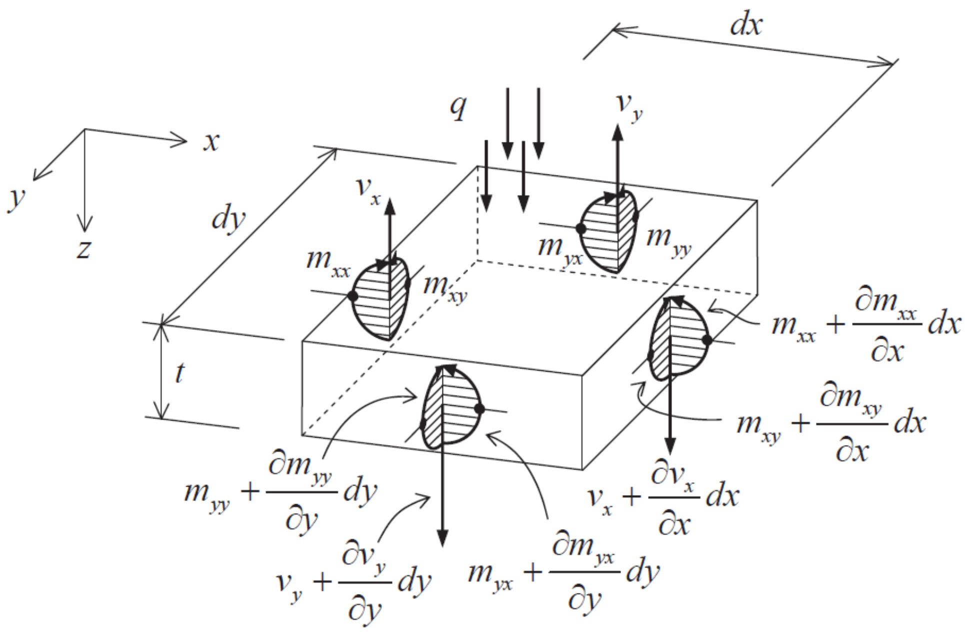


Fig. 14. 29: Infinitesimal plate element

Considering that the equilibrium of forces in *z*-direction has to be satisfied:

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + q = 0 \qquad (14.11)$$

and that the summation of moments from all forces about the *x*-axes and *y*-axis has to be equal to v_x and v_y, respectively:

$$\frac{\partial m_{xx}}{\partial x} + \frac{\partial m_{xy}}{\partial y} = v_x$$

$$\frac{\partial m_{yy}}{\partial y} + \frac{\partial m_{yx}}{\partial x} = v_y \qquad (14.12)$$

Since

$$m_{xy} = m_{yx} \qquad (14.13)$$

this results in the plate equilibrium equation:

$$\frac{\partial^2 m_{xx}}{\partial x^2} + 2\frac{\partial^2 m_{xy}}{\partial x \partial y} + \frac{\partial^2 m_{yy}}{\partial y^2} = -q$$

The solution can be found either by solving analytically the equation (with some further assumptions) or, for example, by using the Finite Element Method (FEM). Engineers generally tended to use tabulated solutions for plates with different boundary and loading conditions, which are provided in textbooks. However, nowadays, the calculations are mostly carried out using FEM analysis.

In two-way spanning slabs, bending moments as well as torsional moments occur. Special attention is to be given to these torsional moments. It appears that the corners of the simply supported slab tend to lift up when the slab is uniformly loaded (Fig. 14. 30).

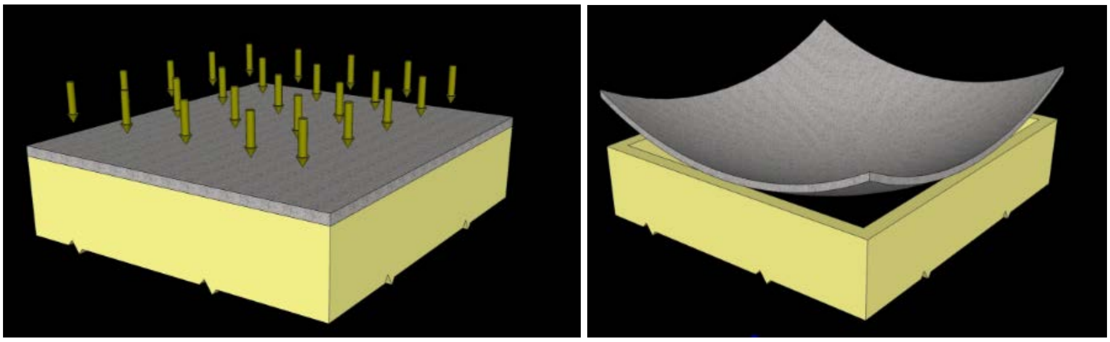


Fig. 14. 30: Lifting-up of the slab at the corners

Lifting up might be prevented, for example, due to a wall built on top of the slab, above a support, or when the slab is supported by and connected to relatively stiff line supports. It is now assumed that all four corners cannot lift up. As a result, bending moments at an angle of 45° relative to the plate edges occur in the four corner parts of the slab. To explain this more in detail, consider the schematic representation in Fig. 14. 31. The slab is assumed to have rigid line supports along the four edges. No fixity is present. The yellow arrows represent the uniformly distributed load acting on the slab (Fig. 14. 31a). The red arrows represent the reaction forces acting at the supports. Let us divide the slab into strips and take one particular strip from this set: the second one from the longer support. (Fig. 14. 31b,c).

At the middle portion of the strip, the inner edge is at a lower level than the outer edge. And towards the ends of the strip, both the edges are at the same level. So the strip is twisted (Fig. 14. 31c). The slab resists the external loads not only by bending but also by so-called torsion. This torsional effect increases towards the ends of the strip. The strips nearer to the two long walls experience this torsion to a greater extent than the strips near the middle of the slab. In the same way, in the other set of strips which are perpendicular to the red strips, the torsional effect will be greater at the ends of those strips which are nearer to the short walls.

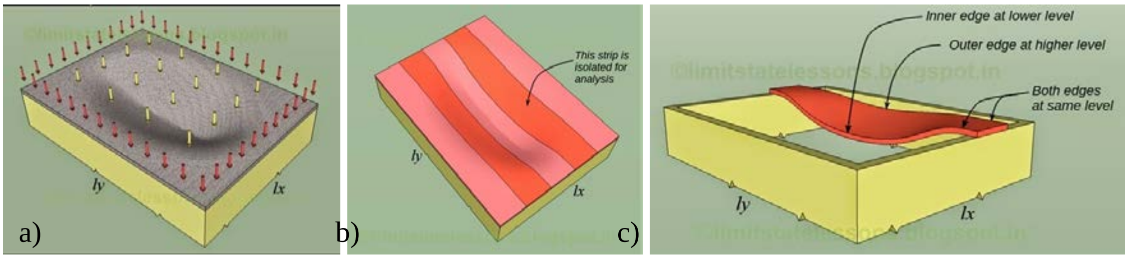


Fig. 14. 31: a) view a two way slab which corners are held down b) strips parallel to l_y and c) analysis of a single strip

The torsional effect will be more pronounced in the four corners of the slab. If torsional moments are larger than the cracking moment, they induce cracking at the top and the bottom side of the slab (Fig. 14. 32). These cracks are in a direction perpendicular to the diagonal of the slab at the top surface and parallel to it at the bottom surface. Therefore, reinforcement to resist this torsion has to be provided. If slabs are calculated according to the theory of elasticity, the total amount of reinforcement used in two orthogonal directions must be based on the so-called ***"reinforcement" bending moments***:

$$m^*_{xx} = m_{xx} \pm m_{xy};$$
$$m^*_{yy} = m_{yy} \pm m_{xy}; \qquad (14.14)$$

where m_{xx} and m_{yy} are the bending moments per unit length in x and y-direction, and m_{yy} is the torsional moment per unit length.

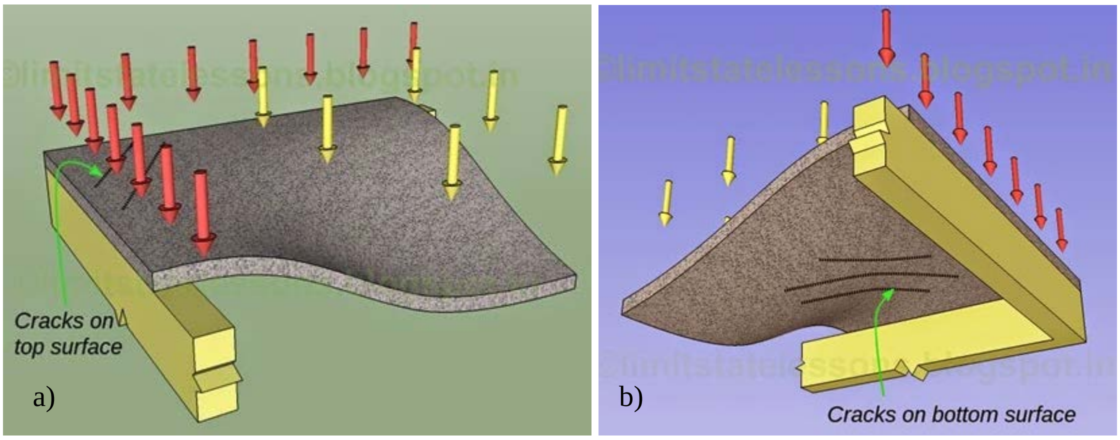


Fig. 14. 32: Cracking at the edges of the slab due to torsion at the a) top and b) bottom surface

Bending and torsional moments

In Fig. 14. 33 the distributions of the bending and torsional moments in the *x*- and *y*-direction are given for a slab with hinged line supports on all four edges, see the Fig. 14. 33 a, b, c and d respectively. In Fig. 14. 34 a small part of the slab is shown, indicating the positive orientations of the moments.

Note that at the location of the maximum bending moments, the torsional moments are zero, and at the location of the maximum torsional moments, the bending moments are zero.

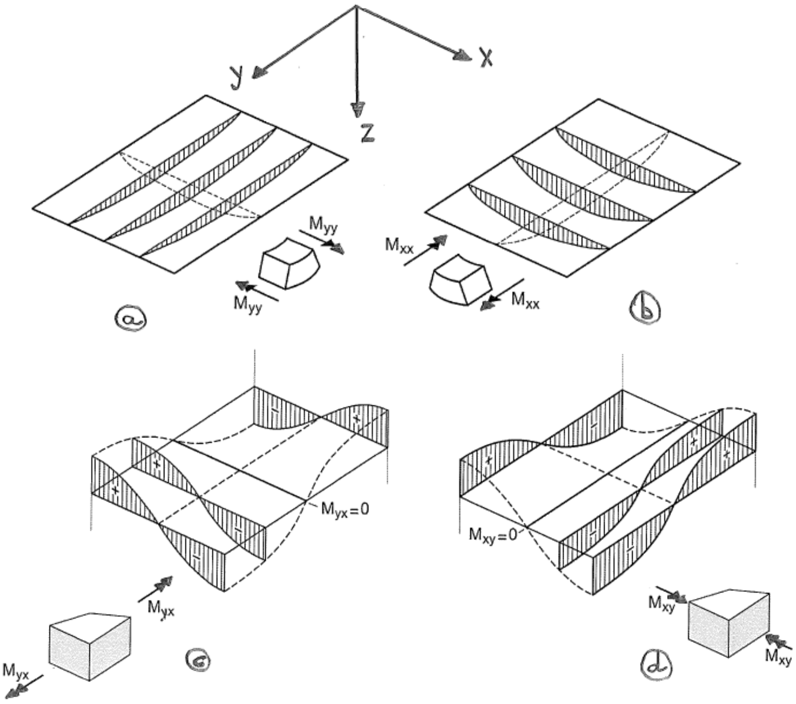


Fig. 14. 33: a. Distribution of the bending moment in the *y*-direction
b. Distribution of the bending moment in the *x*-direction
c. Distribution of the torsional moment in the *x*-direction
d. Distribution of the torsional moment in the *y*-direction

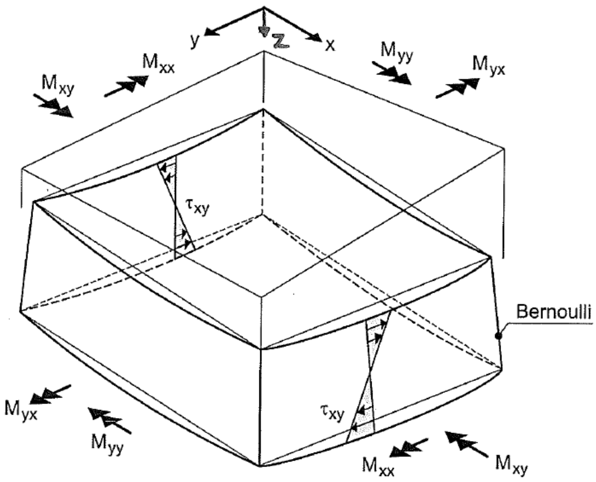


Fig. 14. 34: Positive orientations of bending and torsional moment and deformations of an elementary part of the slab

EN 1992-1-1 presents no tables or figures that can be used to easily calculate the bending and torsional moments that occur in two-way spanning slabs. The designer is referred to textbooks.

The former Dutch code NEN 6720 contained such information in tables. To illustrate the theory, Table 14. 2 (from NEN 6720, table 18 combined with calculated deformations from GTB 2010, chapter 8) presents the bending moments and maximum deflections for a number of slabs having different boundary conditions. Bending moments are expressed using coefficients. In these coefficients the bending moments per unit width as obtained from a linear elastic calculation are incorporated. The coefficients hold for the bending moment in the so-called middle strips which are assumed to be present over 50% of l_x and l_y, see Fig. 14.35 c and d. The remaining parts of the slab are so-called side strips.

Torsion moments are taken into account separately, by prescribing additional reinforcement to be applied in specific regions of the slab. This reinforcement must be applied in two directions at both top and bottom of a slab. The torsion moment is assumed to be equal to the maximum sagging bending moment in the middle strips. These aspects of torsion will be discussed later.

In calculations, the ideal boundary conditions are considered: with full rotational capacity (simply supported) and with the full fixity (clamped). The coefficients are determined without taking into account bending moment redistribution that might occur due to local cracking; it is a linear elastic result. The slabs are assumed to be loaded with a uniformly distributed load q over the entire slab surface.

Example
As an example a slab with rigid hinged line supports along its four edges is analysed. First, the exact linear elastic moment distributions will be discussed. Next, the simplified method from Table 14. 2 will be presented.

The curved bending and torsional moment distributions found when using the theory of elasticity are shown in Fig. 14. 35a. The bending moments m_{xx} and m_{yy} are zero near the edges. Fig. 14. 35b shows the simplified linear distributions which serve as a base for calculating the reinforcement required. The distribution of the maximum span moments in x- and y-direction (m_{xx} and m_{yy}) and the distribution of the torsional moments in x- and y-direction (m_{yx} and m_{xy}) are presented.

This is slab type I in Table 14. 2. Coefficients are presented for various aspect ratios l_y / l_x. It is assumed that, in this example, the ratio is 1,5, so interpolation between results for ratios 1,4 and 1,6 is required.

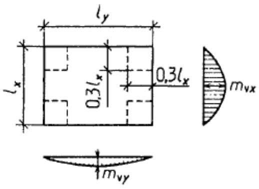
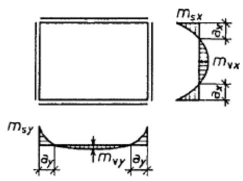
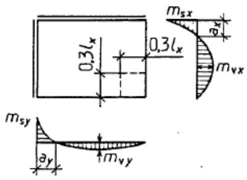
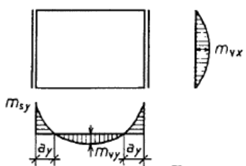
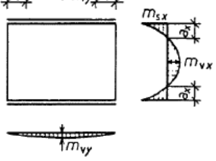
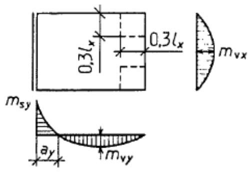
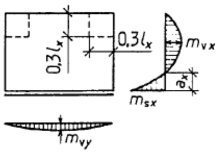
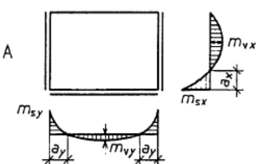
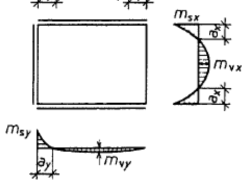
To summarize, Table 14. 2 slab type I; l_y / l_x = 1,5 (v = span = sagging moment):

$m_{vx} = 0,073\, q\, l_x^2$ and $m_{vy} = 0,030\, q\, l_x^2$

It turns out that these table results are close to the exact results given in Fig. 14. 35a:

$m_{vx} = \frac{1}{13,7}\, q\, l_x^2 = 0,073\, q\, l_x^2$ and $m_{vy} = \frac{1}{35,7}\, q\, l_x^2 = 0,028\, q\, l_x^2$

Table 14. 2: Governing bending moment and deflection coefficients per unit of width in the middle strips of slabs on rigid line supports loaded by a uniformly distributed load (NEN 6720).

			l_y/l_x	1,0	1.2	1.4	1.6	1,8	2.0	2.5	3.0
I		m_{vx}	$= 0{,}001\ p_d l_x^2$	41	54	67	79	87	97	110	117
		m_{vy}	$= 0{,}001\ p_d l_x^2$	41	35	31	28	26	25	24	23
		w	$= 0.001 \dfrac{p_d l_x^4}{EI}$	4,1	5,6	7,0	8,2	9,2	10		12.2
II		m_{vx}	$= 0{,}001\ p_d l_x^2$	18	26	32	36	39	41	42	43
		m_{vy}	$= 0{,}001\ p_d l_x^2$	18	16	12	10	10	10	10	10
		m_{sx}	$= -\ 0{,}001\ p_d l_x^2$	51	63	72	78	81	82	83	83
		m_{sy}	$= -\ 0{,}001\ p_d l_x^2$	51	54	55	54	54	53	51	49
		$a_x / l_x =$		0,18	0,19	0,19	0,19	0,19	0,19	0,19	0,19
		$a_y / l_y =$		0,18	0,16	0,15	0.13	0,11	0,10	0.07	0,06
		w	$= 0.001 \dfrac{p_d l_x^4}{EI}$	1,3	1,7	2,1	2,3	2,4	2,5		
III		m_{vx}	$= 0{,}001\ p_d l_x^2$	25	36	45	53	58	62	67	69
		m_{vy}	$= 0{,}001\ p_d l_x^2$	25	23	20	19	18	17	17	17
		m_{sx}	$= -\ 0{,}001\ p_d l_x^2$	68	84	97	106	113	117	122	124
		m_{sy}	$= -\ 0{,}001\ p_d l_x^2$	68	74	77	77	77	76	73	71
		$a_x / l_x =$		0,20	0,22	0,22	0,22	0,23	0,23	0,24	0,24
		$a_y / l_y =$		0,21	0,19	0,17	0,16	0.13	0,12	0,09	0.09
		w	$= 0.001 \dfrac{p_d l_x^4}{EI}$	2,1	3,0	3,6	4,2	4,5	4,9		
IV A		m_{vx}	$= 0{,}001\ p_d l_x^2$	16	28	42	56	69	80	100	112
		m_{vy}	$= 0{,}001\ p_d l_x^2$	29	32	32	30	27	24	20	18
		m_{sy}	$= -\ 0{,}001\ p_d l_x^2$	69	85	97	105	110	112	112	112
		$a_y / l_y =$		0,19	0,19	0,17	0,17	0,16	0,15	0,12	0,11
		w	$= 0.001 \dfrac{p_d l_x^4}{EI}$	1,9	3,2	4,6	6,0	7,2	8,4		
IV B		m_{vx}	$= 0{,}001\ p_d l_x^2$	29	34	38	40	42	42	42	42
		m_{vy}	$= 0{,}001\ p_d l_x^2$	16	14	13	13	13	13	13	13
		m_{sx}	$= -\ 0{,}001\ p_d l_x^2$	69	76	80	82	83	83	83	83
		$a_x / l_x =$		0,19	0,20	0,20	0,21	0,21	0,21	0,21	0,21
		w	$= 0.001 \dfrac{p_d l_x^4}{EI}$	1,9	2,2	2,4	2,5	2,6	2,6		
V A		m_{vx}	$= 0{,}001\ p_d l_x^2$	27	41	54	67	78	89	105	115
		m_{vy}	$= 0{,}001\ p_d l_x^2$	38	37	34	30	27	25	24	23
		m_{sy}	$= -\ 0{,}001\ p_d l_x^2$	91	102	108	111	113	114	114	114
		$a_y / l_y =$		0,21	0,21	0,20	0,18	0,17	0,15	0,13	0,10
		w	$= 0.001 \dfrac{p_d l_x^4}{EI}$	2,8	4,3	5,8	7,1	8,2	9,2		
V B		m_{vx}	$= 0{,}001\ p_d l_x^2$	38	44	52	58	62	65	68	70
		m_{vy}	$= 0{,}001\ p_d l_x^2$	27	21	19	18	17	17	17	17
		m_{sx}	$= -\ 0{,}001\ p_d l_x^2$	91	98	107	113	118	120	124	124
		$a_x / l_x =$		0,21	0,21	0,22	0,23	0,24	0,24	0,24	0,25
		w	$= 0.001 \dfrac{p_d l_x^4}{EI}$	2,8	3,6	4,1	4,6	4,8	5,1		
VI A		m_{vx}	$= 0{,}001\ p_d l_x^2$	18	29	39	47	54	59	66	69
		m_{vy}	$= 0{,}001\ p_d l_x^2$	23	23	20	17	15	14	13	13
		m_{sx}	$= -\ 0{,}001\ p_d l_x^2$	54	72	88	100	108	114	121	124
		m_{sy}	$= -\ 0{,}001\ p_d l_x^2$	60	69	74	76	76	76	73	71
		$a_x / l_x =$		0.19	0,21	0,23	0,23	0,23	0,24	0,24	0,24
		$a_y / l_y =$		0.19	0,18	0,17	0,15	0,14	0,14	0,10	0,08
		w	$= 0.001 \dfrac{p_d l_x^4}{EI}$	1,6	2,4	3,2	3,8	4,3	4,6		
VI B		m_{vx}	$= 0{,}001\ p_d l_x^2$	23	30	35	38	40	41	42	43
		m_{vy}	$= 0{,}001\ p_d l_x^2$	18	15	14	13	13	13	13	13
		m_{sx}	$= -\ 0{,}001\ p_d l_x^2$	60	70	76	80	82	83	83	83
		m_{sy}	$= -\ 0{,}001\ p_d l_x^2$	54	55	55	54	53	53	51	49
		$a_x / l_x =$		0,18	0,19	0,19	0,19	0,20	0,21	0,21	0,21
		$a_y / l_y =$		0,19	0,18	0,14	0,13	0,13	0,10	0,08	0,07
		w	$= 0.001 \dfrac{p_d l_x^4}{EI}$	1,6	2,0	2,3	2,4	2,5	2,6		

Support conditions: See next page

Plate support conditions:

- solid line = line support, no rotational fixity
- two solid lines = line support and fully fixed (no rotation)

m_{vx} is the positive moment per unit length in a cross-section along the longer edge (l_y)
m_{vy} is the positive moment per unit length in a cross-section along the shorter edge (l_x)
m_{sx} is the negative moment per unit length in a cross-section along the longer edge (l_y)
m_{sy} is the negative moment per unit length in a cross-section along the shorter edge (l_x)
p_d is the design value of the uniformly distributed load
w is the maximum deflection

In order to account for the favourable influence of a parallel supporting edge which results in load transfer in *y*-direction too, NEN 6720 states that the reinforcement in *x*-direction over a side strip width of 0,25 l_y along both edges may be reduced by 50%. This is schematically illustrated in Fig. 14. 35c by the 0,5 m_{xx}. The same holds for the reinforcement in *y*-direction, see Fig. 14. 35d. The slab is now in fact split into edge and middle strips.

NEN 6720 states that at least half of the maximum span reinforcement should extend to the support. It has to be noted that not only the table coefficients must be used; additional design rules apply.

As highlighted before, special attention has to be given to the torsional moments m_{xy} and m_{yx}. Torsional moments near plate corner can be replaced by equivalent force couples (
Fig. 14. 36). The equivalent forces counteract each other in the section borders along the entire plate edge However, in the corner, an upward force remains. Corners of the simply supported slab tend to lift up (Fig. 14. 37) when the slab is uniformly loaded and this uplift force in the corner is equal to:

$$R = 2m_{xy} \tag{14.15}$$

Therefore, corner anchoring should be applied to resist this force.

If lifting up is prevented, bending moments directed at an angle of 45° relative to the edges occur in the corner parts of the slabs. As a result, torsional moments are introduced in the corners of a simply supported slab. Analysis of this mechanism leads to the conclusion that the torsional moments in the corner can be sufficiently resisted by applying a square reinforcement mesh having a length 0,3 l_x. Note that this reinforcement needs to be provided in a such a way that cracks due to torsion are crossed by reinforcement (see Fig. 14. 32 and Fig. 14. 38). The mesh must be applied both at the top and bottom of the slab. The amount of reinforcement to be applied in both directions in top and bottom should be equal to the maximum span reinforcement in *x*-direction. This follows from Fig. 14. 35a where it can clearly be seen that the torsional moment at all four corners in both *x*- and *y*- direction is equal to the maximum *x*-direction sagging bending moment.

If the plate is not anchored in the support to resist this force, the corners will lift. If the corners are free to lift, the section forces in the plate will change because these are determined based on the boundary condition that the deflection is equal to 0 along the entire length of all the support edges.

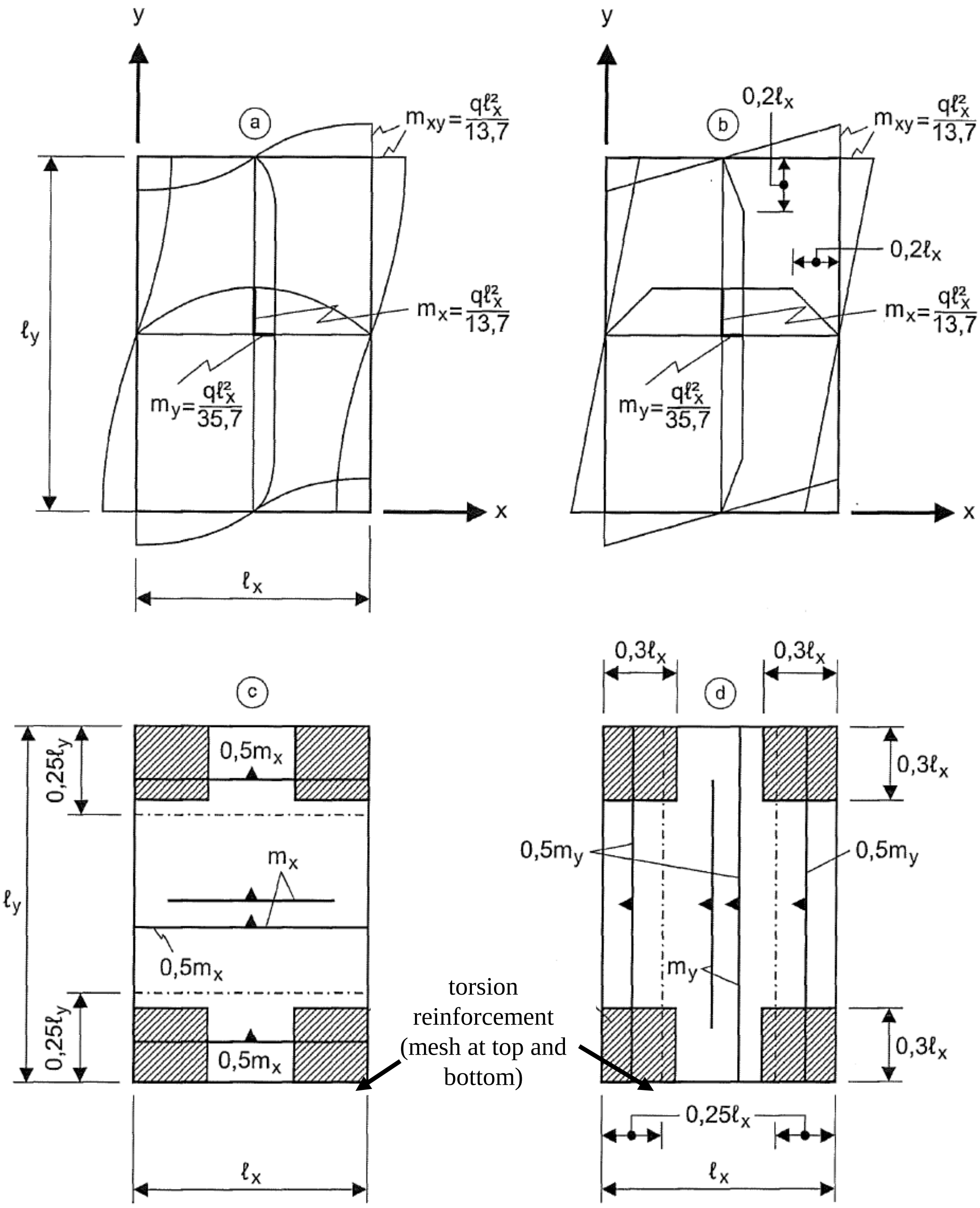


Fig. 14. 35: Slab with rigid line supports along the four edges where $l_y/l_x = 1.5$:

a. Bending and torsional moment distributions

b. Schematized bending and torsional moments distributions

c. Reinforcement in the *x* - direction

d. Reinforcement in the *y* - direction

The aforementioned torsional reinforcement is not required when 1 or 2 of the line supports at their junction in a corner have a restrained rotation. In these cases, a boundary (or the boundaries) prevent(s) the slab from rotating. This can be seen in Table 14. 2 where it is indicated that torsional reinforcement is only required for slab types I, III, VA and VB.

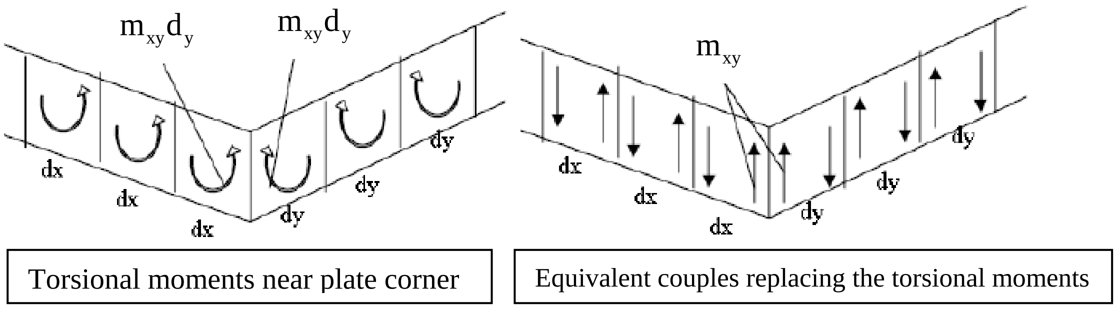


Fig. 14. 36: Effect of torsion on plate corner

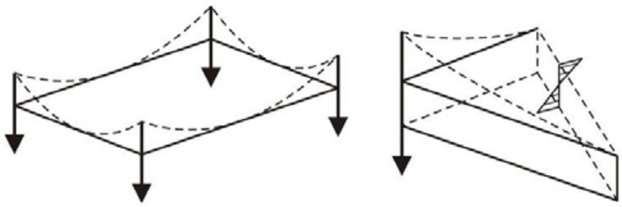
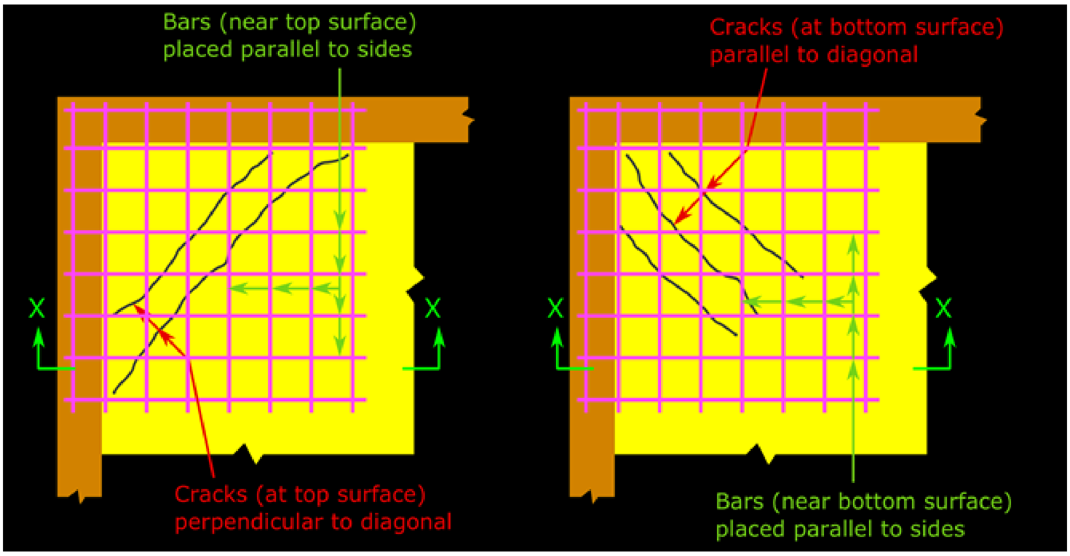


Fig. 14. 37: The corners of a simply supported slab tend to lift up (left) and the deformation is compensated for by bending moments that cause torsion (right)



a) b)

Fig. 14. 38: Reinforcement mesh to ensure torsional moments at the edges at the a) top and b) bottom surface

Note that Table 14. 2 presents bending moment expressions for the middle strips only. The designer, however, has to calculate the reinforcement required in the full slab. For example, the designer has to design the reinforcement in the edge strips too. To enable the designer to do so, NEN 6720 presented a set of design rules. These additional rules are given to derive edge strip and slab corner reinforcement from middle strip results. The designer now has to calculate middle strip bending moments only and can use these to calculate full slab reinforcement.

The following design rules hold (NEN 6720 cl. 7.5.1.4):

- The amount of reinforcement per unit width should not vary over the width of a strip.
- The amount of positive moment (bottom) reinforcement in an edge strip should be at least 50% of the amount of bottom reinforcement applied in the parallel middle strip.
- The amount of negative moment (top) reinforcement in an edge strip should be at least equal to the amount of top reinforcement applied in the parallel middle strip.
- In the corners of free line supported edges mesh top and bottom reinforcement should be applied. The amount of reinforcement required should be based on the maximum amount of bottom reinforcement per unit width applied in a middle strip. The reinforcement should extend over at least $0{,}3l_x$ from the supports. (Note: This reinforcement is required to resist the torsional bending moments that occur in corners where two line supports without rotational restraint meet).

These requirements for the two-way spanning slab design and reinforcement distribution are applied to the example given in Fig. 14. 39 (NEN 6720). In this example, the slab has two rigid line supports (without rotational fixity) and two rigid line supports with rotational fixity.

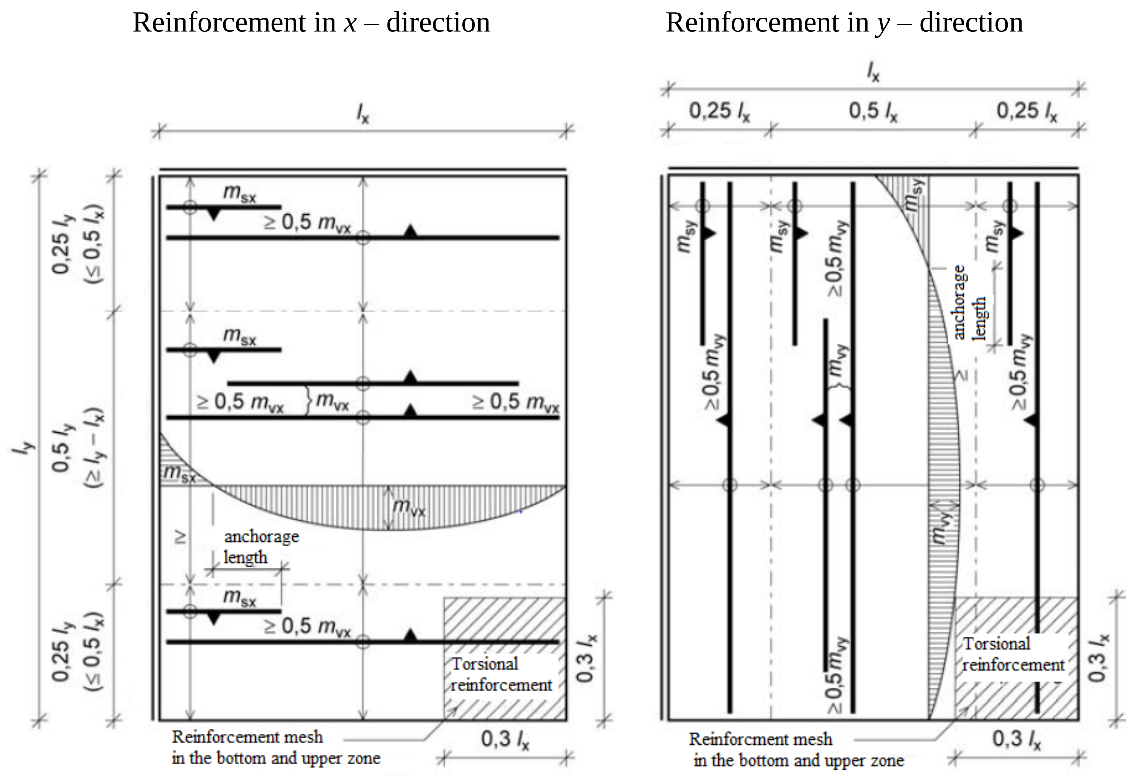


Fig. 14. 39: Example of the requirements for a reinforcement distribution in a two-way slab (designed using bending moment coefficients)

Transition from one-way to two-way slab

To illustrate the use of Table 14. 2 the slab from Fig. 14. 25 is analysed. The bending moment in *x*-direction in the slab is to be calculated. When assuming that the ratio between the longer and shorter span is 3,0 and by using plate type I, it follows that the span bending moment in transverse direction is $m_{xx} = 0{,}023\, q\, l_y^2$. The bending moment in the main load-transfer direction (m_{yy}) is around 5 times higher ($0{,}117\, q\, l_y^2$). The calculated bending moment m_{xx} turns out to be of the same magnitude as the secondary moment in a one-way slab which originates from Poisson’s ratio and the required restrained slab strip deformation.

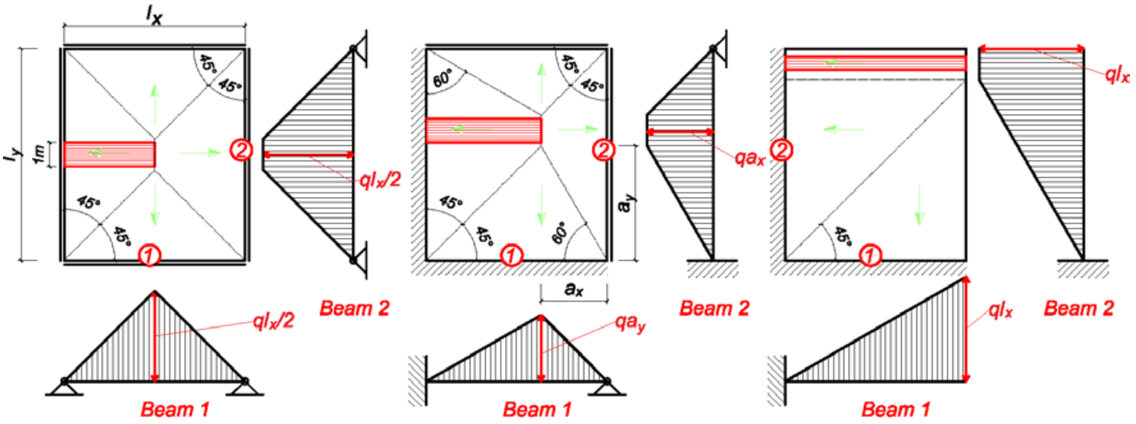
It can also be observed that considering the slab as a one-way slab implies that the moment in the principal direction (equals to $1/8\ q\, l_y^2$) is approximately only 7% larger than according to the theory of elasticity for a two-way slab. All these demonstrate that the major part of the slab acts as a one-way slab, whereas only the end part next to the additional line supports experiences two-way load transfer.

Note: Notations for *x* and *y*-directions are according to Fig. 14. 25 and not Table 14. 2.

Shear stresses and support reactions

In the previous sections attention was given to bending moment reinforcement design. It was demonstrated that coefficients from linear elastic analysis can be used to dimension the reinforcement. The structural engineer, however, also needs information regarding the support reactions and the shear forces. No tables with coefficients are available in the Eurocode, but an option is to model the load transfer in the slab. A major advantage of this approach is that insight into structural behaviour is obtained. Of course, the finite element method can also in this case be used to calculate these forces.

Support reactions and shear forces will now be estimated by modelling the load transfer in a two-way slab. It is often sufficiently accurate to subdivide the slab surface into parts, as shown in Fig. 14. 40. Fig. 14. 40a shows a rectangular slab with rigid hinged line supports along its four edges. The slab is assumed to be uniformly loaded over its full surface. The shaded areas display how the parts of the load are transferred to the edges and load the supports. Fig. 14. 40b shows a similar scheme for a slab that has rigid line supports along two edges and rigid line supports with additional rotational fixity along its two other edges. Note that three different angles are used, namely 30º, 45º and 60º. When two edges that meet in a corner give the same boundary conditions, the 90º corner angle is split in two identical angles, 45º each. When two edges with different boundary conditions meet, the ‘stiff’ edge attracts the major part of the load. Now, the corner angle is split into 60º and 30º angles. The engineer can now construct, starting at the corners, the load transfer. An envelope-like slab surface distribution occurs.



(a) (b) (c)

Fig. 14. 40: Calculation of the reaction forces on rigid line supports of a line supported slab having identical (left) and different (middle, right) support conditions along its four edges

For the two-way slab given in Fig. 14. 40a, where $l_y > l_x$, the maximum reaction force per unit length of support, both in x- and y-direction is:

$$R_{max} = \tfrac{1}{2} q l_x \qquad \text{[kN/m]} \qquad (14.16)$$

The maximum shear stress per unit length in the slab occurs next to these supports (the governing strip is indicated in red):

$$v_{\text{Ed, max}} = \frac{\frac{1}{2} q l_x}{d} \qquad \text{[MPa]} \qquad (14.17)$$

where d is the effective depth (height) of the slab.

Analogously, for the support conditions given in Fig. 14. 40b, the maximum reaction force in beam 2 is:

$$R_{\max} = q a_x = \frac{3}{8} q l_x \qquad \text{[kN/m]} \qquad (14.18)$$

Note, however that this will not be the location of maximum shear stress in the slab. The opposite beam will take more load (labelled in red). The maximum reaction force per unit length of this support is:

$$R_{\max} = q a_y = \frac{5}{8} q l_x \qquad \text{[kN/m]} \qquad (14.19)$$

Therefore, the maximum shear stress per unit length in the slab is:

$$v_{\text{Ed, max}} = \frac{5 q l_x}{8 d} \qquad \text{[MPa]} \qquad (14.20)$$

Take into account that these reaction forces are analogous to the beam with one fixed and one hinged support (third example from Fig. 14. 17).

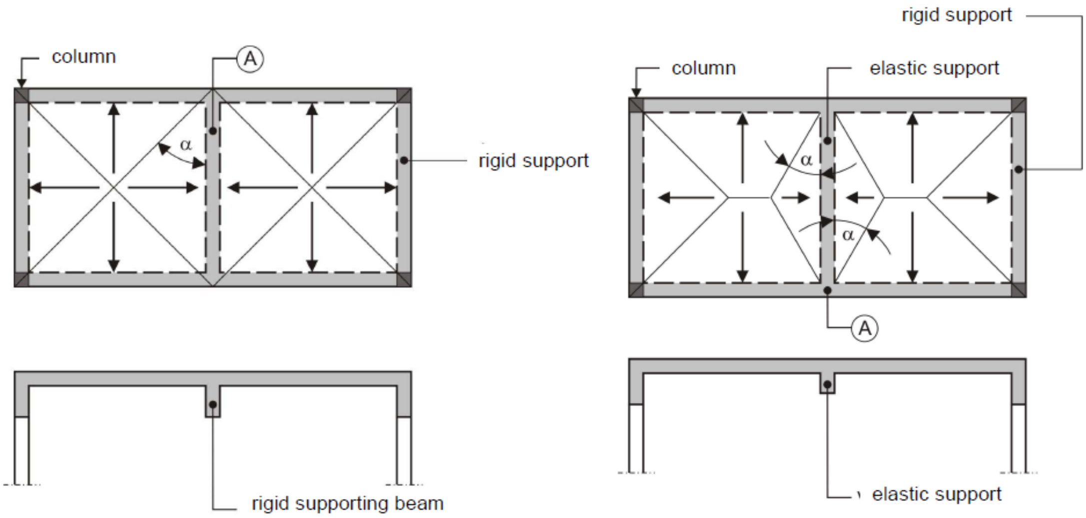
Finally, Fig. 14. 40c gives an example of the load distribution when two edges are free. Note that free edges, as indicated in Fig. 14. 40c, cannot take any load. The procedure for the calculation of the maximum reaction force and shear stress is analogous to the previous examples. The maximum shear stress per unit length in the slab is:

$$v_{\text{Ed, max}} = \frac{q l_x}{d} \qquad \text{[MPa]} \qquad (14.21)$$

Stiffness of supports

In Fig. 14. 41a two slabs are shown. They each have vertically rigid line supports from beams along their three outer sides. There is also a mutual support A. This line support A is from a beam that is assumed to be infinitely stiff in bending. This makes that each of the two slabs has four rigid line supports, distributing each slab in four identical triangles.

The load distribution for each square slab part will now be presented in case the supporting beam A is no longer rigid, but flexible in bending. Fig. 14. 41b shows how a uniformly distributed load on the slabs is transferred to the supports. If beam A acts as a rigid support, the angle describing the load transfer to this beam is $\alpha = 45°$. If beam A acts as a flexible support, this angle is smaller and will be in the range $0^0 < \alpha < 45^0$. So, if the bending stiffness of beam A decreases, the part of the load carried by it decreases too.



(a) (b)

Fig. 14. 41: Slabs supported by a rigid beam A (a) and slabs supported by an elastic beam A (b)

In case beam A is very flexible, α = 0º and no load will be transferred to the beam. The two slabs will now transfer the load as if there is no beam A. As a result, the two slabs act as a single slab and the main load transfer direction will be the shorter span direction.

If the theory of elasticity is used, the load transfer in case of elastic (flexible) line supports is hard to determine since it depends significantly on the ratios between the stiffnesses of the various components (beams, slab, spans). As indicated before, nowadays numerical programs are used (e.g. the Finite Element Method) to calculate the complex actual behaviour.

Continuous two-way slabs

The slabs presented in Table 14. 2 refer to standard cases with specific well defined support conditions (rigid line supports, hinged or with full rotational restraint). Often slabs are continuous and extend over a number of supports. In such cases the rigid supports have a partially restrained rotation.

Similarly as with continuous one-way slabs (multiple spans) and continuous beams with multiple spans, continuous two-way slabs should be designed to withstand the most unfavourable arrangements of loads. There are now multiple spans in two directions. These make that the slab can be modelled at a set of panels. The governing load combination has to be found, taking into account that a live load does not have to act simultaneously on all the panels of a slab. Subsequently, when taking into account the boundary conditions and by combining the diagrams and data from Table 14. 2, the governing bending moments can be calculated for two-way continuous slabs too. In this case the calculation is more complex since load distribution must be found for both directions.

An example is given for a continuous two-way spanning slab with a regularly shaped layout. All the panels of the slab have the same spans in *x*- and *y*-direction. It will be illustrated how the maximum bending moments can be determined, using the data from Table 14. 2.

Maximum positive bending moment:
In Fig. 14. 42 the loading configuration that causes a maximum (positive) moment in *x*-direction in a span is shown. The hatched area indicates where the live load has to be applied to have the maximum positive span moment. The figure also illustrates how this checkerboard load configuration can be divided into two components. The first component is a fully uniformly distributed load on all panels:

$$(\gamma_{\mathrm{G}}\, q_{\mathrm{Gk}}) + (\gamma_{\mathrm{Q}}\, q_{\mathrm{Qk}}) \cdot 0{,}5 \qquad (14.22)$$

Note that this expression corresponds to the *G*+*Q*/2 combination, as used before for continuous one-way slabs. An individual panel can now be regarded as being fully clamped (no rotation at all four edges). The bending moments can be determined using slab type II from Table 14. 2.

The second component of the load consists of an anti-symmetrical load:

$$(\gamma_{\mathrm{Q}}\, q_{\mathrm{Qk}}) \cdot 0{,}5 \qquad (14.23)$$

where this expression corresponds to the *Q*/2 combination, as used for continuous one-way slabs. For this load type the bending moments are zero at the supports, which actually implies that the individual panels can be regarded as being simply supported only along their four edges (see also Fig. 14. 24). The span moments in these panels in both directions follow from slab type I, Table 14. 2.

The sum of the moments in both directions obtained from both load components are the actual bending moments in the span of the panels. They are the basis for the reinforcement design.

In this example the load distribution that results in the maximum sagging bending moment in *x*-direction is identical to the load distribution that gives the maximum sagging bending moment in *y*-direction. This implies that, with regard to the sagging bending moment, identical loading schemes and slab panel types can be used. This is not always the case, as will be shown in the following example where the maximum hogging moment is calculated.

Maximum negative bending moment:
A similar procedure can be used to find the maximum support moments. Fig. 14. 43 presents the loading configuration to have a maximum (negative) moment in *x*-direction at a support. However, now, in order to get the maximum effects, two adjacent panels must be loaded by the live load.

To determine which slab types from Table 14. 2 must be used, the *x*- and *y*-direction boundary conditions must be combined. In each direction, boundary condition information at two panel edges is now obtained. When combining the information regarding all four edges of a panel, the corresponding slab type from Table 14. 2 can be found. It turns out that slab types II and Va from Table 14. 2 must now be combined.

In case the structural engineer has to calculate the maximum negative bending moment in *y*-direction, the previously loaded/unloaded panel scheme has to be changed. In this case, the corresponding load scheme is obtained by rotating the previous one over 90º clockwise.

To summarize:

It is noted that the self-weight is always present. This corresponds to slab type II, Table 14. 2. The live load, however, can be present and is positioned such that one plate (one panel) experiences the most extreme effects.

Maximum span moment:

The load configuration of the live load on the slab should result in a panel with hinged rigid line supports (no rotational restraint) at all its sides. This results in slab type I, Table 14. 2.

Maximum support moment:

The load configuration of the live load should result in a panel with rotational restraint at one side (above the support of interest) and hinges at three sides. This results in slab type Va, Table 14. 2.

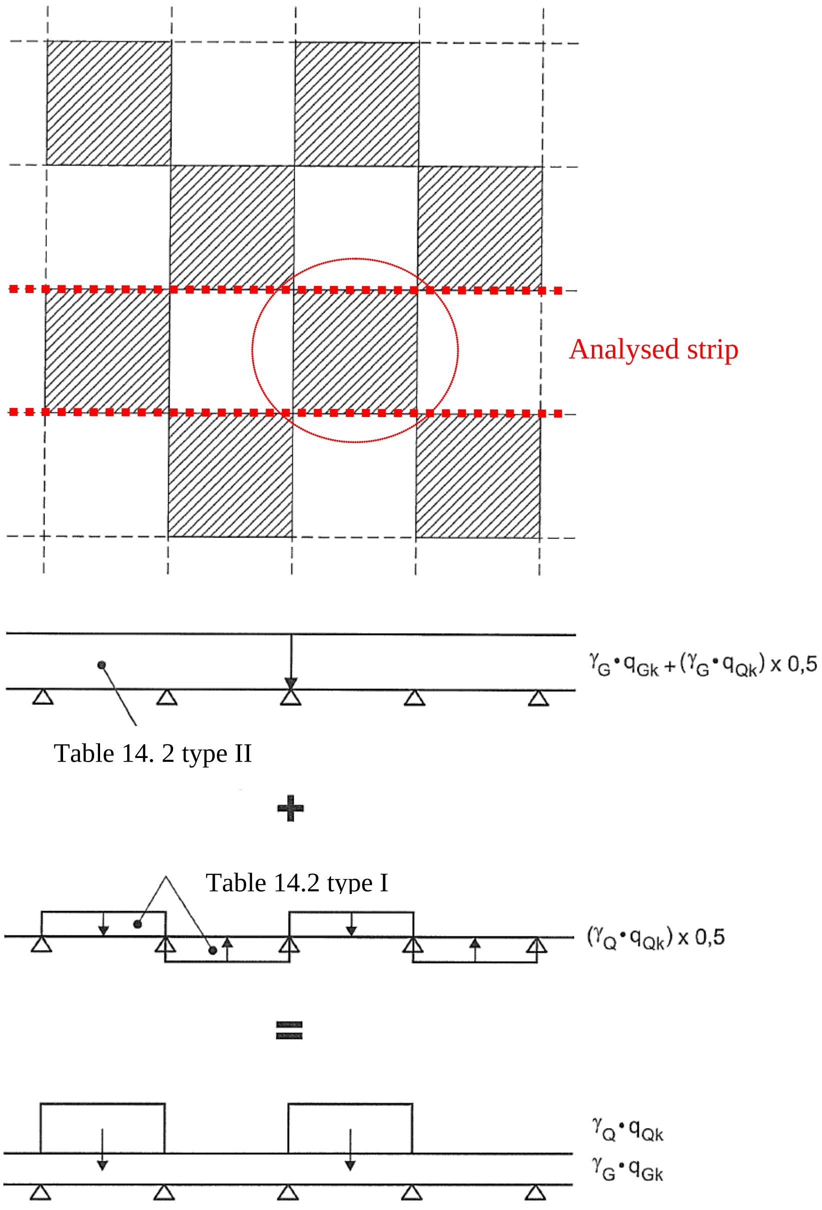


Fig. 14. 42: Alternately loaded panels (checkerboard-type) to determine the maximum span moments. The total load is divided into two known standard load cases from tabulated data (hatched = panel loaded by live load)

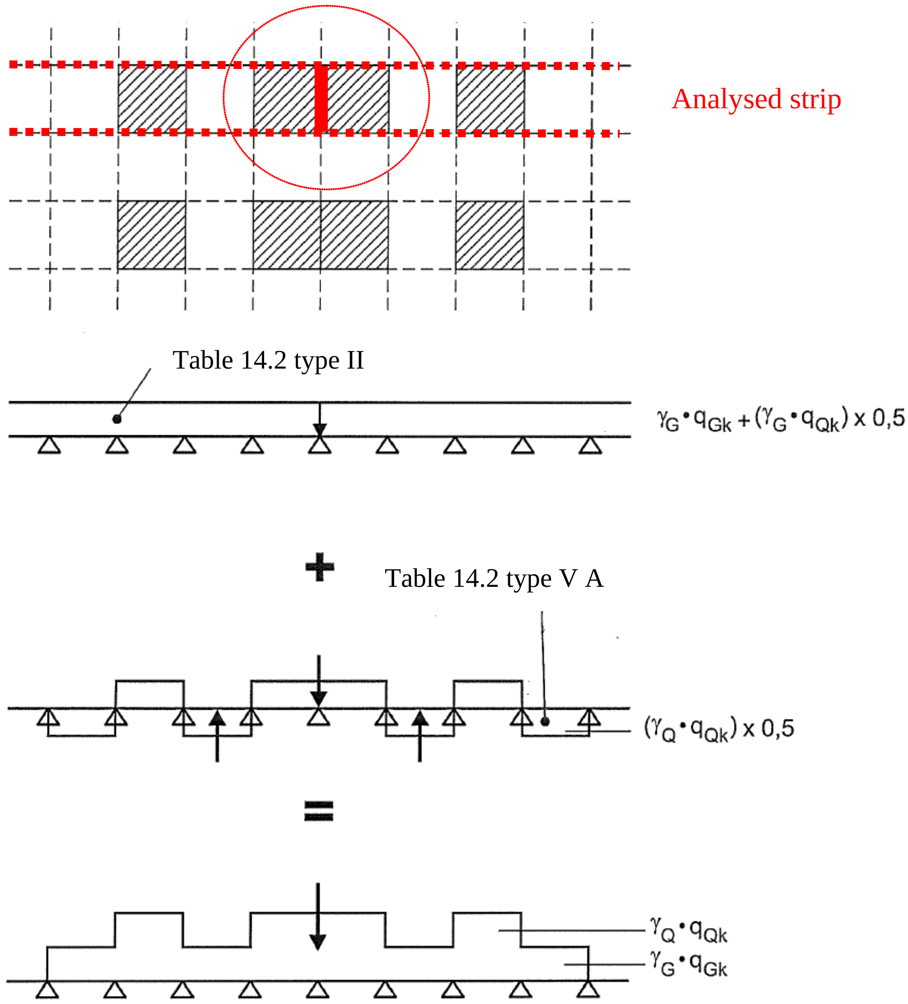


Fig. 14. 43: Alternately loaded panels to calculate the maximum support moments. The total load is divided into two standard cases (hatched = panel loaded by live load)

In a similar way, also the maximum shear forces should be analysed.

Analysis with the strip method or equilibrium method

According to the theory of elasticity and vertical force equilibrium in a slab, the external load is carried by bending and torsional moments. Hillerborg chose a solution where no torsional moments are assumed to be present ($m_{xy} = 0$) and where the load is thus completely carried by bending in x and y-direction:

$$\frac{\partial^2 m_{xx}}{\partial x^2} + \frac{\partial^2 m_{yy}}{\partial y^2} = -q \qquad (14.24)$$

This method is the so-called “strip method”.

The strip method is an approach which offers the structural engineer a certain freedom with regard to the choice of a load transfer model. The slab is assumed to be replaced by a system of separate beams (strips) transferring load in their longitudinal directions.

Freely defined strip method

In case of a rectangular slab, the load can be assumed to be divided into two equal parts: one part is transferred by beams that span in one direction, the second part is transferred by beams that span in the other direction (Fig. 14. 44). Note that in this example, the load transfer is freely chosen and it is not related to the spanning of the slab in x- and y-direction, or boundary conditions.

The primary requirement in this approach is that force equilibrium is present in the system. However, there are some restrictions.

1. The selected load transfer model can result in a load distribution that deviates considerably from the distribution according to the theory of elasticity. In that case, to have ultimate limit state (ULS) load transfer according to the assumed scheme, a redistribution of forces has to take place after cracking has occurred. However, the load transfer mechanism, and, as result, reinforcement distribution, now is not according to the linear elastic load transfer mechanism, but is chosen by the structural engineer. This might result in a cracked stage in which wide cracks and large deflections occur, especially if a qualitatively poor system of load transfer is chosen. A restriction, therefore, comes from serviceability limit state (SLS) considerations.

2. In ULS (ultimate limit states) deformations and crack widths are not, or are of minor interest. The development of a state of equilibrium then is sufficient to meet the requirements from the codes (safety; load bearing capacity). In ULS the designer might, however, have to deal with the aspect that the deformation capacity of the structural components (rotational capacity) has to be high enough to let the failure mechanism develop without premature failure, e.g. by crushing of the concrete. As a result, plastic deformation capacity or plastic rotation capacity checks might be required.

When selecting a load transfer scheme it is, therefore, recommended to account for a force distribution that is identical to, or at least close to that in the elastic stage. Or, in other words, when applying the strip method, an engineer should strive to follow SLS stress flow as much as possible in order to limit the crack widths and be close to force equilibrium from the elastic force distribution.

Note that, if l_x is not equal to l_y, and the type of support is not the same at all the four sides, the load transfer from Fig. 14. 44 (i.e. the uniform load is divided in two equal parts) will not be the most optimal. As demonstrated before, when the ratio l_y/l_x increases, more load gets transferred in the x-direction (shorter span), and the slab will behave more like a one-way slab. This will be further elaborated.

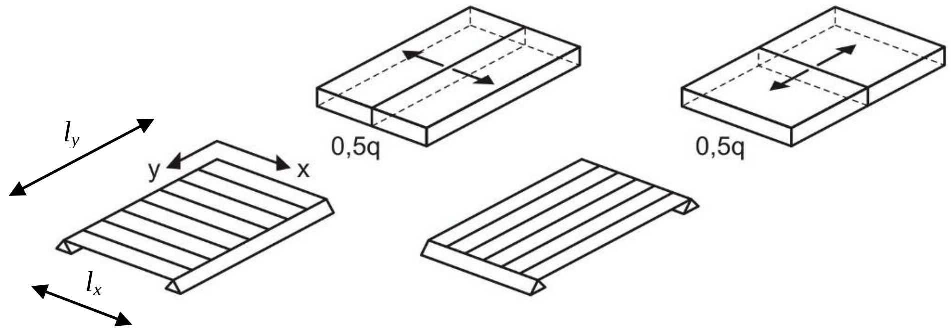


Fig. 14. 44: Load transfer in two directions by strips acting independently one from the other

Strip method based on deflection compatibility

Fig. 14. 45 illustrates how two separated load transfer mechanisms can be defined in a rectangular slab that is simply supported (stiff line supports) along its four edges. The slab is subjected to a uniformly distributed load q. To visualize the flexural performance of this slab, think of a set of parallel strips, in each of the two directions, intersecting each other. From compatibility requirements, at the intersection of the two strips, the deflection of both strips has to be equal. This should in theory be valid for any strip. This requirement will however not be met since we would have to solve a huge set of compatibility requirements. We would then in fact return to the linear elastic load distribution approach. Therefore, in the strip method approach the calculation is simplified and the focus is on only one or a few compatibility requirements. In this example, let's consider the maximum deflection of the middle strips only; the strips halfway the edges.

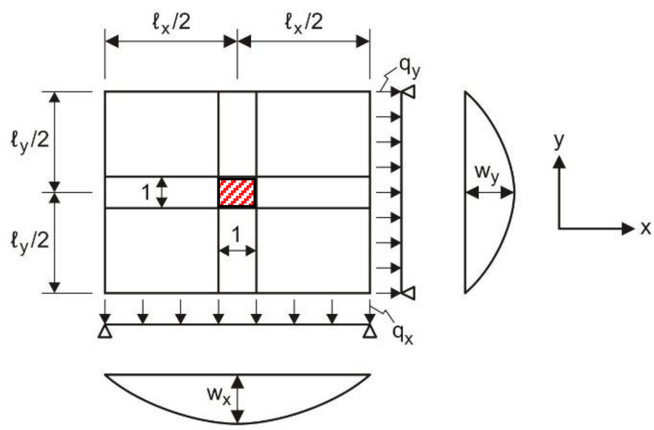


Fig. 14. 45: Compatibility of the deflection of the middle strips in x- and y-direction

Evidently, part of the load is carried by one set of strips and transmitted to the edge supports, and the remaining part of the load is transferred by the other set of strips. In *x*-direction a part q_x of the load is assumed to be transferred. The deflection at mid-span position of the strip in *x*-direction is:

$$w_x = \frac{5}{384}\frac{q_x\, l_x^4}{(E_c I)_x} \tag{14.25}$$

In a similar way for the strip in *y*-direction it holds:

$$w_y = \frac{5}{384}\frac{q_y\, l_y^4}{(E_m I)_y} \tag{14.26}$$

The load equilibrium condition requires that:

$$q_x + q_y = q \tag{14.27}$$

Compatibility at the position where the strips meet requires that:

$$w_x = w_y \tag{14.28}$$

If it is assumed that the slab material is isotropic, the bending stiffness per unit width of both strips is the same ($(E_c I)_x = (E_c I)_y$):

$$q_x\, l_x^4 = q_y\, l_y^4 \tag{14.29}$$

When considering that

$$q_x = q - q_y \tag{14.30}$$

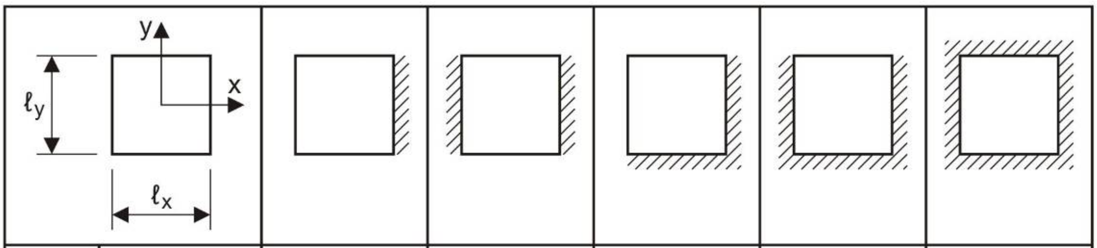
the following equations for the load transfer contributions q_x and q_y of the strips are obtained:

$$\begin{aligned} q_y &= \frac{l_x^4}{l_x^4 + l_y^4}\, q = k_y\, q \\ q_x &= \frac{l_y^4}{l_x^4 + l_y^4}\, q = k_x\, q \end{aligned} \tag{14.31}$$

where k_x and k_y are the load distribution factors. k_x indicates the portion of the total load transferred in the *x* – direction, whereas k_y indicates the portion of the load transferred in the *y* – direction. For other, more or less standard, support conditions the corresponding values can be derived in the same way. Table 14. 3 contains the results from these calculations.

In Fig. 14. 46 the load distribution factor k_x of a rectangular simply supported slab (first slab in Table 14.3) is presented as a function of the ratio l_y / l_x. It can be seen that for aspect ratios $l_y / l_x >$ 1,5 and $l_y / l_x < {}^2/_3$ the load transfer primarily takes place in one direction. In these cases the slab can be regarded as being a one-way spanning slab, where the detailing of the reinforcement in transverse direction is based on 20% of the maximum span moment in the principal direction (which follows from Poisson’s ratio).

Table 14. 3: Strip method: Load distribution factors for different support conditions (solid line = rigid line support; line with shaded area = line support and rotational fixity)



$\mathbf{k_x} =$	$\frac{\ell_y^4}{\ell_x^4+\ell_y^4}$	$\frac{5\ell_y^4}{2\ell_x^4+5\ell_y^4}$	$\frac{5\ell_y^4}{\ell_x^4+5\ell_y^4}$	$\frac{\ell_y^4}{\ell_x^4+\ell_y^4}$	$\frac{2\ell_y^4}{\ell_x^4+2\ell_y^4}$	$\frac{\ell_y^4}{\ell_x^4+\ell_y^4}$
	$\mathbf{k_y} = 1 - k_x$					

The strip method is particularly useful in non-standard cases, such as irregular slab lay-outs, irregular load configurations, slab openings, etc. A proper load transfer mechanism for such a slab can be prepared by using information from Table 14. 3 as input.

Although torsional reinforcement in the corners is not necessary to satisfy the equilibrium conditions, it is recommended to be applied in order to prevent the occurrence of wide cracks.

Note: With the strip method, based on compatibility of deflections, only in the case that l_y is equal to l_x, and boundary conditions on all four sides are identical, the load transfer is such that $q_y = q_x$. Generally, as already indicated, the strip method also gives the designer freedom to define a load transfer model such that deflection compatibility is not complied with.

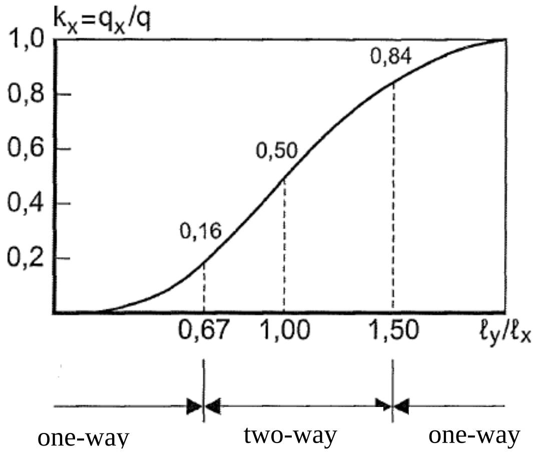


Fig. 14. 46: Load distribution factor k_x of a rectangular simply supported slab as a function of the ratio between the spans

In general it can be stated that designing according to the theory of elasticity is most economic. The difference between the solution according to the theory of elasticity and a solution that follows from the strip method can be reduced when the load transfer mechanism selected is more close to the linear elastic one. This is illustrated by the following example.

Example

Consider a quadratic slab that is simply supported along its four edges.
When the slab is designed according to the theory of elasticity the maximum sagging (span) bending moment m_x is (see Table 14. 2 slab type I; l_y / l_x = 1,0; v = span cross-section):

$$m_{vx} = 0{,}041\, q\, l_x^2 \tag{14.32}$$

This bending moment is assumed to be present over the full width of the middle strip (strip width 0,5 l_y). In both edge strips (each having a width 0,25 l_y) the positive bending moment (bottom) reinforcement should be at least 50% of the reinforcement in the middle strip (NEN 6720 requirements to Table 14. 2). The total span moment on which the reinforcement must be based is:

$$M_{xx} = \sum m_{xx,i}\, l_{y,i} = 0{,}041\, q\, l_x^2 \cdot \tfrac{1}{2} l_y + 2 \cdot \left(\tfrac{1}{2} \cdot 0{,}041\, q\, l_x^2\right) \cdot \tfrac{1}{4} l_y = 0{,}031\, q\, l_y\, l_x^2 = 0{,}031\, q l^3$$

When analysed with the strip method, the least complicated method of load transfer for this slab is to divide the load using two perpendicular strips which transfer it to the supports, as shown in Fig. 14. 47. This system is considered as *Solution 1*.

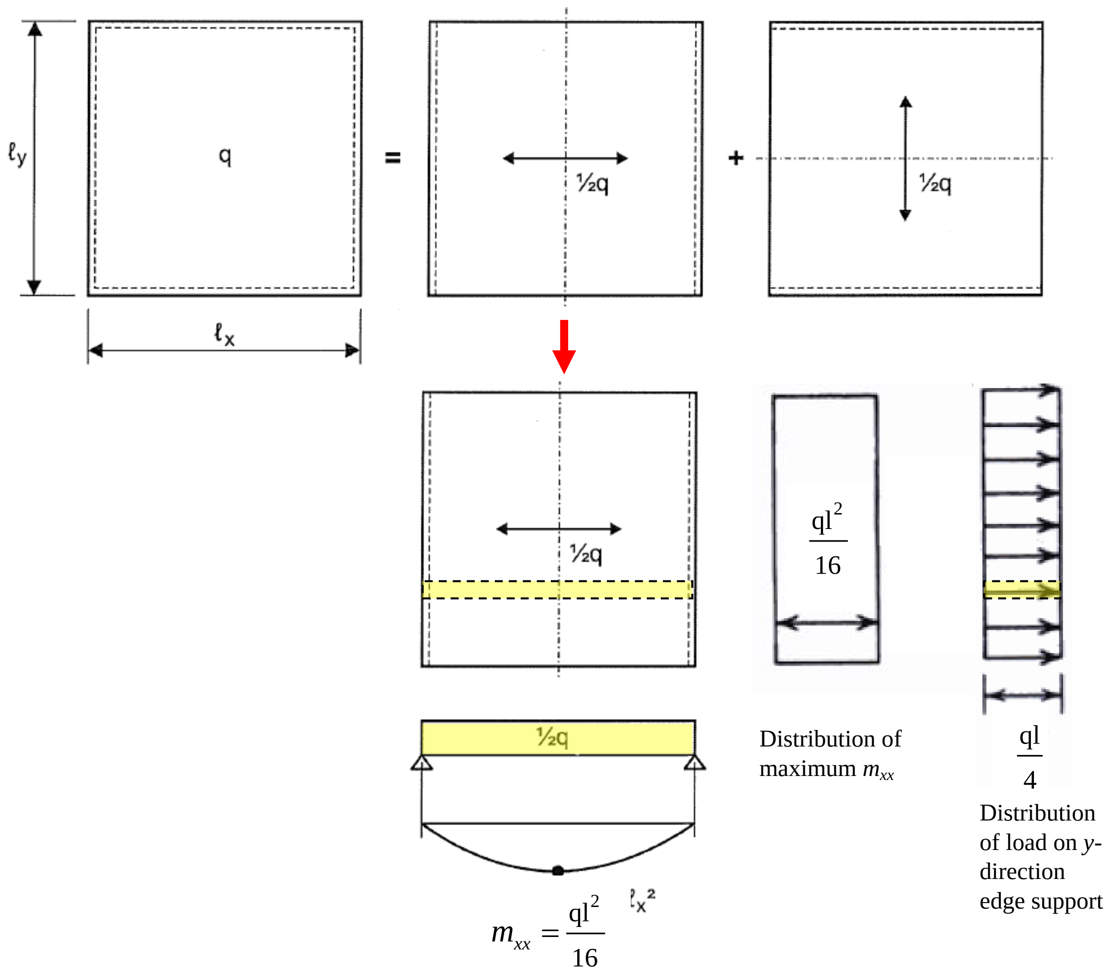


Fig. 14. 47: System of load transfer according to the strip method for a quadratic (l_x=l_y=l) slab simply supported along all edges, *Solution 1*

The maximum bending moment per unit width of each of the strips occurs at mid-span:

$$m_{xx} = \tfrac{1}{8}(\tfrac{1}{2}q)\,l_x^2 = \tfrac{1}{16}\,q\,l_x^2 = 0{,}0625\,q\,l_x^2 \qquad (14.33)$$

This bending moment is present over the total width l_y since identical strips are assumed to be present over the full width of the slab. The total bending moment, considering that $l_x = l_y = l$ is:

$$M_{xx} = m_{xx}\,l_y = 0{,}0625\,q\,l_y\,l_x^2 = 0{,}0625\,q\,l^3 \qquad (14.34)$$

The distribution of the load acting on the edge support is also shown in Fig. 14. 47. This load is simply the end reaction (support reaction) of the strips that are supported by a beam or wall.

The results indicate that the load transfer mechanism from Fig. 14. 47 requires two times as much reinforcement as the bending moment from the tabulated data from Table 14. 2 which is based on the theory of elasticity (0,0625 versus 0,031).

The load transfer mechanism from Fig. 14. 47 will be improved now. The result is shown in Fig. 14. 48 and it is denoted as *Solution 2*.

The support reactions of the middle strip spanning in *x*-direction:

$$V_{\mathrm{Ed,A1}} = \tfrac{1}{4}l_x \cdot \tfrac{1}{2}q \cdot \tfrac{1}{2}l_y + \tfrac{1}{4}l_x \cdot q \cdot \tfrac{1}{2}l_y = \tfrac{3}{16}q\,l_y l_x \qquad (14.35)$$

The total bending moment at mid-span of the middle strip:

$$M_{xx1} = (\tfrac{3}{16}\,q\,l_y\,l_x)\cdot\tfrac{1}{2}l_x - (\tfrac{1}{8}q\,l_y\,l_x)\cdot\tfrac{3}{8}l_x - (\tfrac{1}{16}\,q\,l_y l_x)\cdot\tfrac{1}{8}l_x = \tfrac{5}{128}q\,l_y\,l_x^2 \qquad (14.36)$$

The support reactions of the two edge strips spanning in *x*-direction:

$$V_{\mathrm{Ed,A2}} = \tfrac{1}{4}l_x \cdot q \cdot \tfrac{1}{4}\,l_y = \tfrac{1}{16}\,q\,l_y\,l_x \qquad (14.37)$$

The bending moment at mid-span of the two edge strips together:

$$M_{xx2} = (\tfrac{1}{16}q\,l_y\,l_x)\cdot\tfrac{1}{2}l_x - (\tfrac{1}{16}q\,l_y l_x)\cdot\tfrac{3}{8}l_x = \tfrac{1}{128}q\,l_y\,l_x^2 \qquad (14.38)$$

Considering that l_x=l_y=l the total moment over the whole span in x - direction is:

$$M_{xx,tot} = (\tfrac{5}{128} + \tfrac{1}{128})\,q\,l^3 = \tfrac{3}{64}q\,l^3 = 0{,}047ql^3 \qquad (14.39)$$

The load transfer system from Fig. 14. 48 (*Solution 2)* turns out to be a better alternative than the system from Fig. 14. 47 *(Solution 1)*; the total midspan bending moment over full plate width is reduced to 75% of its original value. However, the result is still about 50% greater than the total bending moment according to the theory of elasticity.

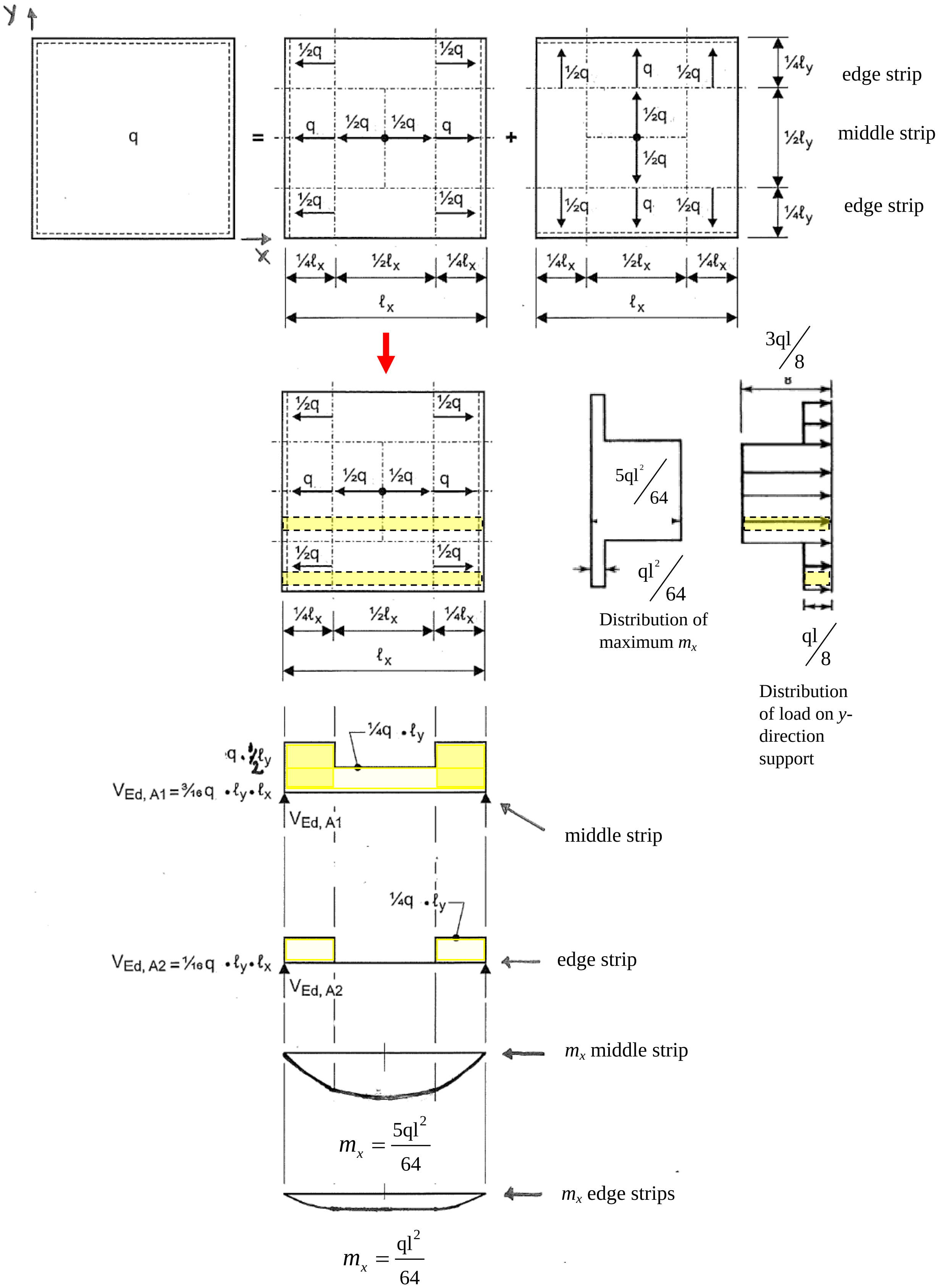


Fig. 14. 48: Alternative system of load transfer according to the strip method (line supported slab, l_x=l_y=l), *Solution 2*

The load transfer system is further improved and this solution is denoted as *Solution 3*. The load is now divided into triangular regions by diagonal lines and is transferred to the nearest support, as indicated in Fig. 14. 49. Each strip therefore carries a uniform load per unit area over the end regions. The distribution of the load acting on the edge supports of the slab is also shown.

The maximum *x*-direction bending moment is a function of *y* and rises sharply (parabolic) to a peak at the slab centre. The distribution of the *y*-direction bending moments is similar to the *x*-direction bending moment distribution. The maximum bending moment per unit width is:

$$m_x = \frac{ql^2}{8} \tag{14.40}$$

The total bending moment over the slab cross-section is:

$$M_x = \frac{1}{3} l \frac{ql^2}{8} = \frac{1}{24} ql^3 = 0.042 ql^3 \tag{14.41}$$

Solution 3 matches with the yield line theory (exact) solution. From all the three solutions based on the strip method it turns out to be the most economical one. Compared to *Solutions 1 and 2* it requires around 33% and 11% less reinforcement, respectively. However, in *Solution 3*, the bending moment varies continuously across the slab, requiring (theoretically) a continuously varying bar spacing, which is very impractical and not efficient from an economical point of view.

Obviously, for all four solutions, the reinforcement in the *y*-direction is identical to the reinforcement in *x*-direction as half of the load is transferred in this direction.

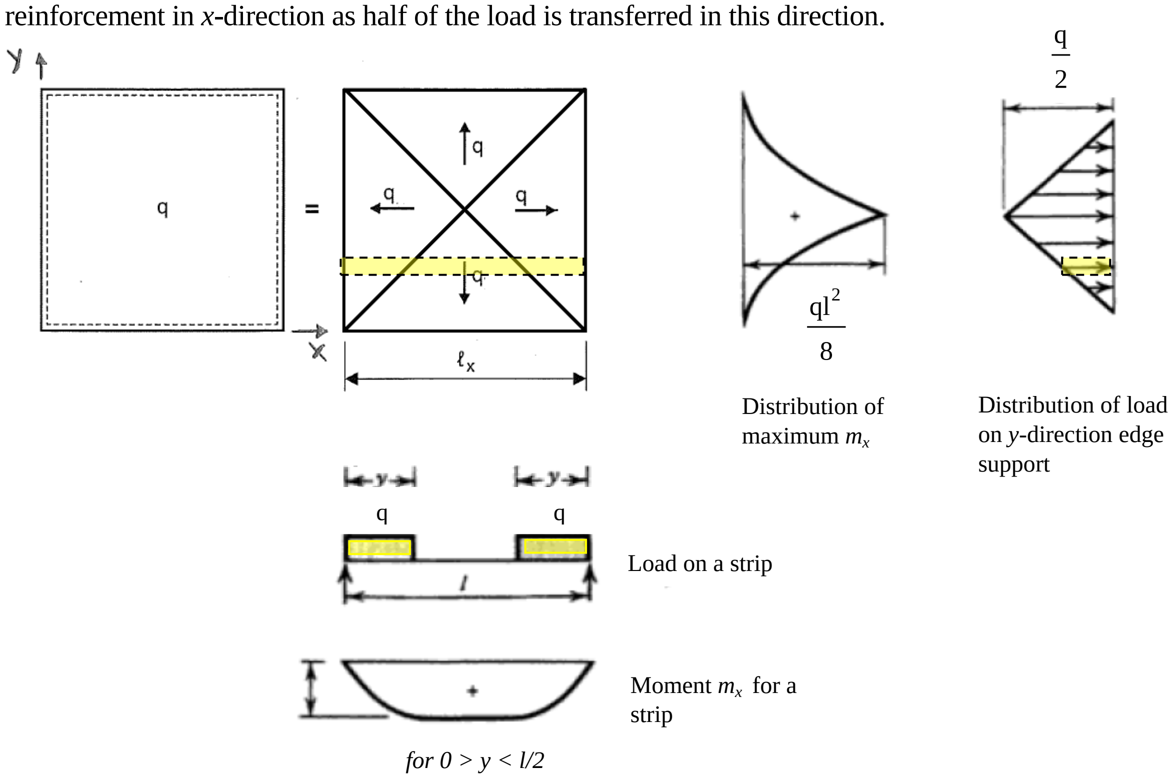


Fig. 14. 49: Alternative system of load transfer according to the strip method (line supported slab, l_x=l_y=l), *Solution 3,* matching with the yield line theory

Note:
These three solutions with strip models illustrate two features of this method. The first is the ease in which the bending moments in the slab and the loads on the supporting systems can be obtained. The second is the variety of bending moment and load distributions possible, depending on the assumed load transfer direction(s). This indicates that all three design methods, together with the result from the theory of elasticity, result in highly different amounts of reinforcement.

It is, however, stressed that the previous analyses with the theory of elasticity only concerns the reinforcement due to bending. The actual required amount of reinforcement comes from the amounts required both for bending and torsion. In the three cases considered, the strip method does not require torsional moment reinforcement (torsion is not accounted for), whereas Table 14. 2 data indicate that the four corners are to be provided with mesh reinforcement at top and bottom.

In other words, according to the theory of elasticity, the total load on the slab is carried not only by the bending moments which occur in two directions, but also by torsional (twisting) moments. Therefore, the maximum bending moments in the slabs designed using the theory of elasticity, are smaller ($0,042\, q\, l_x^2$) than computed for the sets of unconnected strips loaded by $q/2$ ($0,0625\; q\, l_x^2$, *Solution 1*). In this case, the twisting moments reduce the bending moments by about 35%. Since part of the load is now transferred by torsion, the torsional moments require reinforcement too.

14.2.3 Flat slab

When two –way slabs are supported only by columns we refer to a flat slab. The deformed shape of a uniformly loaded flat slab, compared to a uniformly loaded two-way slab with rigid, fully fixed line supports is shown in Fig. 14. 50. Note that both slabs have the same load, thickness and span. The flat slab is supported by four columns; one column in each corner.

Columns are modelled as stiff supports. The deformation of the floor system along the column lines alters the distribution of moments in the slab panel itself and a number of new aspects are introduced.

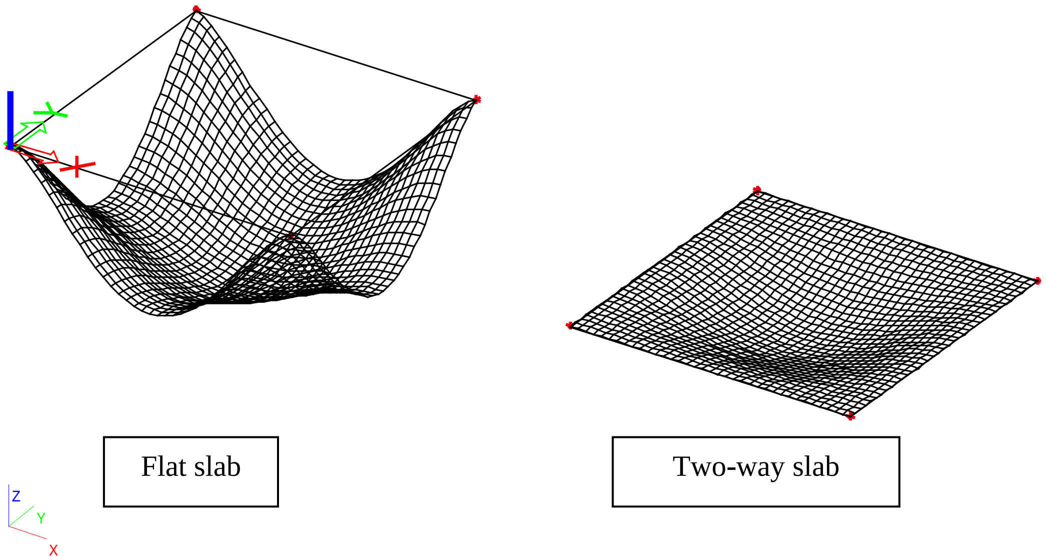


Fig. 14. 50: Deflected shape of uniformly loaded flat slab compared to that of the two way slab

One of these aspects are the column support reactions. The slab load is locally transferred to the column by shear stresses. Local shear at the columns is therefore often critical for flat slabs (
Fig. 14. 51). The effect of concentrated loading on slabs is referred to as punching shear. If the shear capacity is too low, a local punching failure may happen.

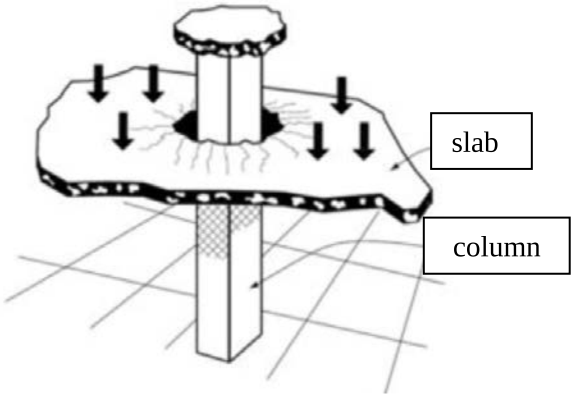


Fig. 14. 51: Punching shear

Shear capacity can be increased ina number of ways. For instance by the application of higher concrete strenght class and/or shear reinforcement and/or increase of the cross-section transferring the shear force. An option to increase the cross section is to increase the column dimensions. However, an increase of the shear capacity is required at the slab-column connection only. It is not required over full column height. Therefore, attention focuses on the support area only. The dimensions of the support area can be increased with capitals and/or drop panels.
Fig. 14. 52 shows two flat slabs; one with and one without drop panels. A drop panel increases slab thickness locally, thus increasing the area of the cross-section loaded in shear.

The flat slab is a type of slab that is often aplied since it has, compared with a slab supported by beams, a number of advantages:

- the flat soffit allows for a simple form- and falsework and reduces the time required to erect the building;
- the flat soffit makes that the slab reinforcement can be provided fast and without problems;
- the flat soffit of the slab eases the installation of equipment (pipes, cables etc.) underneath;

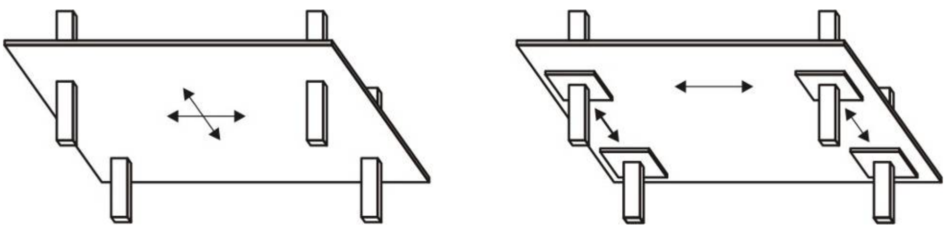


Fig. 14. 52: Solid slab without (left) and with (right) drop panels

As this is a special type of two-way slab, a flat slab can also be analysed using the theory of elasticity, the strip method, the yield line theory and FEM analysis.

Analysis with the strip method (equilibrium method)

The load transfer in a solid flat slab, see Fig. 14. 53b, is very similar to the one in a slab supported by relatively shallow beams on four sides, see Fig. 14. 53a. In both cases the load is assumed to be first transferred from the slab parts to the “beams”. In the case of Fig. 14. 53a, these are the visible beams in which a part of the slab is incorporated (by means of the so-called effective width b_{eff}; NEN-EN 1992-1-1 fig. 5.3). In Fig. 14. 53b these beams are not visible since they are "hidden" in the slab as fictitious "effective" beams or "column strips" (EN 1992-1-1 fig. I.1). Column strips in flat slabs behave as beams.

The beams are supported by columns at the intersection of their centrelines. If the surface load is applied, this load is transferred by imaginary slab strips l_a in the short direction and l_b in the long direction, similar as with two-way slabs explained previously (Fig. 14. 46a). The portion of the load that is carried by the long strips l_b is transferred to the beams *B1* spanning in the short direction of the panel. The portion carried by the beams *B1* plus the part carried directly in the short direction by the slab strips l_a sum up to 100% of the load applied to the panel. Similarly, the short – direction slab strips, l_a, transfer a part of the load to the long direction spanning beams *B2.* That load, plus the load carried directly in the long direction by the slab, also is 100% of the applied load. It is clear that with the column-supported construction, the full load is carried in both directions, jointly by the slab and its supporting beams.

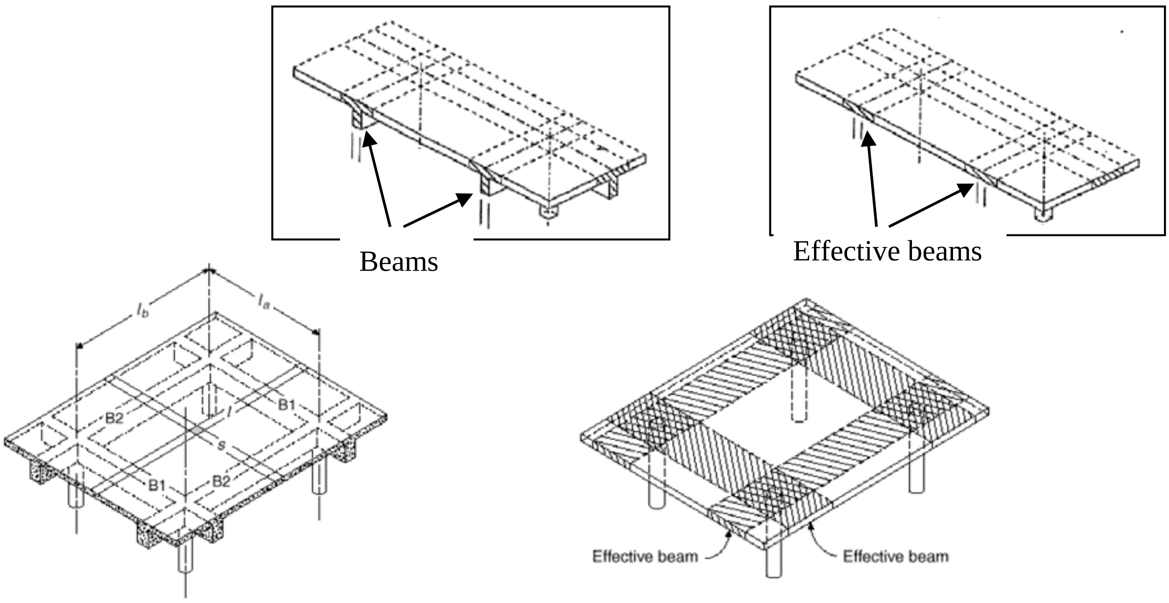


Fig. 14. 53: A flat slab (left) compared with a solid slab supported by beams (right)

Fig. 14. 54 shows how the flat slab, just as the slab having rigid line supports, is subdivided into strips. Column strips and middle strips are to be distinguished. The chosen width of these strips is open for discussion. For example, the column strips as well as the middle strips have widths of 0,5 l_x or 0,5 l_y according to the former Dutch code NEN- 6720. NEN-EN 1992-1-1 (Fig. 14. 55) proposes a different definition of strip width with a strip having a width of 0,5 l_y in both directions, where l_y is the shorter span. This definition is not used in The Netherlands since available tabulated bending moment data are based on the strip layout from Fig. 14. 54.

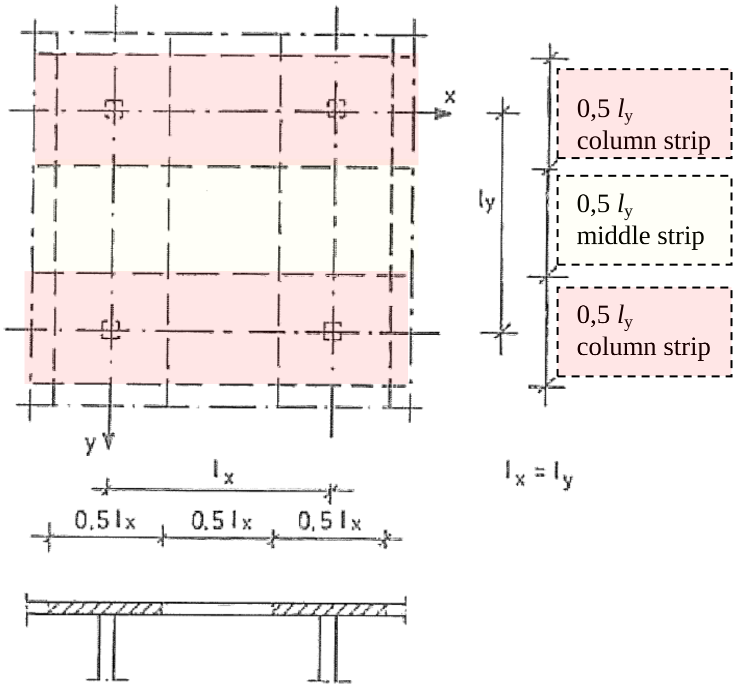


Fig. 14. 54: Subdividing a solid slab into strips (according to the former Dutch code NEN 6720)

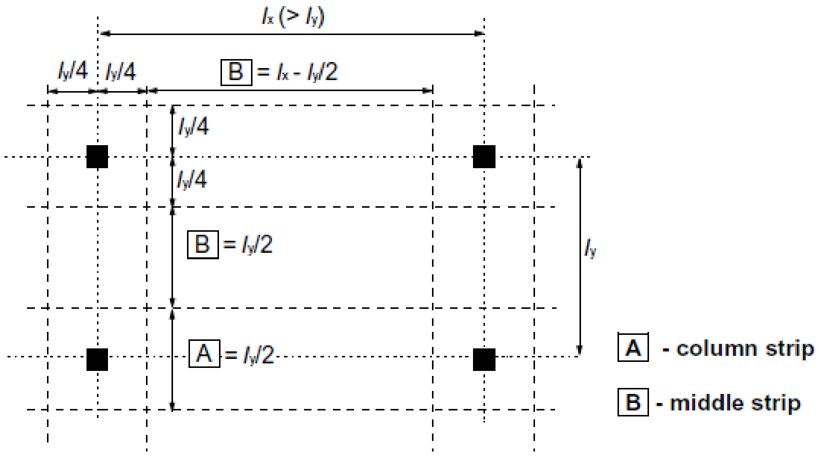


Fig. 14. 55: Division of panels in flat slabs (according to NEN-EN 1992-1-1)

Fig. 14. 56 illustrates how a uniformly distributed load q is transferred from the middle strip to a column strip ("hidden beam") and how the loads from columns strips are transferred to the columns. Note that a column strip not only has to transfer the load directly acting on it, but is also loaded by a middle strip that transfers its load to column strips. The column strips are the supports of the middle strips.

In Fig. 14. 57 the static system and the loads on the middle strip and the column strip are shown schematically. Note that clamped boundary conditions at the connection between the slab and the column are assumed.

As already indicated, the total load that has to be transferred in the x-direction is equal to q, being the total load on the flat slab. The same holds for the y-direction.

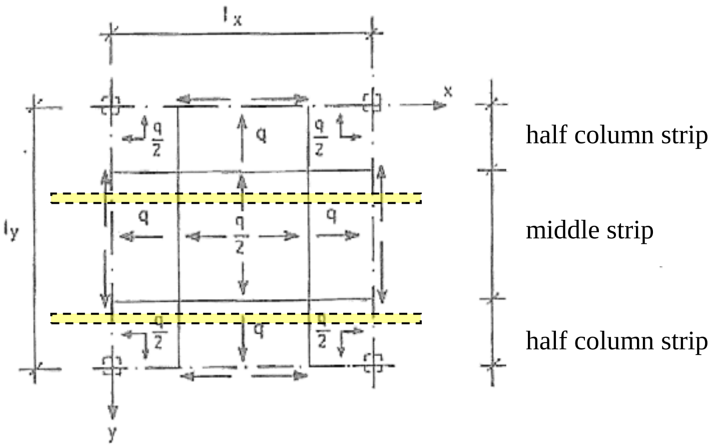


Fig. 14. 56: Schematised load transfer

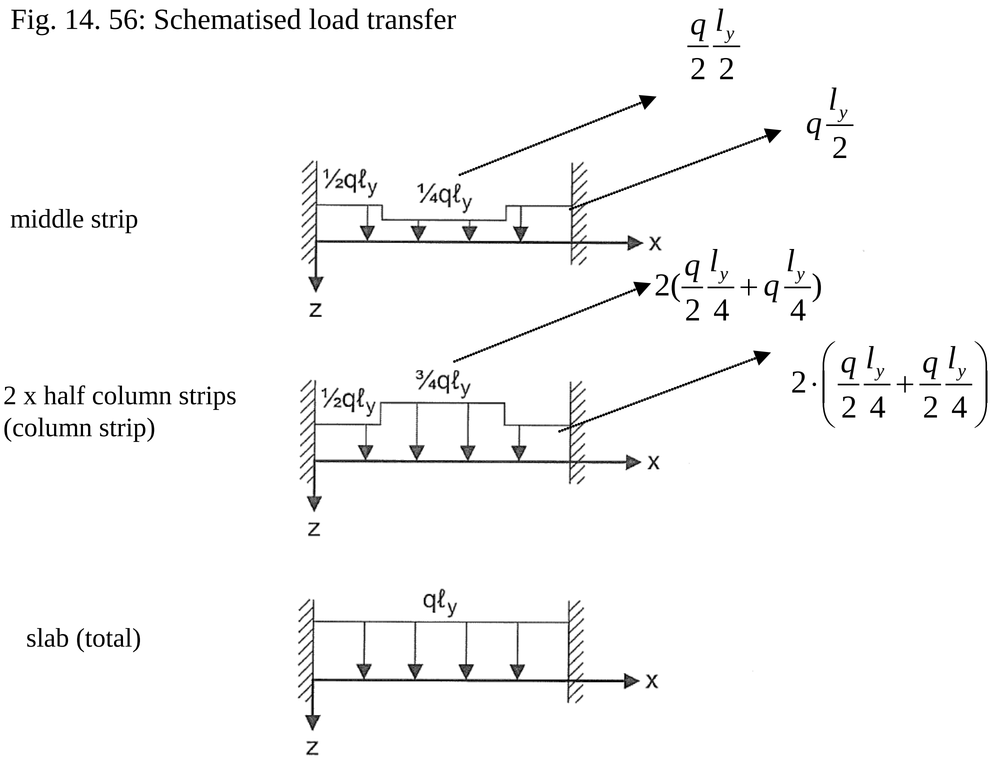


Fig. 14. 57: Schematized illustration of the load transfer to the middle strip and the column strips

For a square panel ($l_x = l_y = l$) the bending moments at mid-span and at a support (absolute value) are the same in x and y-direction for the middle strip (width 0,5l) and the column strip (total width $2 \cdot {}^1/_4\, l = 0{,}5\, l$):

middle strip: $M_{support} = \dfrac{21}{768} ql^3$ $M_{span} = \dfrac{9}{768} ql^3$

column strip: $M_{support} = \dfrac{43}{768} ql^3$ $M_{span} = \dfrac{23}{768} ql^3$

As a check, consider the bending moment diagram for the direction of span l_x. In this direction, the slab may be considered as a wide, flat beam of width l_y. Accordingly, the load per unit length of the span is ql_y. In any span of a continuous beam, the sum of the midspan positive bending moment and the negative bending moments at the adjacent supports is equal to the midspan positive moment of a correspondingly simply supported beam.

As a result:

$$M = (M_{\text{support}} + M_{\text{span}})_{\text{middle strip}} + (M_{\text{support}} + M_{\text{span}})_{\text{column strip}} = \frac{1}{8} q l^3 \qquad (14.42)$$

which is equal to the well-known mid span bending moment of a correspondingly simply supported beam.

From these bending moments it appears that the bending moments in the column strip are about 70% of the total bending moment $M_{\text{total}} = M_{\text{support}} + M_{\text{span}} = {}^{1}/_{8}\, ql^3 = {}^{96}/_{768}\, ql^3$; the bending moments in the middle strip are about 30% of total moment. This holds for both the negative (support) and positive (span) bending moments. According to the EC 2, the bending moments should be apportioned as given in Table 14. 4.

Table 14. 4: Simplified apportionment of bending moments for a flat slab

	Negative moments	**Positive moments**
Column Strip	60 - 80%	50 - 70%
Middle Strip	40 - 20%	50 - 30%
Note: Total negative and positive moments to be resisted by the column and middle strips together should always add up to 100%.		

In this example, a very approximate method was used for calculating the effects in the flat slab by simplifying it to a continuous beam analysis.

EC 2 (Annex I) gives recommendations for idealizing the three-dimensional column-plate structure as plane frames in two directions, where the stiffness of the members is calculated from their gross cross-section. This is called equivalent frame analysis and it is the most common method for the determination of load actions in the panels and columns of a flat slab (Fig. 14. 58).

In order to determine maximum negative and positive moments in the slab at the positions of the columns and the mid-span cross-section, respectively, the live load has to be placed unfavourably with respect to each load case. In the old days, this was a rather time consuming task, and therefore several approximate methods were often used:

- One alternative was to analyse the slab as a continuous beam (simply supported by the columns), with successive approximations for the column moments (beam method).
- Another even more simple alternative was an approximation of the beam method, based on bending moment coefficients for outer and inner spans. This method is valid provided there certain ratio limits between neighbouring spans are met.

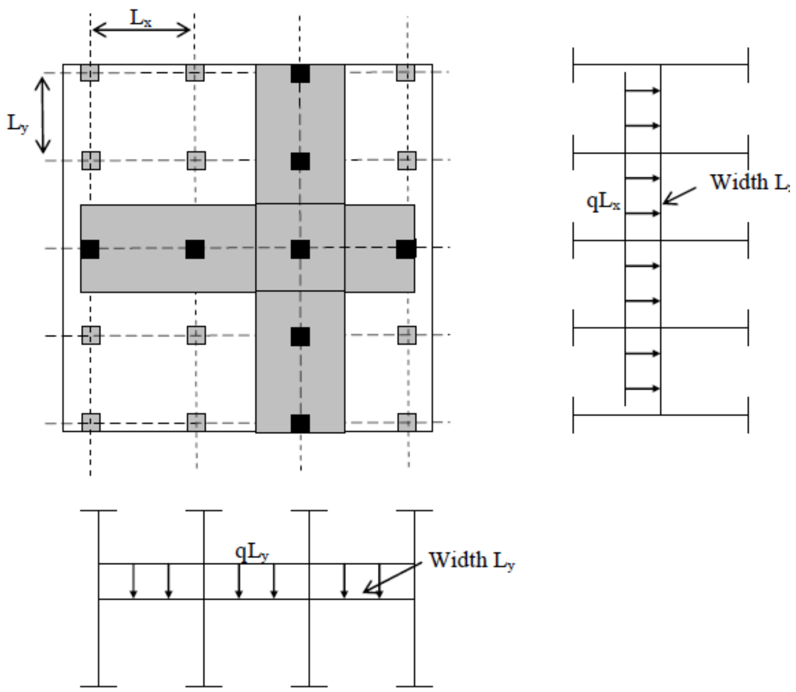


Fig. 14. 58: Equivalent frames from a flat slab consisting of a number of panels and columns

With today's availability of computer programs, static analyses of the frames can be carried out by using an available computer program for plane frames. However, an engineer should always check results by hand performing approximate calculations.

Theory of elasticity

Similarly as with a two-way slab with line supports, also for a flat slab the theory of elasticity might be applied. For the flat slab the columns are the only rigid supports. This implies that, when focusing on the linear elastic distribution of the bending moment over column strip width, it turns out that high (negative) support bending moments occur at the position of the column. Due to cracking a certain level of bending moment redistribution will occur. The reinforcement required is therefore distributed over column strip width such that at the centre part, close to the column, more reinforcement is provided than at the outer parts of the column strip. This has a positive influence on crack width control and on the punching shear resistance (EN 1992-1-1 cl. 6.4).

The width of the central part of the column strip with concentrated reinforcement depends on the geometry of the structure, see Fig. 14. 59. In this figure, the width of the strip to be provided with concentrated reinforcement is indicated. Its width depends on column width and slab height.

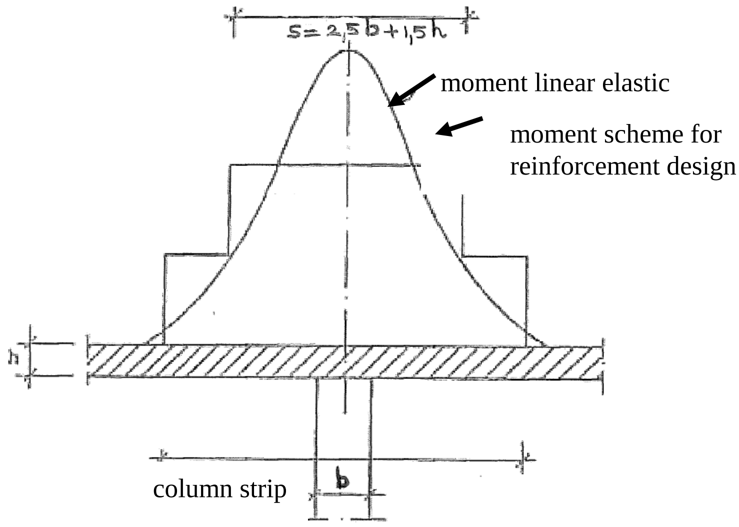


Fig. 14. 59: Distribution of the negative moment over the width of the column strip (NEN 6720)

In Fig. 14. 60 the distribution of the total support bending moment M_s in the column strip over the width of the column strip (0,5 l_x) and the (concentrated) reinforcement strip (*s*) is schematically shown: 40% of the moment is transferred by the reinforcement strip only, whereas 60% is transferred by the full column strip.

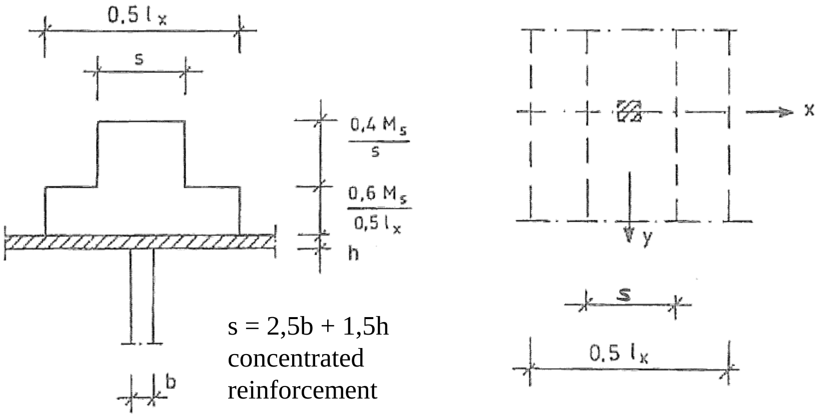


Fig. 14. 60: Distribution of the negative bending moment over the width of a column strip (left) and the width of the strip over which part of the reinforcement is concentrated (right)

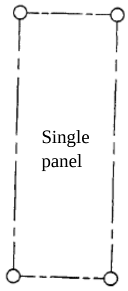
EN 1992-1-1 cl. 9.4.1 (2) gives a different approach to distribute the reinforcement over the column strip. Namely, it states that half of the reinforcement required to resist the full negative moment (from the sum of the two half panels at each side of the column) should be placed in a width equal a sum of 0,125 times the panel width on either side of the column. In case of panels with equal widths, half of the reinforcement should be placed in a strip having a width 0,25 l_x (half of the column strip width, see Fig. 14. 60).

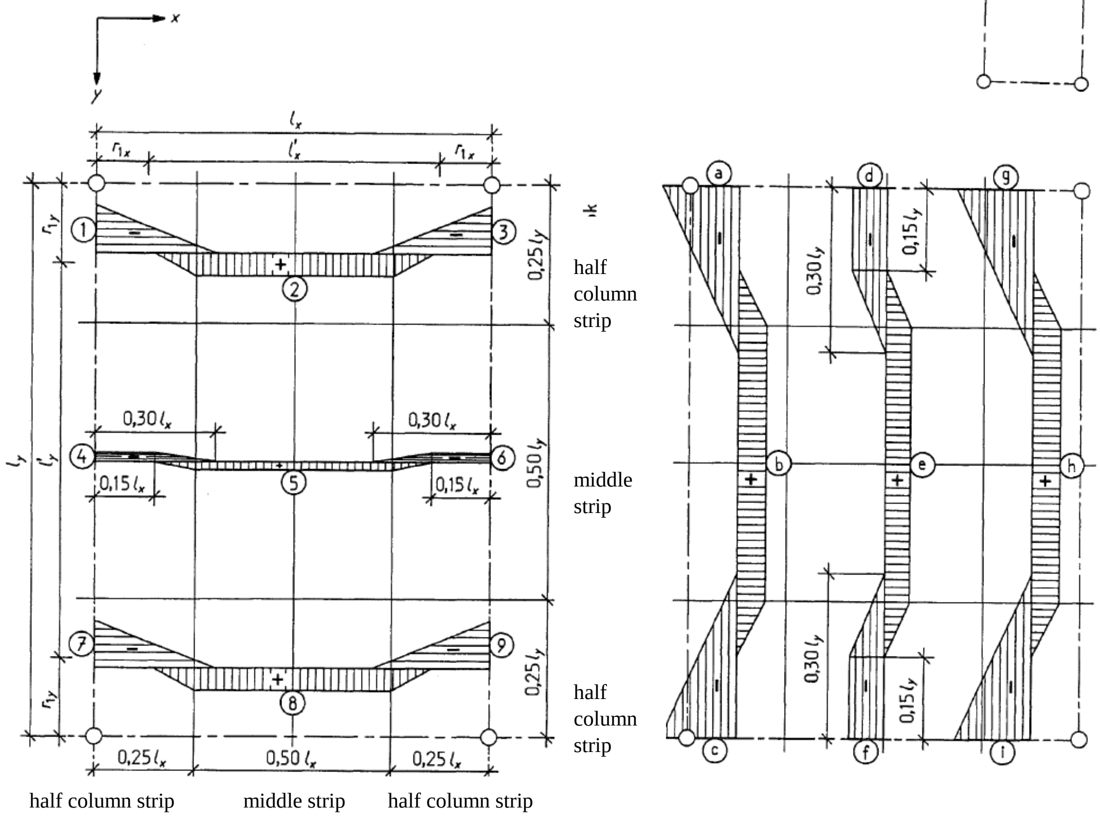
To avoid that designers have to make calculations for each standard situation, tables are developed (enabling a so-called ‘direct design method’). The former Dutch code NEN 6720 contains a set of such tables. The use of these tables is restricted by certain conditions. An important requirement is that the load should be uniformly distributed on each panel of the slab. Furthermore, there are requirements with respect to the ratio between the magnitude of the uniform loads on adjacent panels. There are also requirements with respect to the ratio between adjacent span lengths.

To derive the tabulated data the columns are regarded as hinged point supports that do not transfer a bending moment. Table 14. 5 shows an example of one of the tables available in NEN 6720.
The tables are based on a subdivision of each panel in column strips and middle strips in both load transfer directions. This subdivision results in nine sections in each panel. For all these sections the reinforcement moments (bending moment plus torsional moment) are specified. It is noted that the data are based on the results of sets of linear-elastic finite element calculations. These results were combined such that hand calculations could be performed. That is why it was assumed that bending moments are constant over strip width.

Table 14. 5: Reinforcement moments in a flat slab, continuous along all four edges (NEN 6720 table 19). Schematic presentation of the positive and negative reinforcement moments in two directions (figure) and their magnitude, depending on the ratio between both spans (table)

Schematized reinforcement moments per unit width
Panel continuous at all four edges
Full rotational fixity, no vertical support





l_y/l_x	$m^*_{xx} = 0{,}001\ p_d\ l_x^2 \times$									$m^*_{yy} = 0{,}001\ p_d\ l_x^2 \times$								
	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f	g	h	i
1,0	−132	+ 54	−132	− 40	+ 34	− 40	−132	+ 54	−132	−132	+ 54	−132	− 40	+ 34	− 40	−132	+ 54	−132
1,2	−140	+ 61	−140	− 34	+ 30	− 34	−140	+ 61	−140	−180	+ 69	−180	− 67	+ 53	− 67	−180	+ 69	−180
1,4	−146	+ 67	−146	− 28	+ 25	− 28	−146	+ 67	−146	−235	+ 89	−235	−102	+ 77	−102	−235	+ 89	−235
1,6	−151	+ 73	−151	− 22	+ 21	− 22	−151	+ 73	−151	−296	+112	−296	−145	+104	−145	−296	+112	−296
1,8	−155	+ 78	−155	− 18	+ 17	− 18	−155	+ 78	−155	−363	+140	−363	−195	+134	−195	−363	+140	−363
2,0	−159	+ 81	−159	− 14	+ 14	− 14	−159	+ 81	−159	−437	+171	−437	−251	+167	−251	−437	+171	−437

Irregular load patterns

If the loads on the various panels do not meet the requirements, the following approach can be used:

- The effect of a higher load at a single panel (q_a) relative to the load on all other panels (q_b), can be taken into account in a practical approximate way, see Fig. 14. 61. This approach can be used as an alternative to the system with the checkerboard load configuration (Fig. 14. 42). Take notice that here, Δq is only added in the panel with a relatively higher load, whereas in the checkerboard configuration $Q/2$ is added and subtracted from certain panels.
- The difference between the uniform load on the panel considered and the load on all the other panels is subdivided into the parts Δq_x and Δq_y that are transferred in the *x*- and *y*-direction respectively. Now the specified coefficients from Table 14. 5 can be used or the strip method can be used to calculate the bending moments in the slab.
- In this way not only the additional span moments and support moments are obtained, but (with a good approximation) the additional loading (reactions *R*) on the "hidden beams" as well. After the calculation of the bending moments, the required additional reinforcement can be determined.

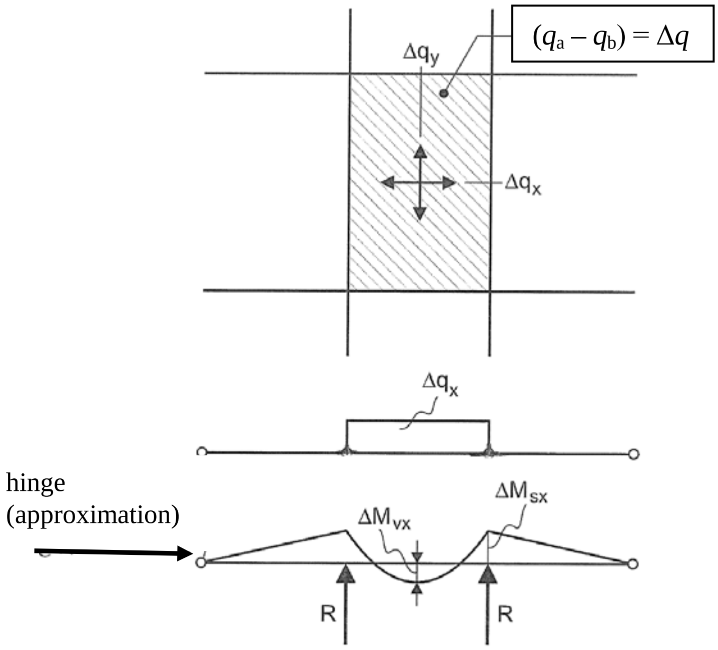


Fig. 14. 61: Practical approach to account for the additional load on one panel from a flat slab

Comparison of a two-way slab with line supports and flat slab - FEM analysis

Note that the bending moment diagrams and therefore also the resulting reinforcement distribution are different for a two-way slab with line supports and a flat slab. Fig. 14. 62 shows the result of a FEM calculation where the bending moments in a two-way slab with fixed line supports and a flat slab are compared. In a flat slab, the edge strips (column strips) are the strips taking the larger portion of the load, and therefore have more reinforcement compared to the middle strip. On contrary, in a two-way slab, the largest portion of the load is carried by the middle strip.

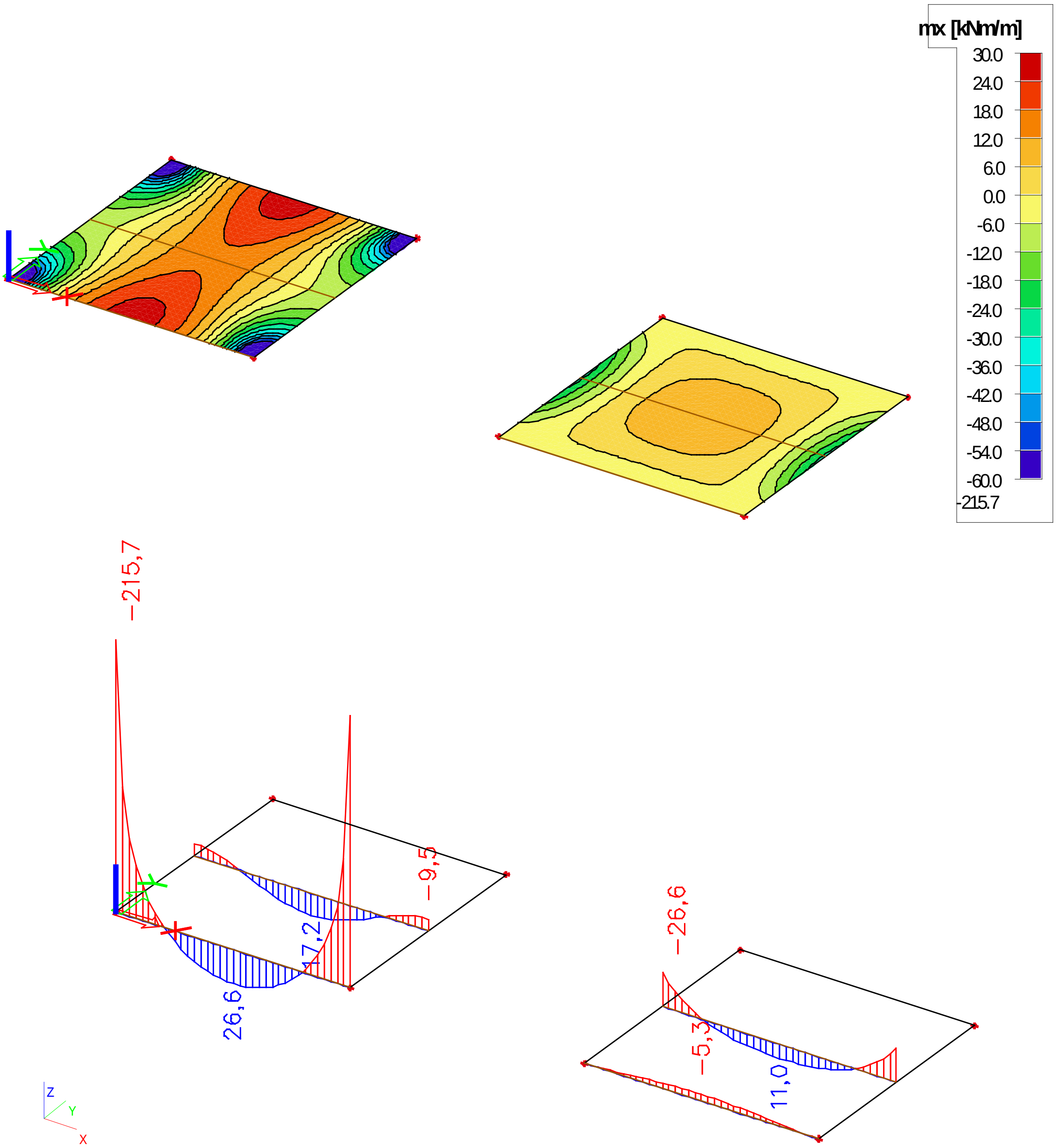


Fig. 14. 62: Moment distribution in a panel from a continuous flat slab (left) and a two-way slab with fixed (no rotation) line supports (right)

Punching shear stress

Due to the high reaction forces at the columns, local shear at the columns is often critical for flat slabs. If the shear capacity is too low, local punching failure may happen. This requires special considerations with respect to flat slab design and will be separately dealth with in Secion 14.4.

14.3 Design of the slab

14.3.1 Introduction

The design of the slab starts with determining the thickness of the slab, the distribution of reinforcement and the required material properties of the slab in order to satisfy:

- Ultimate Limit State (ULS) criteria (related to the capacity of the structure, e.g. bending moment resistance, shear resistance)
- Serviceability Limit State (SLS) Criteria (related to functionality of the structure, e.g. crack widths and deflections).

14.3.2 Estimation of slab thickness

In general a slab is designed in such a way that the maximum deflection, relative to a horizontal line between the supports, under the quasi-permanent load combination, does not exceed w_{max} = $l / 250$ = 0,004 l. Quasi-permanent load is self-weight plus a “permanent” part of the live load (often 40% – 50%).

Usually, this is the governing criterion for the design of the slab. This requirement follows from appearance and general utility (EN 1992-1-1 cl. 7.4.1 (4)).

If the slab is designed with an upward camber (w_c), this camber may be taken into account when calculating the deflection w_{max}. It is advised to make the camber not larger than 0,004 l (EN 1992-1-1 cl. 7.4.1. (4)).

As a complementary requirement the following rules are used in engineering practice for the so-called additional deflection:
$w_{add} \leq 0{,}003\, l$ for slabs in general
$w_{add} \leq 0{,}002\, l$ for slabs which carry partition walls that are susceptible to cracking

The “additional” deflection is defined as the total deflection under the quasi-permanent load combination ($w_{tot,qp}$) at time $t = \infty$, minus the immediately occurring deflection, which takes place when the formwork is removed at time $t = 0$ (w_{on}) (note: partition walls are not constructed yet):

$$w_{add} = w_{tot,qp} - w_{on} = \left(w_{max} + w_c\right) - w_{on} \tag{14.43}$$

If no camber is applied $w_{max} = w_{tot,qp}$.

To calculate the deflection $w_{tot,qp}$ long-term loading has to be taken into account. This implies that the quasi-permanent load combination is to be assumed as permanently acting on the slab. In this calculation, creep has to be taken into account when calculating the bending stiffness of the slab $(E_cI)_\infty$. The deflection $w_{tot,qp}$ has two components. One component comes from the increase of the load from selfweight G to the quasi-permanent load combination, in which a part of Q is present too. The second component is from long-term loading. The following vertical deflections can be defined (Fig. 14. 63):

w_c = camber
w_1 = deflection due to selfweight
w_2 = deflection due to longterm instead of short term selfweight loading
w_3 = deflection due to load increase from selfweight to quasi-permanent loading
w_{tot} = total deflection (total deformation)
w_{max} = total deflection (visible) relative to a line between the supports

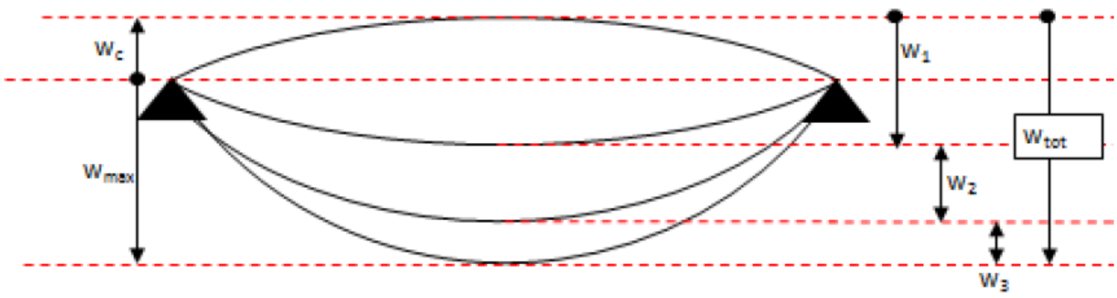


Fig. 14. 63: Definitions of vertical deflections

For cost-efficiency reasons, the slab should preferably be designed in such a way that no shear reinforcement is required.

Note:
According to NEN-EN 1990, 6.5.3(2), the combination of actions for the Servicability Limit State are defined either as:

- characteristic combination which is normally used for irreversible limit states, or when somewhere in the construction the yield limit of reinforcement is reached and deformation stays permanent even if load is removed. This combination results in the most strict requirements.
- frequent combination which is normally used for reversible limit states
- quasi-permanent combination which is normally used for long-term effects and the appearance of the structure

The Dutch national annex (NEN-EN 1990, National Annex, A1.4.3(3)) in certain situations (e.g. with floors that carry partition walls susceptible to cracking) uses a maximum additional deflection, w_{add}, that is calculated for the frequent ($G + \psi_1 Q$) instead of the quasi-permanent load combination. Then the more strict regulation applies as $\psi_2 \leq \psi_1$.

Rule of thumb for the thickness of one-way slab

In the following, the derivation is given on how to calculate the slab thickness for which the deflection will not be governing. For the design of a slab the maximum allowable deflection (w_{max}) according to NEN-EN 1992-1-1 cl. 7.4.1 can be used as a criterion. This recommendation requires that the total deflection (w_{max}) is not larger than 0,004 l:

$$w_{max} = 0,004\,l \tag{14.44}$$

with l being the span of the slab.

Consider a simply supported one-way slab as given in Fig. 14. 64.

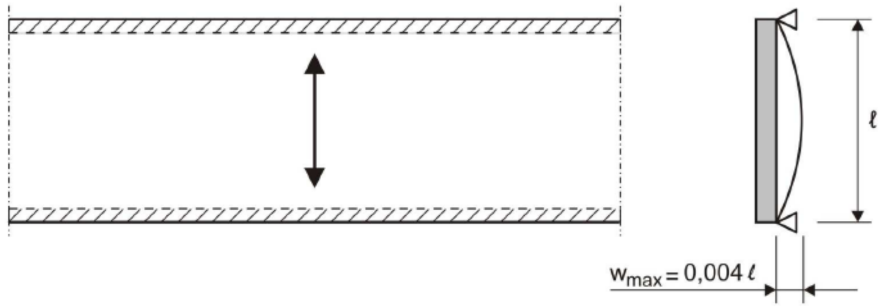


Fig. 14. 64: Slab with two simple line supports spanning in one direction

As shown before in the load transfer analysis, a one-way slab with hinged line supports can be approximated with the simply supported beam. The deflection of a statically determinate beam on two supports under a uniformly distributed load q is now assumed to be equal to the limit value of the total deflection:

$$w_{max} = \frac{5\,q\,l^4}{384\,E_c\,I} \tag{14.45}$$

If the total deflection under the quasi-permanent load combination ($w_{tot,qp}$) should not be larger than w_{max} = 0,004 l, this results in:

$$0,004\,l = \frac{5\,(q_G + \psi_2\,q_Q)\,l^4}{384\,E_c\,I} \tag{14.46}$$

where:

q_G is the permanent load per unit length or unit area;
ψ_2 is the factor to calculate the quasi-permanent part of a live load;
q_Q is the live load per unit length or unit area.

With this equation a rule can be derived to determine the slab thickness, which will generally not violate the condition $w_{max} \leq 0,004\,l$.

For the slab considered, having line supports on two sides and being subjected to a uniformly distributed load, the maximum bending moment per unit width in the serviceability limit state, under the quasi-permanent load combination (q_p) is:

$$M_{\mathrm{qp}} = \tfrac{1}{8}\ (q_{\mathrm{G}} + \psi_2\, q_{\mathrm{Q}})\, l^2 \tag{14.47}$$

which can also be written as:

$$0{,}004\, l = \frac{5\, M_{\mathrm{qp}}\, l^2}{48\, E_{\mathrm{c}}\ I} \tag{14.48}$$

where

$$\frac{M_{\mathrm{qp}}}{E_{\mathrm{c}}\ I} = \kappa \tag{14.49}$$

At mid-span the curvature κ (which follows from a cross-sectional analysis) is Fig. 14. 65:

$$\kappa = \frac{\varepsilon_{\mathrm{s}}}{(d - x)} = \frac{\sigma_{\mathrm{s}}}{E_{\mathrm{s}}\ (d - x)} \tag{14.50}$$

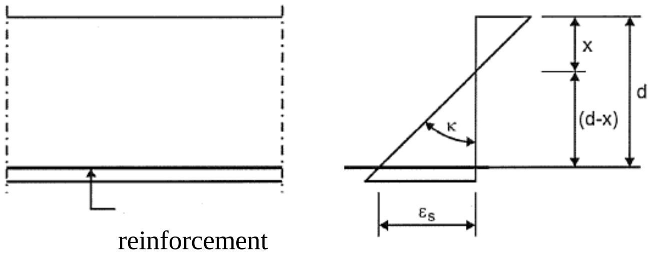


Fig. 14. 65: Curvature (κ) of a cracked reinforced concrete cross-section

Rewriting the expressions results in:

$$0{,}004\, l = \frac{5}{48} \cdot \frac{\sigma_{\mathrm{s}}\, l^2}{E_{\mathrm{s}}\, (d - x)}$$

The requirement with regard to the effective depth of the cross-section is:

$$d \geq \frac{5}{48} \cdot \frac{\sigma_{\mathrm{s}}\, l}{E_{\mathrm{s}} \left(1 - \dfrac{x}{d}\right) \cdot 0{,}004} \tag{14.51}$$

For a rectangular cracked cross-section the relative concrete compression zone height only depends on the Young's moduli of concrete and steel and the amount of reinforcing steel:

$$\frac{x}{d} = -\alpha_{\mathrm{e}}\, \rho_l + \sqrt{(\alpha_{\mathrm{e}}\, \rho_l)^2 + 2(\alpha_{\mathrm{e}}\ \rho_l)} \tag{14.52}$$

where:

α_{e} is the Young's modulus ratio between steel and concrete, $E_{\mathrm{s}} / E_{\mathrm{c}}$;

ρ_l is the reinforcement ratio based on the effective depth d of the cross-section:

$$\rho_l = \frac{A_{\mathrm{s}}}{b\, d}$$

Note:

Equation (14.52) is derived assuming that the concrete is cracked and behaves linear elastic in compression. The concrete is assumed to have no tensile strength. Linear elastic behavior in compression mostly holds since the calculation of deflection is in the serviceability limit state, not in the ultimate limit state.

In Table 14. 6 the ratio *x* / *d* is given for slabs made of concrete having a strength class C25/30 and for various reinforcement ratios (ρ).

E_{cm} = 31000 N/mm^2 for concrete C25/30 (EN 1992-1-1 table 3.1). It should be noted that this is the Young's modulus in case of short-term loading. Creep of concrete (EN 1992-1-1 cl. 3.1.4) results in a time-dependent increase of the concrete compressive strains (EN 1992-1-1 cl. 5.8.6 (4)). This effect can be incorporated in the analysis by using a fictitious Young's modulus:

$$E_{c,eff} = \frac{E_{cm}}{1+\varphi_{eff}} \tag{14.53}$$

where φ_{eff} is the effective creep coefficient of the concrete.

Assuming that, for instance φ_{eff} ~2,7 results in $E_{c,eff} \approx$ 9000 N/mm². With E_s = 200·10^3 N/mm²:

$$\alpha_e = \frac{E_s}{E_{c,eff}} = \frac{200000}{9000} = 22,2$$

The relation between *x* / *d* and ρ_l is given in Table 14. 6 and can approximately be described by:

$$\frac{x}{d} \sim 0,2 + 0,3\,\rho_l$$

where ρ_l is in %.

Table 14. 6: Relation between *x* / *d* and the reinforcement ratio ρ_l

ρ_l (%)	*x* / *d* [-]
0,2	0,26
0,3	0,30
0,4	0,34
0,5	0,37
0,6	0,38

As a result:

$$1 - \frac{x}{d} = (0,8 - 0,3\,\rho_l)$$

If this expression is substituted in eq. (14.51), it is found that:

$$d \geq \frac{5}{48} \cdot \frac{\sigma_s\, l}{200 \cdot 10^3 \cdot (0,8 - 0,3\rho_l) \cdot 0,004}$$

or: $$d \geq \frac{\sigma_s\, l}{7680 \cdot (0,8 - 0,3\rho_l)} \quad (\rho_l \text{ in } \%) \qquad (14.54)$$

However, no account has been taken yet of the stiffening effect of the concrete in the tension zone between the cracks (the so-called tension stiffening). This effect results in an increase of the stiffness by about 20%. If in eq. (14.54) the number 7680 is replaced by 9200 (≈ 1,20 * 7680), then it is found that:

$$d \geq \frac{\sigma_s\, l}{9200\,(0,8 - 0,3\rho_l)} \quad (\rho_l \text{ in } \%) \qquad (14.55)$$

The reinforcement ratio in slabs is mostly low (0,15-0,30%). When using a value ρ_l = 0,2% eq. (14.55) becomes (rounded values):

$$d \geq \frac{\sigma_s\, l}{7000} \quad \text{for slabs} \qquad (14.56)$$

Note, that the similar estimation can be made for beams. Considering that the reinforcement ratio of beams is higher than that of slabs (0,6-1,2%), and therefore assuming a value ρ_l = 0,8% the result is:

$$d \geq \frac{\sigma_s\, l}{5000} \quad \text{for beams} \qquad (14.57)$$

The eqs. (14.55 and 14.56) are derived for simply supported beams and slabs having an effective span l_{eff} that is here denoted as l. The boundary conditions of beams and slabs can differ from the assumed two simple supports: The slab or beam can be continuous or the type of support might diff, for instance clamped/fixed. Equations (14.55 and 14.56) can then still be used, but the effective span has to be adjusted. It is then not based on the actual span, but on the distance between the points where the bending moment $M = 0$.

Table 14. 7 presents some cases for which the deflection can be calculated using a fictitious effective span l_{eff} as presented (l is the actual span). When drawing the bending moment lines for a uniform load and marking the points of zero bending moment, the table values can be derived.

Equation (14.56) and Table 14. 7 can be replaced by a practical table to determine the slab thickness.
Table 14. 8 gives these practical minimum values for l / d of a slab.

For example, the steel stress, σ_s , in SLS (= Serviceability Limit State) can be assumed to be in a range 250-330 MPa. With the value of σ_s ~ 280 MPa the following relation is derived:
$d \geq \frac{l}{25}$, which corresponding to the value given in Table 14. 8.

Note:
The span l is in Table 14. 8 always the distance between the supports and not the distance between points of zero bending moment. It is therefore independent of the type of structural system (boundary conditions) since the influence of the latter is incorporated in the slenderness ratio.

Table 14. 7: Effective span (l_{eff}) for a number of structural systems having different boundary conditions

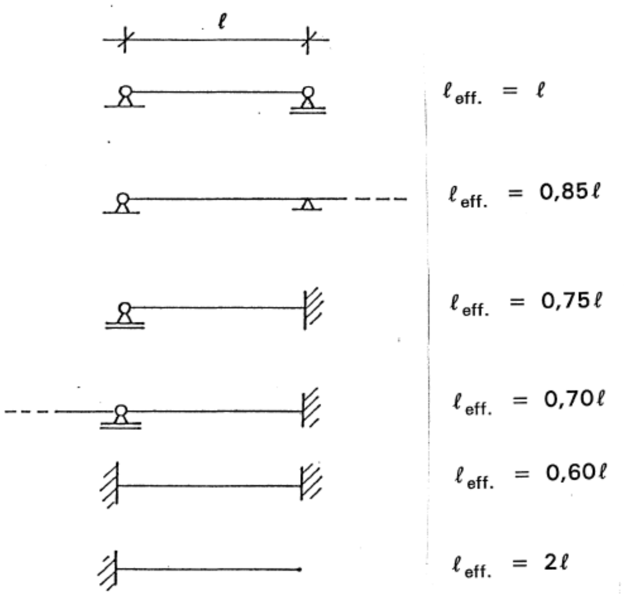
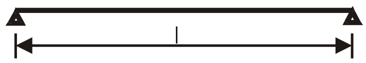
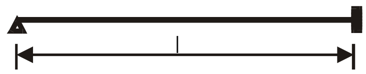
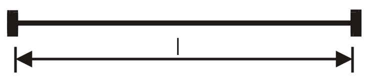


Table 14. 8: Estimation of the slenderness ratio of slabs

Scheme	***l / d* (*l* ≤ 7,0 m)**	***l / d* (*l* ≥ 7,0 m)**
 slabs simply supported at both sides (pinned supports).	25	175 / *l*
 slabs simply supported at one side and fixed or continuous at the other side	32	225 / *l*
 slabs fixed or continuous at both sides	35	245 / *l*

By using this practical approach the designer can quickly estimate the slab thickness which in most cases satisfies the demands for the maximum deflection, without making extended calculations.

Example for determination of the thickness of a one-way slab

The use of the ratio between *l* and *d* as presented in Table 14. 8 is now demonstrated by an example. The slab considered is simply supported at two sides and has a span of 5,0 m. The effective depth of the slab (d) should then be:

$$d \geq \tfrac{1}{25} \cdot 5000 = 200 \text{ mm} \tag{14.58}$$

Bar diameter is 10 mm and concrete cover is 20 mm. The total depth of the slab then is:

$$h = d + c_{\text{nom}} + \tfrac{1}{2}\text{Ø} = 200 + 20 + 5 = 225 \text{ mm} \tag{14.59}$$

Note:

The effective depth is the distance from the outer compression fibre to the centre of the tensile reinforcement. The total depth is the full depth of the slab, so d plus the distance of the centre of the reinforcement to the outer tension fibre. The concrete cover on the reinforcement (c_{nom}) has to be taken into account when calculating h.

Permanent loads		
self-weight slab:	0,225 * 25 =	5,6 kN/m²
floor finishing:		<u>0,4 kN/m²</u>
		6,0 kN/m²
Live load		4,0 kN/m²

The design load (ULS = ultimate limit state; load factor 1,2 for permanent and 1,5 for live loads):

$$q_d = 1{,}2\, q_{G,k} + 1{,}5\, q_{Q,k} \tag{14.60}$$
$$q_d = (1{,}2 \cdot 6{,}0) + (1{,}5 \cdot 4) = 13{,}2 \text{ kN/m}^2$$

The load to be used to determine the stress in the steel reinforcement in the SLS condition regarding the deflection is:

$$q = q_{G,k} + \psi_2\, q_{Q,k} = 6{,}0 + (0{,}5 \cdot 4{,}0) = 8{,}0 \text{ kN/m}^2 \tag{14.61}$$

Note that for live load: $\psi_2 = 0{,}5$ (for the quasi-permanent value).

The steel stress caused by this quasi-permanent SLS load can be approximated by assuming that the ratio between the steel stresses in SLS and in ULS ($f_{yd} = 435$ N/mm^2) is the same as the ratio between their loads. The result is:

$$\sigma_{s,qp} = (8{,}0/13{,}2) \cdot f_{yd} = (8{,}0/13{,}2) \cdot 435 = 264 \text{ N/mm}^2 \tag{14.62}$$

When substituting this value in eq. (14.56) the result is a required effective depth:

$$d = \frac{\sigma_{s,qp}\, l}{7000} = \frac{264 \cdot 5000}{7000} = 189 \text{ mm} < 200 \text{ mm} \tag{14.63}$$

Note:
The steel stress in a cracked concrete cross-section in SLS must be (in most cases) calculated using linear-elastic behaviour of concrete in compression. In ULS, concrete demonstrates elasto-plastic behaviour too (Fig. 14. 66).

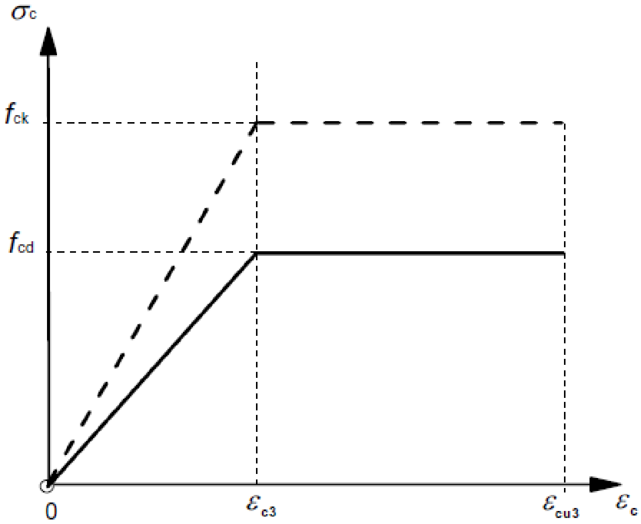


Fig. 14. 66: Bi-linear stress-strain relation of concrete (EN 1992-1-1 fig. 3.4)

The amount of reinforcement actually provided in the slab ($A_{s,prov}$ as used in NEN-EN 1992-1-1) will mostly differ from the amount of reinforcement theoretically required in ULS ($A_{s,req}$ in NEN-EN 1992-1-1) since a number of bars having specific bar diameter(s) and bar spacing must be chosen. In case of slabs, reinforcement labour costs are reduced by choosing meshes from a standard assortment.

Both aspects make that a more exact SLS cross-sectional analysis might result in a different steel stress than the simple approach used in the previous example. The result, however, is often sufficiently accurate in a preliminary design.

It can be derived that at a steel stress of 240 N/mm^2 in SLS, the limit value d = 180 mm is calculated, since:

$$(240 / 264) \cdot 189 \text{ mm} = 172 \text{ mm} \sim 180 \text{ mm} \qquad (14.64)$$

When using d = 180 mm as found with the rule of thumb, the longitudinal reinforcement ratio (ρ_l) is:

$$\rho_l = \frac{A_s}{b\,d} = \frac{M_{Ed}}{z\ f_{yd}} \cdot \frac{1}{b\,d} = \frac{\frac{1}{8}\,b\,q_d\,l^2}{z\ f_{yd}} \cdot \frac{1}{b\,d} =$$
$$= \frac{\frac{1}{8} \cdot 1000 \cdot 12{,}96 \cdot 10^{-3} \cdot (5{,}0 \cdot 10^3)^2}{(0{,}9 \cdot 180 \cdot 435)} \frac{1}{(1000 \cdot 180)} = 0{,}32 \cdot 10^{-2} = 0{,}32\% \qquad (14.65)$$

Note:
The internal lever arm z is assumed to be z = 0,9 d. The exact ratio between z and d ($z = d - x/3$, see Fig. 14. 65) depends on the reinforcement ratio, but an assumed internal lever arm ratio of 0,9 is most often quite accurate.

Thickness of a one way slab according to Eurocode rules

From the previous findings it appears (eq. (14.55)) that the reinforcement ratio (ρ_l) is important when determining the thickness of the slab. In the derivation of this expression the effect of the strength class (in fact the Young's modulus of the concrete) is neglected. As a conservative approach a low strength class, namely C25/30, was assumed. The slab thickness can also be estimated using the slenderness rules from the Eurocode.

In the Eurocode an expression is given in which the reinforcement ratio ($\rho = A_s/A_c$) as well as the strength class and the favourable action of the concrete in between the cracks (tension stiffening) is included. The result of a parameter analysis is as follows (EN 1992-1-1 eq. (7.16.a-b)):

$$\frac{l}{d} = K \cdot \left[11 + 1.5\sqrt{f_{ck}} \cdot \frac{\rho_o}{\rho} + 3.2\sqrt{f_{ck}} \left(\frac{\rho_o}{\rho} - 1 \right)^{3/2} \right] \qquad \text{for } \rho \leq \rho_o \qquad (14.66a)$$

$$\frac{l}{d} = K \cdot \left[11 + 1.5\sqrt{f_{ck}} \cdot \frac{\rho_o}{\rho - \rho'} + \frac{1}{12}\sqrt{f_{ck}} \sqrt{\frac{\rho'}{\rho_o}} \right] \qquad \text{for } \rho > \rho_o \qquad (14.66b)$$

where:

l / d	is the limit value of the slenderness ratio;
K	is a parameter that depends on the static scheme (see Table 14. 9);
ρ_o	is the reference reinforcement ratio = $10^{-3}\sqrt{f_{ck}}$;
ρ	is the required tension reinforcement ratio at mid-span to resist the bending moment due to the design loads (at support for cantilevers);
ρ'	is the required compression reinforcement ratio at mid-span to resist the bending moment due to design loads (at support for cantilevers).

These equations are based on the requirement that $\delta \leq l/500$, where δ is the defection after the construction.

Note:
Compression reinforcement is not often used. Only in slabs that are extremely slender sometimes compression reinforcement is used to reduce deflections and/or to increase rotational capacity.

When deriving the equations (14.66a-b), a steel stress of 310 N/mm^2 was assumed under the quasi-permanent load combination. In Table 14. 9 the factor K is given for concrete elements with different support conditions.

Table 14. 9: Factor *K* for different support conditions (EN 1992-1-1 table 7.4N)

K	Structural system
1,0	Simply supported slab spanning in one or two directions
1,3	End span of a one-way continuous slab or a two-way spanning slab continuous over one long side
1,5	Interior span of a one-way or two-way spanning slab
1,2	Slab supported by columns without beams (flat slab) (based on the longest span)
0,4	Cantilever

Fig. 14. 67 shows the results of the equations (14.66a-b) in a graph. In this case a two-way slab, simply supported at its ends, is assumed (K = 1,0), without compressive reinforcement. With the increase of concrete compressive strength, the slenderness ration of the slab will increase. The slenderness ratio also increases with the lower reinforcement ratios. This is also indicated by expression 14.8 showing that the slenderness increases if the reinforcement ratio decreases. This follows from the reduced compression zone height, resulting in an increase of internal lever arm.

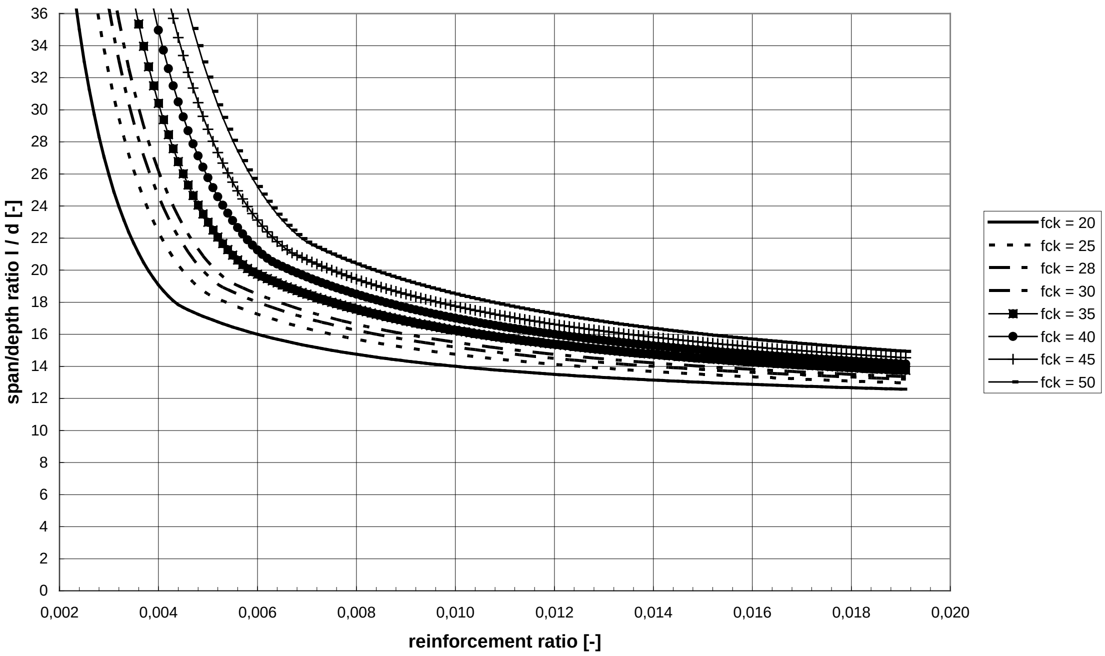


Fig. 14. 67: Relationship between the basic slenderness ratio and the reinforcement ratio

Deflection and thickness of a two-way slab

For two-way slabs with different span ratios and boundary conditions, deflections can be determined based on the theory of elasticity. Tables for moments and deflections for various support conditions can be found in different textbooks. Table 14. 2 shows bending moments and deflections of two-way line supported slabs. Take a look at an uncracked rectangular plate with hinged line supports, subjected to uniformly distributed load q, for span ratios L_x/L_y ranging from 1 to ∞.

The table shows that for a span ratio of 3 the deflection is:

$$w = 12,2 \cdot 0,001 \frac{qL^4{}_x}{EI} = 0,0122 \frac{qL^4{}_x}{EI} \qquad (14.67)$$

The thickness of the slab can now be determined such that the deflection should generally satisfy the rule:

$$w < w_{max} = l/250 = 0,004\, l. \qquad (14.68)$$

Note that for a one-way uncracked slab, the deflection of a simply supported strip with unit width is:

$$w = \frac{5}{384} \frac{qL^4{}_x}{EI} = 0,013 \frac{qL^4{}_x}{EI} \qquad (14.69)$$

Hence, similarly to the bending moments (see section Transition from one-way to two-way slab, page 30), the one-way assumption for L_y/L_x = 3,0 will over-estimate the deflection with approximately 6% .

For the ratio L_y/L_x = 2,0, considering the slab as a one-way instead of a two-way slab (theory of elasticity) results in overestimating deflection by approximately 30% (coefficient is 10 instead of 13). This implies that the common practice approach, namely assuming that a slab may be considered as a one-way slab if the span ratio is larger than 2,0, is rather conservative for span ratios between 2,0 and 3,0[1]. This further demonstrates the advantage of applying the theory of elasticity for plates, which might give a more economical solution with regard to the thickness requirements.

Example: Deflection check for a two-way slab – Serviceability Limit State (SLS)

Note however, that calculating deflections by the theory of elasticity requires that the material is homogenous and isotropic. These requirements are not satisfied for a reinforced concrete slab which is cracked at SLS. Therefore, some approximations have to be made in order to calculate the deflections of a reinforced concrete slab still using the theory of elasticity.

Fig. 14. 68 shows a typical crack pattern at ULS for a simply supported two-way slab (seen

[1] Note that Timoshenko and Woinowsky-Krieger, in ”Theory of Plates and Shells” conclude that one-way assumption is relevant for span ratios larger than 3,0. Therefore, it might be considered to calculate a slab as being two-way if Ly/Lx < 3,0, even though EC2 cl. 5.3.1(5) states that a slab with Ly/Lx > 2,0 can be considered to be a one-way slab.

from bottom face). Cracking in the middle of the slab starts perpendicular to the principal load-carrying direction. It is therefore, reasonable to assume that the plate stiffness is dominated by the one-way acting plate strips in this direction (x-direction, marked in red).

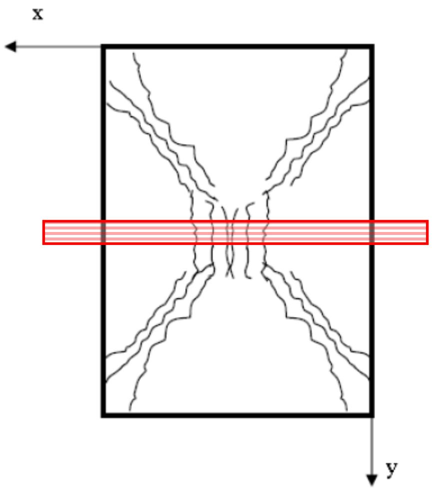


Fig. 14. 68: Typical crack pattern in simply supported two-way slab

For a cracked cross-section, the height of compression zone (x) and moment of inertia (I_{cx}) of a 1 m wide plate strip in x-direction (b = 1m) can be derived based on principles of mechanics and diagrams of concrete and steel strains and stresses in the cross-section:

$$I_{cx} = \frac{M}{\sigma_c} x = \frac{1}{2} x \left(d_x - \frac{x}{3} \right) bx = \frac{1}{2} a_x{}^2 (1 - \frac{a_x}{3}) d_x{}^3 b \tag{14.70}$$

where d_x is the effective depth of cross section and a_x is the compression zone depth factor (which determines the height of the neutral axis) in x-direction:

$$a_x = \frac{x}{d_x} = -\alpha_e \rho + \sqrt{(\alpha_e \rho)^2 + 2\alpha_e \rho} \tag{14.71}$$

with $\alpha_e = \frac{E_s}{E_{cm}}$

and the reinforcement ratio per unit width in x-direction:

$$\rho = \frac{A_{s,x}}{d_x} \tag{14.72}$$

where $A_{s,x}$ is the reinforcement area in the x-direction [mm^2/m]

From the moment of inertia of a cracked cross-section, an effective thickness of a cracked 1 m wide plate strip can be calculated as:

$$h_{eff} = \sqrt[3]{12 I_{cx}} \tag{14.73}$$

Deflection at the midspan section for the actual span ratio can be determined from Table 14. 2:

$$w = 0.001 \frac{p l_x^4}{E_{c,eff} I_{cx}} \gamma \qquad (14.74)$$

Note that a similar calculation can be performed for checking the maximum deflection in cracked, one-way slabs. Calculations of deflections can also be done using FEM analysis. Young modulus of concrete, $E_{c,eff}$, should be estimated based on long-term loading effects (see eq. 14.53).

Deflection and thickness for a flat slab

Similarly as with one-way and two-way slabs, also in flat slabs, the slab thickness has a major influence on the deflections and therefore, it is also important to choose the slab thickness and reinforcement ratio based on deflection calculations. Note, however, that with the flat slabs, punching might be also governing for the design and determination of the slab thickness.

For flat slabs, the thickness is recommended to be chosen in the order of magnitude $h \sim L/25$, for normal live loads and span lengths $\leq 7{,}2$ m [14.1]. If the live load is close to the self-weight, a larger slab thickness should be considered since otherwise large reinforcement quantities are needed to limit the deflections to acceptable sizes. For smaller spans, e.g. $\leq 7{,}2$ m, a thickness of $L/30$ is probably sufficient [14.1].

For a flat slab, the equivalent frame analysis might be used. Since transverse distribution of moments in the equivalent frame analysis is based on elastic theory solutions, the same distribution assumptions can be used for analysis in SLS. Fig. 14. 69 shows how the deflection of the slab can be calculated. The approach is based on calculating two deflections using the strip method. One deflection is calculated using a column strip in *x* direction and a middle strip in *y* direction. The second deflection is calculated using a column strip in *y* direction and a middle strip in *x* direction. The compatibility requirement is that both calculations result in the same deflection.

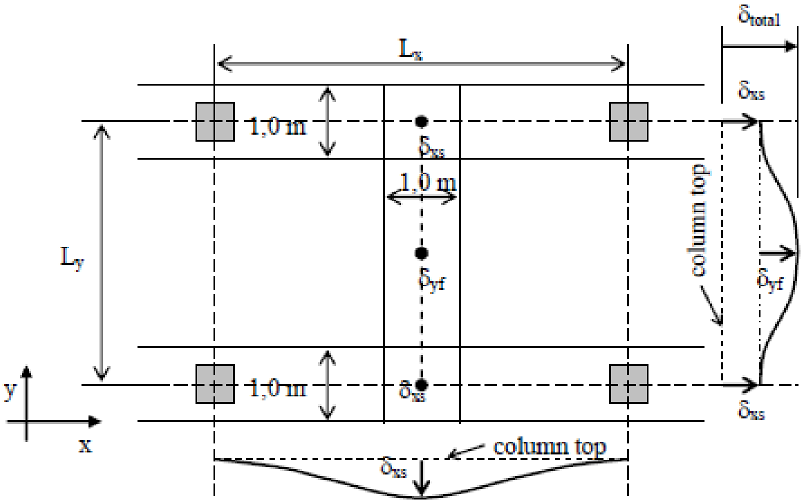


Fig. 14. 69: Principle for deflection calculation in a flat slab panel [14.1]

14.3.3 Bending moment resistance

Once the thickness is determined, the design of the slab starts with the cross-sectional analysis (Fig. 14. 70), similarly as the design of a beam. The amount of reinforcement required should be calculated taking into account ULS loads and a non-linear relation between concrete compressive stress and strain. Crack widths and deflection should be calculated taking into account SLS loads and, in general, considering concrete to be linear elastic in compression.

For a slab transferring load in one direction the following design procedure is used:
In the main load bearing direction the reinforcement is calculated in the same way as in a beam. In the transverse direction, around 20% of the reinforcement required in the main direction is applied (as secondary transverse reinforcement). This is a conservative approach, since actually, when the cracking moment (M_{cr}; r = rupture) is reached, the stiffness of a cross-section is reduced. That is why Poisson's ratio of cracked concrete is lower compared to the ratio of uncracked concrete. As a result, the bending moment in the main direction could be reduced. The method chosen is therefore conservative and the result is sufficiently accurate.

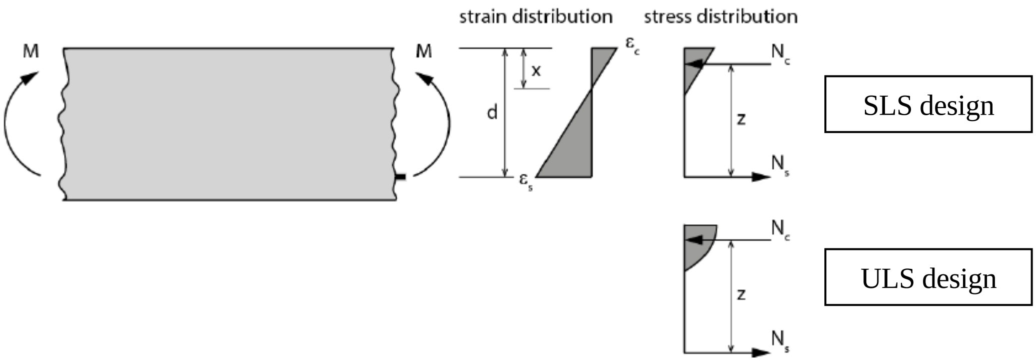


Fig. 14. 70: Cross-sectional analysis of one-way slab (starts analogous to the design of a beam)

For a slab transferring load in two directions, as already shown (Eq. (14.14)), the amount of reinforcement should be calculated based on the "reinforcement" bending moments in two directions, taking into account both bending moments and torsional moments Fig. 14. 71:

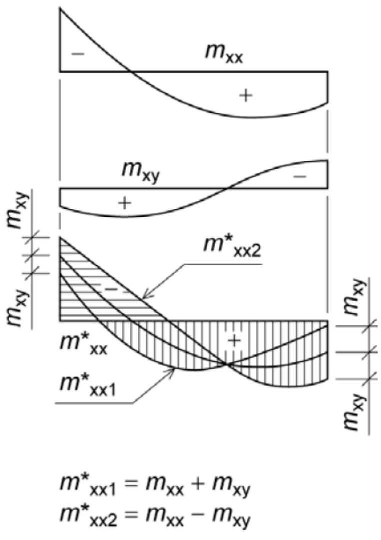


Fig. 14. 71: Reinforcement bending moments in x-direction.

The amount of reinforcement can be also be determined using the bending moments calculated by the strip method. In the strip method it is assumed that the slab takes the load only by bending, without torsion. Note that the assumed load transfer scheme should comply with the actual load transfer scheme. If that is not the case, the SLS behaviour of the slab might not meet code requirements (e.g. crack width).

14.3.4 Shear resistance

From the point of view of costs it should be avoided that a slab requires shear reinforcement. This is because shear reinforcement leads to more demanding workmanship and therefore higher costs which by far overweight savings on concrete. This is why the requirement that no shear reinforcement is needed is sometimes governing for the determination of the slab thickness. One can easily verify whether the effective depth of the slab (d) satisfies the rules with regard to the design value of the shear resistance of a slab without shear reinforcement ($\nu_{Rd,c}$) (EN 1992-1-1 cl. 6.2.2). The calculation is as follows.

The design value of the shear resistance of a concrete structure without shear reinforcement has a minimum value that can always be applied (EN 1992-1-1 eq. (6.3N)):

$$v_{\mathrm{Rd,c}} = v_{\min} = 0.035\, k^{3/2} \sqrt{f_{\mathrm{ck}}} \tag{14.75}$$

where k represents the size effect (EN 1992-1-1 cl. 6.2.2):

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2,0 \tag{14.76}$$

where d is in mm.

Equation (14.75) presents a minimum value. NEN-EN 1992-1-1 presents another expression that might result in a higher value for $v_{\mathrm{Rd,c}}$ (EN 1992-1-1 eq. (6.2.a)):

$$v_{\mathrm{Rd,c}} = 0,12k(100\rho f_{\mathrm{ck}})^{\frac{1}{3}} + k_1\sigma_{\mathrm{cp}} \tag{14.77}$$

where σ_{cp} is the concrete stress in the cross-section from an axial load and/or prestressing. According to Dutch National Annex, k_1 is 0,1 for compression and 0,5 for tension.

However, to use this equation, the amount of longitudinal (bending) reinforcement, represented by percentage of reinforcement ρ , must be known.

Shear reinforcement is not required if the design value of the shear stress ($v_{\mathrm{Ed}} = V_{\mathrm{Ed}}/(b\, d)$) is less than $\nu_{\mathrm{Rd,c}}$. Table 14. 10 contains the $v_{\mathrm{Rd,c}}$ values calculated with eq. (14.75).

Table 14. 10: Minimum shear stress resistance $v_{Rd,c}$ (in N/mm^2) of a concrete element not provided with shear reinforcement

Strength class	d (mm)	200	225	250	275	300	350
	k	2,00	1,94	1,89	1,85	1,82	1,76
	f_{ck} (N/mm^2)	$v_{Rd,c}$ (N/mm^2)					
C20/25	20	0,44	0,42	0,41	0,39	0,38	0,36
C25/30	25	0,49	0,47	0,46	0,44	0,43	0,41
C28/35	28	0,52	0,50	0,48	0,47	0,45	0,43
C30/37	30	0,54	0,52	0,50	0,48	0,47	0,45
C35/45	35	0,59	0,56	0,54	0,52	0,51	0,48

14.3.5 Reinforcement detailing aspects

In order to improve the robustness of slabs, it is in general required that a part of the tensile reinforcement in the span continues to the supports (EN 1992-1-1 cl. 9.3.1.2). This reinforcement ensures that there is always a residual bearing capacity to reduce the impact of possible damage.

EN 1992-1-1- cl. 9.3.1.2 (1) states that at least 50 % of the span reinforcement must extend on to the supports of a simply supported slab, see Fig. 14. 72 [14.4]. In case of a continuous slab, 25 % of the span reinforcement must be present at the intermediate supports, see Fig. 14. 73 [14.4]. At an end support of an end span of a continuous slab, also 25 % of the span reinforcement is required.
Both Fig. 14. 72 and Fig. 14. 73 also present some information on the length over which top support reinforcement is at least required. Part of the lengths presented in Fig. 14. 73 is based on experience from engineering practice. These dimensions should be checked for each situation.

Note:
Also the simply supported slab from Fig. 14. 72 has to be provided with top support reinforcement. This is to resist possible negative bending moments at the supports when partial fixity occurs. In reality, the connection between the slab and the edge beam is partially fixed. The degree of fixity depends on the bending stiffness of the beam or a wall supporting the slab. For practical calculations, it is usually assumed that the slab edge is simply supported. Therefore, if the partial fixity is not taken into account in the calculation, at least 15 % of the maximum positive span moment should be resisted at the support, see NEN-EN 1992-1-1 cl. 9.3.1.2 (2). This reinforcement should extend over at least 0,2 times the length of the span.

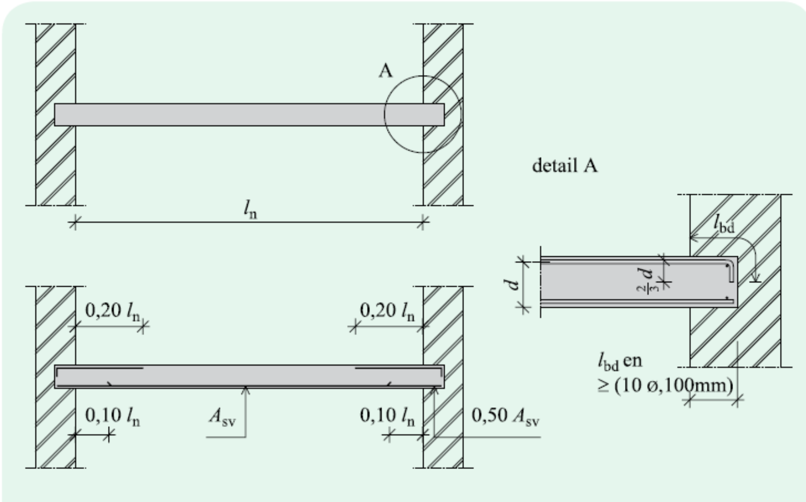


Fig. 14. 72: Requirements on reinforcement at the supports (masonry walls in this case) of a simply supported slab [14.4]

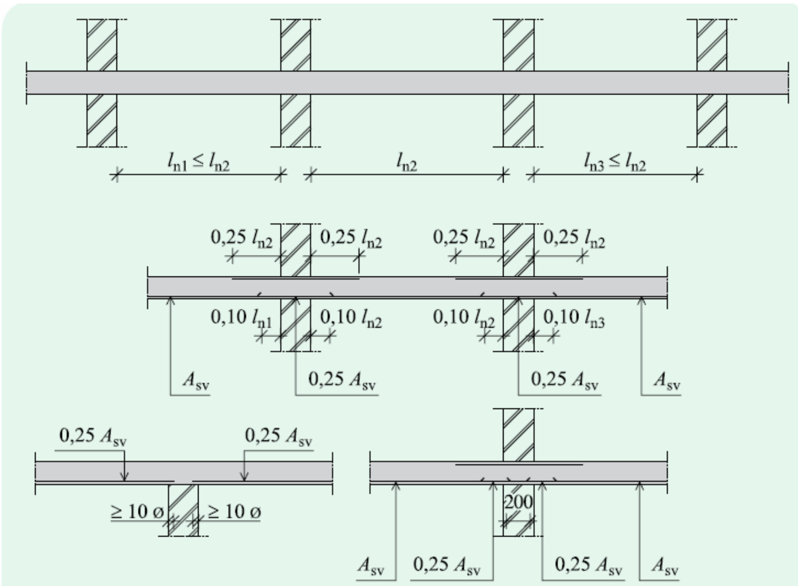


Fig. 14. 73: Requirements on continuity reinforcement in the mid span and top reinforcement of a continuous slab [14.4]

Such a small ("accidental") fixity moment occurs for example when the end of the slab is supported by a continuous wall, see Fig. 14. 74.

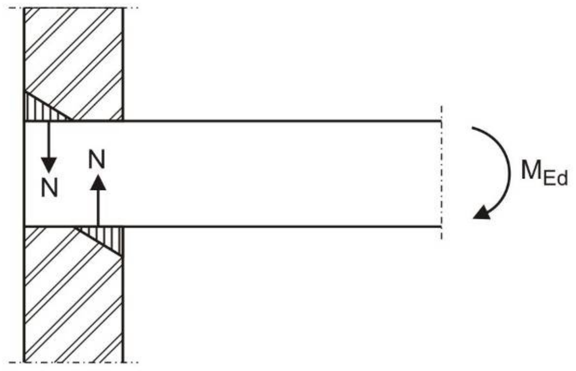


Fig. 14. 74: The development of an (“accidental”) fixed end moment when partial fixity occurs

The fixity caused by such a wall is limited, but it is difficult to calculate the actual fixed end bending moment. By applying a minimum bending moment a practical solution is found to determine the amount of reinforcement required.

The maximum spacing of the reinforcing bars has to be limited. The maximum spacing (*s*) for the main reinforcement is min. (2*h* ; 250 mm) (EN 1992-1-1 cl. 9.3.1.1 (3)). In case of a decreasing moment the maximum spacing is min. (3*h* ; 400 mm) (EN 1992-1-1 cl. 9.3.1.1 (3)).
The maximum spacing for the transverse reinforcement is min. (3*h* ; 400 mm) (EN 1992-1-1 cl. 9.3.1.1 (3)).

At free (unsupported) edges the longitudinal reinforcement must be enclosed, see Fig. 14. 75 (EN 1992-1-1 fig. 9.8). This can be achieved by extending and bending the transverse reinforcement (Fig. 14. 75, top) but this is complex. In most cases hairpins are used (Fig. 14. 75, bottom).

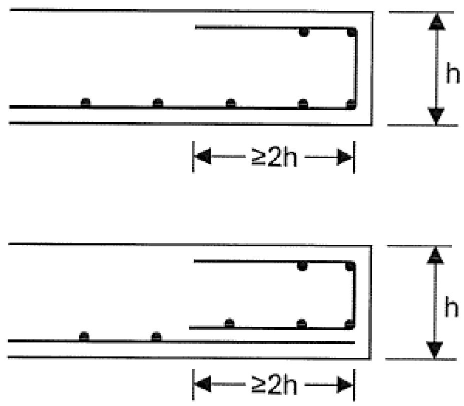


Fig. 14. 75: Enclosing reinforcement at free edges of a slab (adopted fig. 9.8, NEN-EN 1992-1-1)

14.4 Punching shear in flat slabs

In the design of concrete slabs, shear is generally not critical when slabs carry distributed loads and are supported by beams or walls. However, in slabs with concentrated loads and flat slabs, the maximum shear force per unit length can be relatively high in a cross-section close to the loaded area. The effect of concentrated loading on slabs is referred to as punching shear. Punching shear can result from a concentrated load or reaction force acting on a relatively small area. In civil engineering, punching shear is usually considered in:

- Slab-column connections in flat slabs floors
- Column-footing connections in a foundation
- External loads such as wheel loads.

As already indicated, in flat slabs the bearing capacity of the slab around the column can be a critical point. The local combination of very high shear stresses and normal compressive stresses (from bending) can lead to failure of concrete in the area where the column support reaction is transferred. This type of failure is referred to as "punching shear" failure, see Fig. 14. 76.

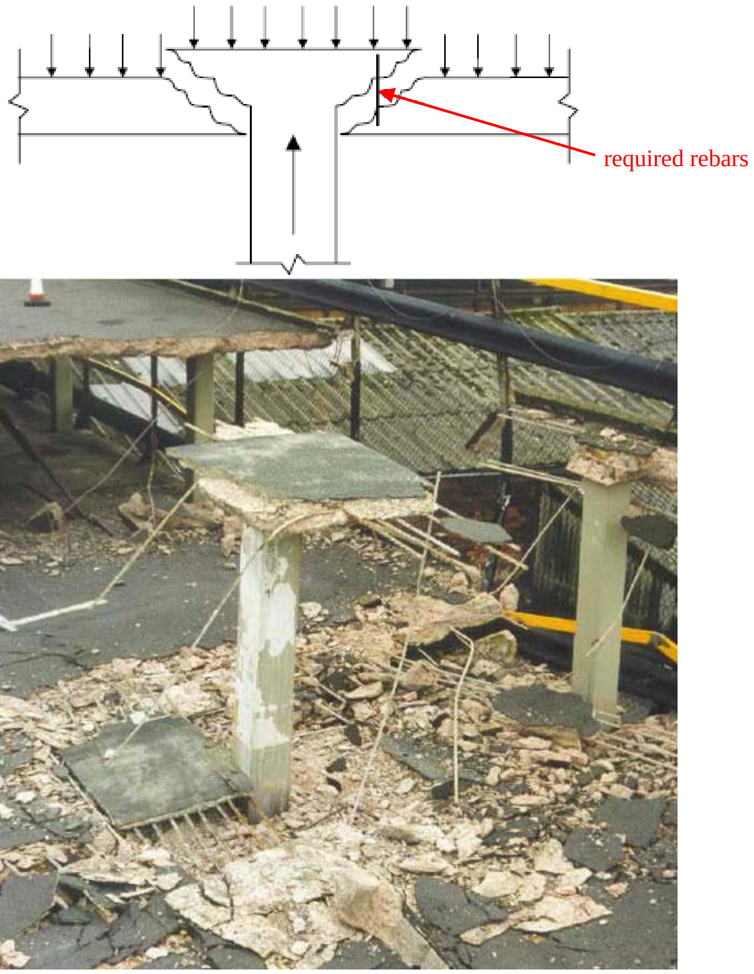


Fig. 14. 76: Punching shear failure mechanism in a flat slab and the functioning of shear reinforcement (if required) (top) and an example of the actual failure of a flat slab

At first the mechanism leading to punching will be described, see Fig. 14. 77 and Fig. 14. 78. In the beginning circular shaped (tangential) cracks arise along the perimeter of the column. These are caused by the large radial bending moments occurring in this region. Then, cracks arise in radial direction. At further increasing load new circular cracks develop at a shorter distance from the column. At a certain load an inclined shear crack occurs in the interior part of the slab. This crack may open and propagate further when a part of the slab moves in vertical direction relative to the surrounding slab. This is only possible when the segments between the radial cracks mutually rotate. This rotating will be counteracted by the reinforcement crossing the cracks. Finally, the column punches through the slab, while pushing away a conical part of the slab. Failure due to punching might occur prior to the yielding of bending moment reinforcement.

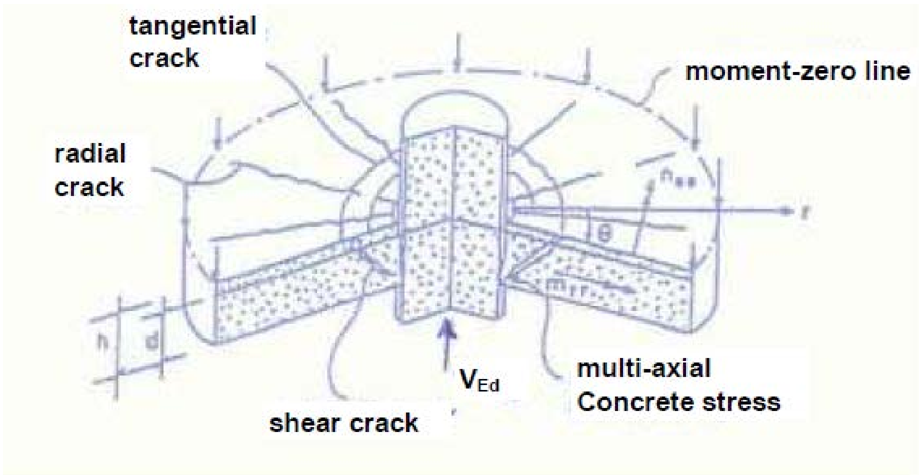


Fig. 14. 77: Crack development prior to punching shear failure

Based on the observed failure mechanism it is concluded that the following factors play a role:

- concrete strength;
- slab thickness;
- dimensions of the directly loaded area (or supporting area);
- slab reinforcement.

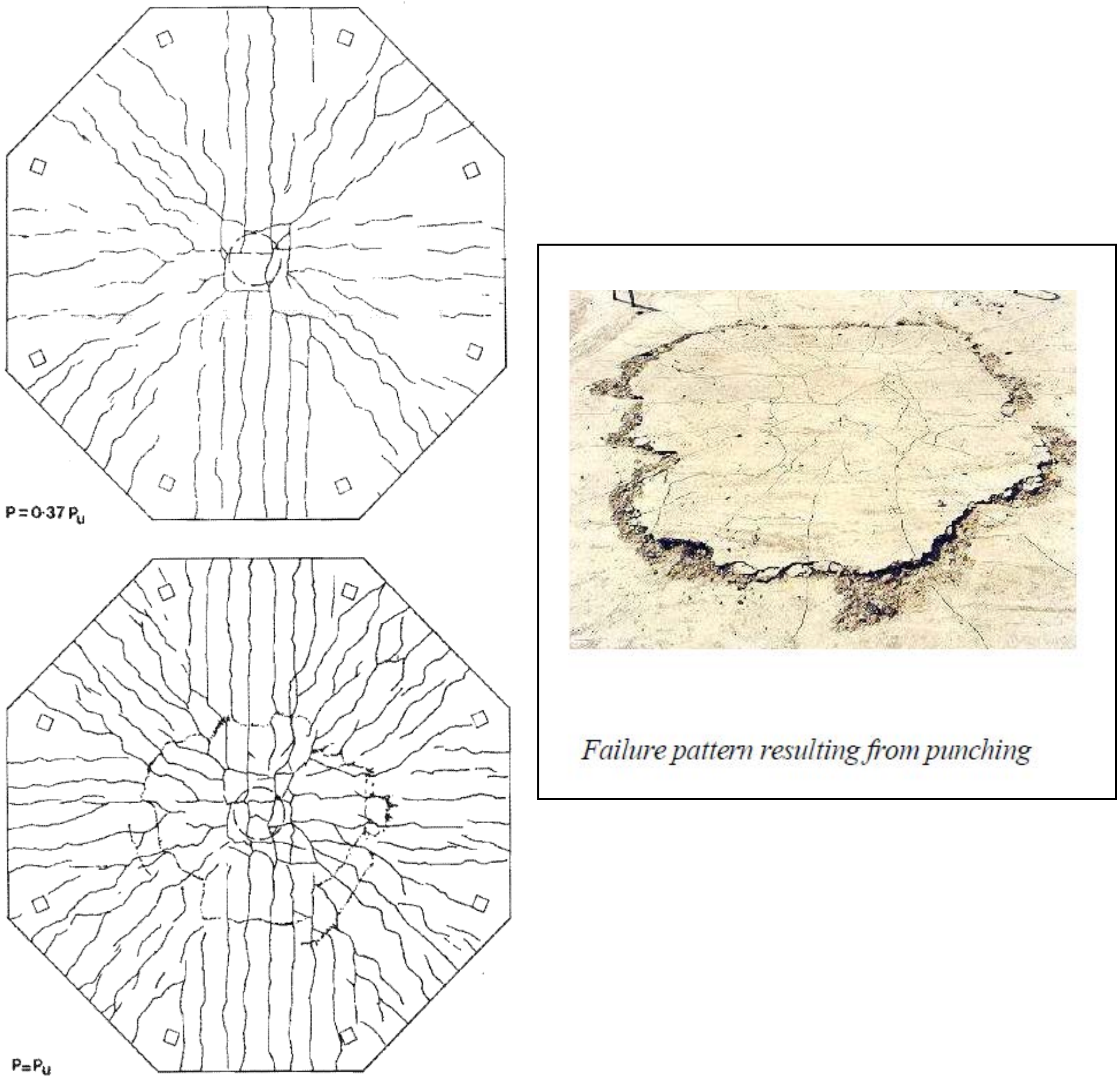


Fig. 14. 78: Top view of a concrete slab with internal column at a load of 0,37V_{Rd} and V_{Rd} (left; V_{Rd} = punching shear resistance, cracking of a symmetrical two-way slab, slab P2 from [14.2]) and after failure (right)

14.4.1 Centrically loaded column

Introduction

Since the punching mechanism is very complex, a simplified model is used. In the model the conical failure surface is replaced by a vertical shear plane at a distance of 2*d* from the loaded area, see Fig. 14. 79 (EN 1992-1-1 fig. 6.12). The circumference of this plane is denoted as the first control perimeter u_1.

In case of a centrically applied load a uniformly distributed vertical shear stress (v_{Ed}) is assumed to be present through the effective depth *d* of the slab. The average shear stress in the first control perimeter is the basis for punching shear design. The design value of the punching shear stress in the first periphery follows from the basic control perimeter and the design value of the punching shear force:

$$v_{\mathrm{Ed}} = \frac{V_{\mathrm{Ed}}}{u_1\ d} \tag{14.78}$$

where:

v_{Ed} is the design value of the punching shear stress;
V_{Ed} is the design value of the punching shear load;

Centrically loaded circular column

For a circular cross-section of a column the expression for the length of the basic control perimeter is:

$$u_1 = \pi\ (c + 4\ d) \tag{14.79}$$

where:

c is the diameter of the column;

d is the effective depth of the slab

At the position of a column, a slab is reinforced in two directions. NEN-EN 1992-1-1 weighs the effective depths in both directions (y and z) (EN 1992-1-1 eq. (6.32)):

$$d = d_{\mathrm{eff}} = \frac{d_{\mathrm{y}} + d_{\mathrm{z}}}{2} \tag{14.80}$$

No punching shear reinforcement is required if the design value of the punching shear stress (v_{Ed}) is smaller than or equal to the limit value of the punching shear resistance stress ($v_{\mathrm{Rd,c}}$) (EN 1992-1-1 eqs. (6.47) & (6.3N):

$$\begin{aligned} v_{\mathrm{Rd,c}} &= 0{,}12\ k\ \left(100 \rho_l\ f_{\mathrm{ck}}\right)^{1/3} + k_1 \sigma_{\mathrm{cp}} \\ v_{\min} &= 0{,}035\ k^{3/2}\ f_{\mathrm{ck}}^{1/2} + k_1 \sigma_{\mathrm{cp}} \end{aligned} \tag{14.81}$$

where :

k is a size effect parameter: $k = 1 + \sqrt{\dfrac{200}{d}} \leq 2{,}0$ (d in mm)

ρ_l is the weighed reinforcement ratio: $\rho_l = \sqrt{\rho_{\mathrm{lx}}\ \rho_{\mathrm{ly}}} \leq 0{,}02$

σ_{cp} is the normal concrete stress: $\sigma_{\mathrm{cp}} = \dfrac{\sigma_{\mathrm{cy}} + \sigma_{\mathrm{cz}}}{2}$

σ_{cy} , σ_{cz} are the normal concrete stresses in the critical sections in y- and z-direction (positive for compression).

k_1 = 0,1 for compression (Dutch National Annex)

= 0,5 for tension (Dutch National Annex)

Note:
Equation (14.18) presents the expressions used to calculate the shear resistance, see NEN-EN 1992-1-1 cl. 6.2.2. The only difference with the shear resistance is that in the punching shear expressions three variables come from a two-dimensional approach, namely the effective depth, the reinforcement ratio and the axial in-plane stress.

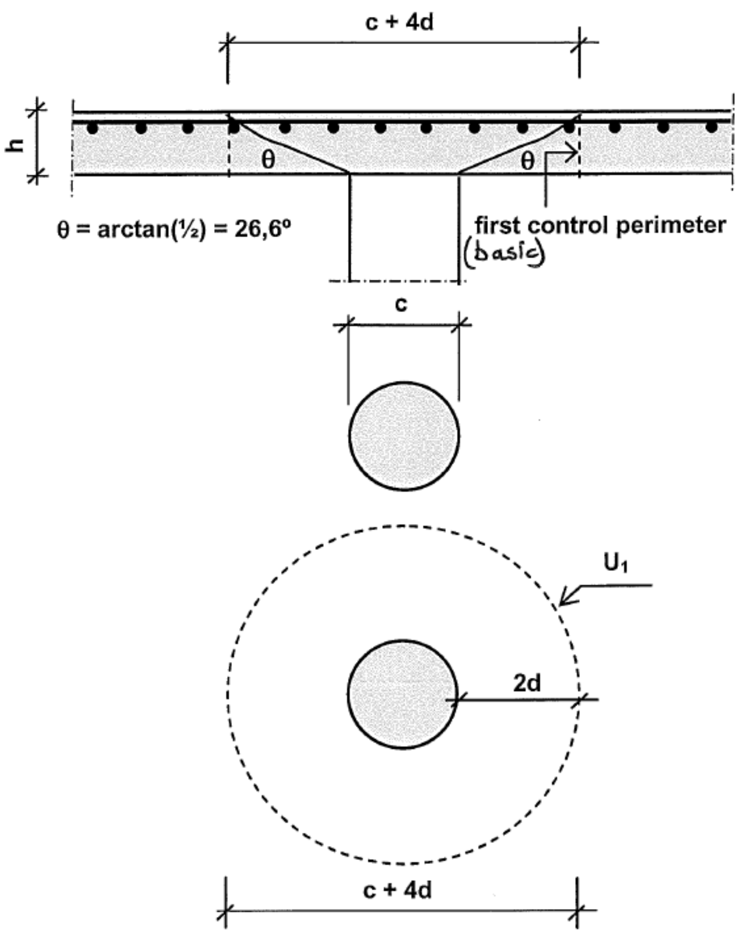


Fig. 14. 79: The basic (or: first) control perimeter (u_1) for a circular cross-section (h = slab height; d = effective slab depth) (EN 1992-1-1 fig. 6.12)

Equation (14.18) is an empirical expression that is the result from the evaluation of a large number of experiments. The parameters used in the expressions have the following effects on the punching shear failure mechanism:

ρ_1 : the basic reinforcement ratio above the support counteracts the rotation of the segments (see Fig. 14. 78) and consequently delays punching failure;

f_{ck} : at a higher concrete strength class the cracks will develop later (increase of concrete tensile strength) which causes punching failure to occur at a higher load;

k : a size effect factor which, just as for shear, expresses that the failure load does not proportionally increase with the slab depth;

σ_{cp} : if the slab is under in-plane compression, for example by prestressing, the cracks will develop later which results in a delay of punching failure. Under tension, cracks will develop sooner which reduces the punching failure load.

If $v_{Ed} > v_{Rd,c}$ structural measures should be taken. These measures can consist of:

1. Increase of concrete strength class, $f_{ck} \uparrow$
2. Increase of reinforcement ratio, $\rho \uparrow$
3. Increase of the slab thickness $d \uparrow$
    a. Locally with the drop panel
    b. Globally, by increase of the slab thickness
4. Increase of the loaded area
    a. Locally with the column head
    b. Globally, by increase of the column dimensions
5. The application of punching shear reinforcement (hooks, stirrups or dowels).

Note that a designer often tries to avoid the use of punching shear reinforcement in slabs.

Centrically loaded non-circular columns

In case of columns having a non-circular cross-section the basic control perimeter is defined in another way. Fig. 14. 80 (EN 1992-1-1 fig. 6.13) demonstrates how the basic control perimeter is derived for different shapes of the loaded area.

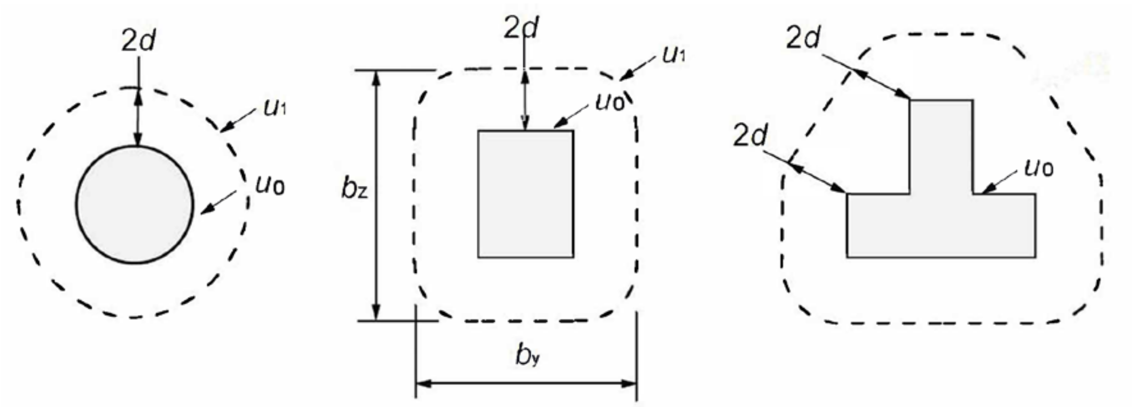


Fig. 14. 80: Typical basic control perimeters around loaded areas

For a rectangular cross-section of a column (see Fig. 14. 81) the size of the basic control perimeter is:

$$u_1 = 2\,(c_1 + c_2) + 4\,\pi\,d \qquad (14.82)$$

where:

c_1 , c_2 are the column dimensions;

d is the effective slab depth.

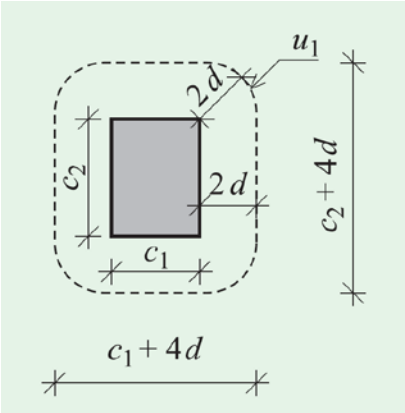


Fig. 14. 81: Basic control perimeter for a rectangular cross-section

14.4.2 Eccentrically loaded columns

Introduction

When the support reaction is eccentric relative to the centre of gravity of the basic control perimeter, not only a shear force but also a bending moment acts. This will occur, for example, when:

- only a part of the slab is loaded,
- adjacent spans have different lengths
- the framework provides stability
- the columns are near an edge or at the corner of a slab.

Fig. 14. 82 (EN 1992-1-1 fig. 6.15) gives information on the perimeter for loaded areas close to or at an edge or corner of a slab. Two aspects contribute to the bending moment. First, the bending moment in the column connections at both top and bottom fiber level of the slab. The greater the column stiffness, the higher the column bending moments. Second, the eccentricity of the basis control perimeter relative to the axis of the column. In case there is an eccentricity, even the normal force from an axially loaded column only, results in a bending moment in the perimeter cross-section.

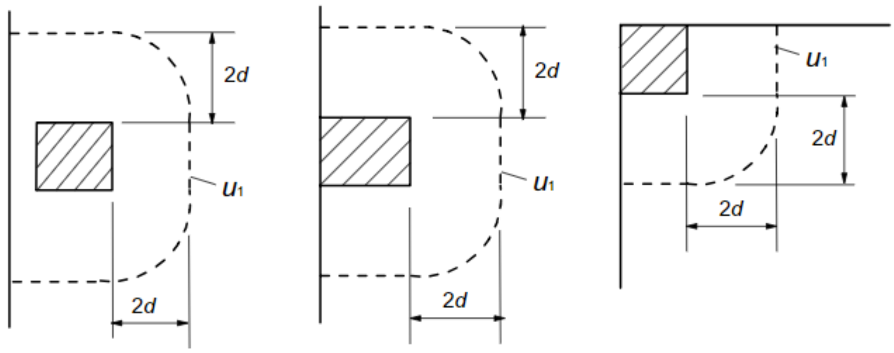


Fig. 14. 82: Basic control perimeters for loaded areas close to or at edge or corner

The bending moment contributes to the shear stresses in the concrete. See Fig. 14. 83 for the assumed plastic shear stress distribution in case of pure bending (EN 1992-1-1 fig. 6.19).

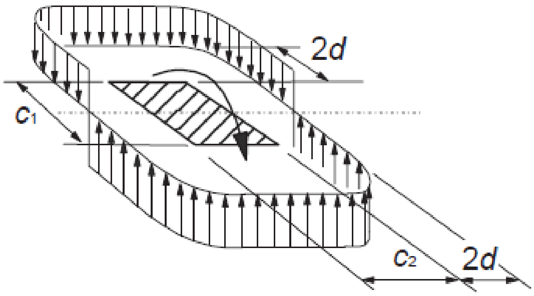


Fig. 14. 83: Assumed plastic shear distribution due to a bending moment at a slab - internal column connection.

In a structure where the lateral stability comes from the frame action between the slabs and the columns, the frame action provides the stability of the building ("sway frames"). Relatively large moments occur in the connections between columns and slabs since all horizontal loads, e.g. wind loads, must be resisted by the frame. The contribution of the eccentricity moment should then be accurately calculated since now, in most cases, the eccentricity of the punching shear force will not be toward the interior of the slab.

In a structure where the lateral stability does not depend on the frame action between the slabs and the columns, the stability is provided by specific stability elements such as shear walls resulting in "non-sway frames" or "braced frames". The contribution of the eccentricity moment can be calculated accurately, but the Eurocode provides easier, safe (upperbound) approximation of the effect of bending moments to shear stresses.

Internal column with bending about one axis

Accurate approach

For uni-axial bending (see Fig. 14. 84) the design value of the punching shear stress in the basic perimeter is (EN 1992-1-1 eq. (6.38)):

$$v_{\mathrm{Ed}} = \beta \, \frac{V_{\mathrm{Ed}}}{u_1 \, d} \tag{14.83}$$

where

v_{Ed} is the design value of the punching shear stress;

V_{Ed} is the design value of the punching shear force;

u_1 is the basic control perimeter;

d is the effective slab depth.

The variable β is used to take into account the influence of the bending moment on the developed shear stress (EN 1992-1-1 eq. (6.39)):

$$\beta = 1 + k \, \frac{M_{\mathrm{Ed}}}{V_{\mathrm{Ed}}} \, \frac{u_1}{W_1} \tag{14.84}$$

where:

k is a coefficient dependent on the ratio between the column dimensions c_1 and c_2 (EN 1992-1-1 table 6,1; Table 14. 11);

W_1 corresponds to a distribution of shear as illustrated in Fig. 14. 83 and is a function of the first control perimeter u_1 (W_1 = linear surface moment of u_1).

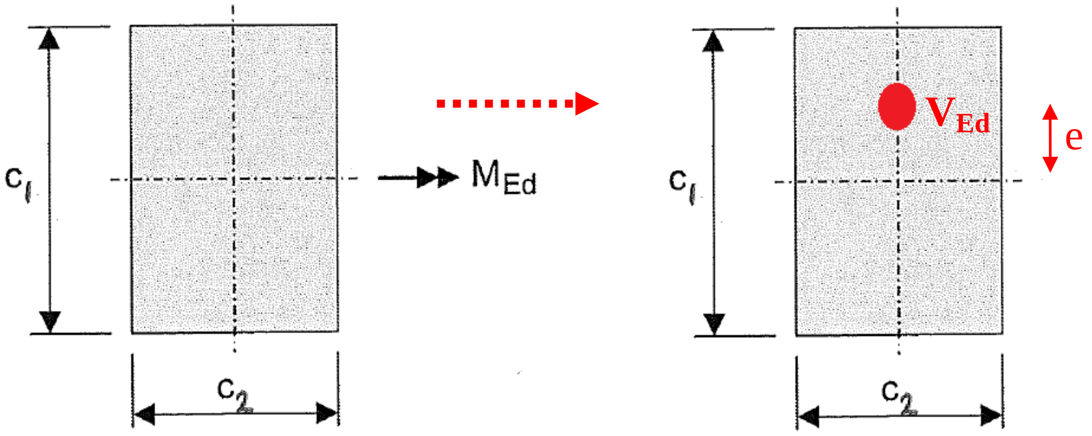


Fig. 14. 84: Definition of column dimensions for a column loaded in bending around one axis (EN 1992-1-1 fig. 6.19)

W_1 of the basic control perimeter is:

- for a rectangular column (Fig. 14. 84; NEN-EN 1992-1-1 eq. (6.41)):

$$W_1 = \frac{c_1^2}{2} + c_1\, c_2 + 4\, c_2\, d + 16\, d^2 + 2\pi\, d\, c_1 \qquad (14.85)$$

c_1 is the column dimension parallel with the eccentricity of the load;

c_2 is the column dimension perpendicular to the eccentricity of the load.

- for a circular cross-section of the column (D = column diameter):

$$W_1 = (D + 4d)^2 \qquad (14.86)$$

The factor k reflects that the bending moment transferred by the column (Fig. 14. 85) does not only cause shear forces but also bending and torsional moments in the slab. This implies that only a part of the bending moment contributes to vertical shear stresses.

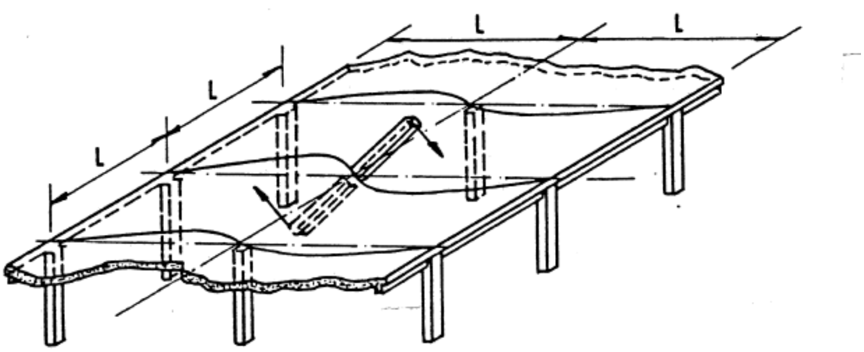


Fig. 14. 85: The transfer of a bending moment from a column into a slab [14.3]

For rectangular loaded areas the values for k are given in Table 14. 11.

Table 14. 11: Values for k with respect to rectangular loaded areas (EN 1992-1-1 table 6.1)

c_1 / c_2	≤ 0,5	1,0	2,0	≥ 3,0
k	0,45	0,60	0,70	0,80

Internal column with bi-axial bending

For bi-axial bending (see Fig. 14. 86), the design value of the punching shear stress in the basic control perimeter is:

$$v_{\mathrm{Ed}} = \beta \frac{V_{\mathrm{Ed}}}{u_1\, d} \qquad (14.87)$$

where:

$$\beta = 1 \; + \; k_1 \frac{M_{\mathrm{Ed},1}}{V_{\mathrm{Ed}}} \cdot \frac{u_1}{W_1} \; + \; k_2 \frac{M_{\mathrm{Ed},2}}{V_{\mathrm{Ed}}} \cdot \frac{u_1}{W_2} \qquad (14.88)$$

For a rectangular column W_1 and W_2 are:

$$W_1 = \frac{c_1^2}{2} + c_1\, c_2 + 4\, c_2\, d + 16\, d^2 + 2\pi\, d\, c_1 \qquad (14.89)$$

$$W_2 = \frac{c_2^2}{2} + c_1\, c_2 + 4\, c_1\, d + 16\, d^2 + 2\pi\, d\, c_2 \qquad (14.90)$$

The expression for W_2 is easily constructed from the expression for W_1 by interchanging c_1 and c_2.

Note:
This must also be accounted for when determining the factors k_1 and k_2.

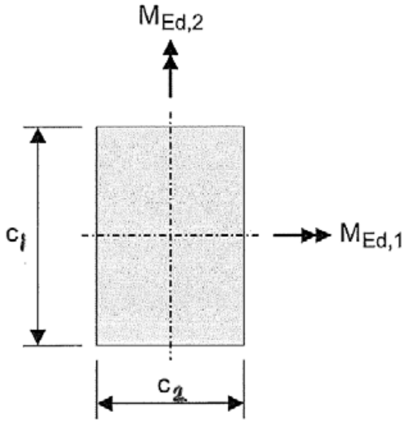


Fig. 14. 86: Bi-axially loaded column

For a circular column (diameter D) bi-axially loaded $W_1 = W_2$ and (EN 1992-1-1 eq. (6.42):

$$\beta = 1 + 0,6\pi \cdot \frac{e}{D+4d} \tag{14.91}$$

where e is the eccentricity of the load.

For an interior bi-axially loaded rectangular column the factor β can be approximated (EN 1992-1-1 eq. (6.43)):

$$\beta = 1 + 1,8\sqrt{\left(\frac{e_y}{b_z}\right)^2 + \left(\frac{e_z}{b_y}\right)^2} \tag{14.92}$$

where:

e_y , e_z are the eccentricities M_{Ed} / V_{Ed} along the y and z axes respectively;

b_y , b_z are the dimensions of the basic control perimeter (EN 1992-1-1 fig. 6.13, Fig. 14. 80):

$$b_y = c_2 + 4d$$

$$b_z = c_1 + 4d$$

For a square column ($c = c_1 = c_2$) the approximate expression for β is:

$$\beta = 1 + \frac{1,8}{c + 4d}\sqrt{e_y^2 + e_z^2} \tag{14.93}$$

Edge column

For an edge column distinction should be made with respect to the direction of the eccentricity of the axial force. The eccentricity can be toward the interior (in Fig. 14. 87: to the right) or the exterior of the slab (in Fig. 14. 87: to the left). If the eccentricity is directed interior, the following approximate expression for β can be used:

$$\beta = \frac{u_1}{u_1^*} \tag{14.94}$$

where u_1* is the reduced basic control perimeter, see Fig. 14. 87 (EN 1992-1-1 fig. 6.20a). This perimeter is the basic control perimeter u_1 from which two line segments are removed (each of length $\geq 0,5c_1$ or $\geq c_1 - 1,5d$).

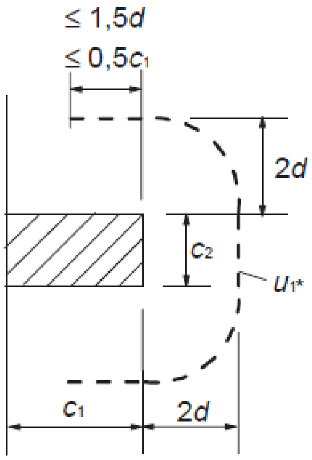


Fig. 14. 87: Reduced basic control perimeter u_1* for a rectangular edge column

If the eccentricity of the axial force in an edge column is directed interior (in Fig. 14. 87: to the right) and an additional eccentricity resulting from a moment about an axis perpendicular to the edge of the slab is present, β can be determined by adding both components that contribute to the shear stress in the slab (EN 1992-1-1 eq. (6.44)):

$$\beta = \frac{u_1}{u_1^*} + k\frac{u_1}{W_1}e_{\text{par}} \tag{14.95}$$

where

e_{par} is the eccentricity parallel to the edge of the slab resulting from a moment about an axis perpendicular to the slab edge

W_1 is calculated for the basic control perimeter u_1 for a rectangular edge column as given in Fig. 14. 87: $W_1 = \frac{c_2^2}{4} + c_1c_2 + 4c_1d + 8d^2 + \pi dc_2$

If the eccentricity of the force is not interior but exterior to the slab, the simplified reduced basic control perimeter approach cannot be used. The designer then has to use the original basic expression for β:

$$\beta = 1 + k\frac{M_{\text{Ed}}}{V_{\text{Ed}}} \cdot \frac{u_1}{W_1} \tag{14.96}$$

The W_1 should be calculated with respect to the centroid of the basic control perimeter. This calculation results in complex expressions and is not elaborated in this textbook.

Corner column

Just as for the edge column, the direction of the eccentricity of the column force is important with regard to the expressions to use. For an interior directed eccentricity the following expression for β may be used:

$$\beta = \frac{u_1}{u_1^*} \tag{14.97}$$

where u_1* is the reduced basic control perimeter, see Fig. 14. 88(EN 1992-1-1 fig. 6.20b).

Note:
The expression is the same as for an edge column. The difference between an edge and a corner column is in the definition of u_1*, see Fig. 14. 88.

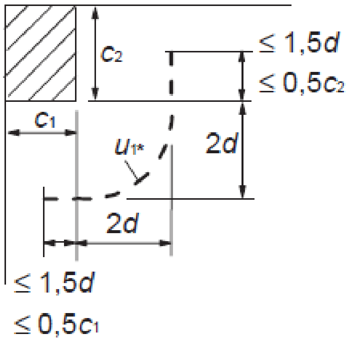


Fig. 14. 88: Reduced basic control perimeter u_1* for a rectangular corner column

If the eccentricity of the axial force is exterior, the reduced control perimeter cannot be used. Just as for the edge column then the following expression for β must be used:

$$\beta = 1 + k \frac{M_{\mathrm{Ed}}}{V_{\mathrm{Ed}}} \cdot \frac{u_1}{W_1} \tag{14.98}$$

Also now the W_1 must be calculated for the basic control perimeter. The calculation becomes even more complex than for the edge column and is not elaborated in this textbook.

Standard β values

Easy (approximate) approach

If the lateral stability does not depend on the frame action between the columns and the slabs, and when the adjacent spans do not differ in length by more than 25%, approximate values for β may be used, see Fig. 14. 89 (EN 1992-1-1 fig. 6.21N).

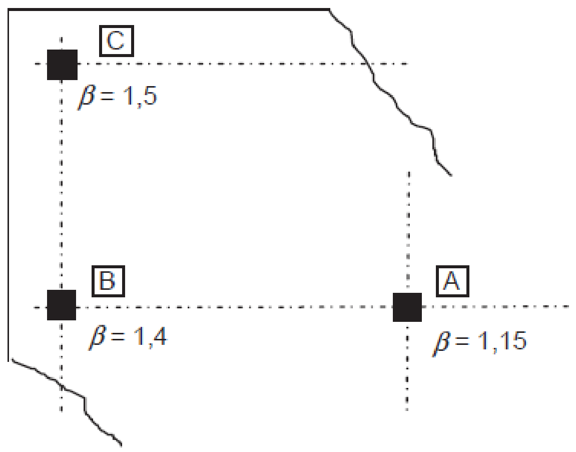


Fig. 14. 89: Apporximated safe values for β (EN 1992-1-1 fig. 6.21N)

The expression for the design value of the punching shear in the basic control perimeter is unaltered:

$$v_{Ed} = \beta \frac{V_{Ed}}{u_1 \, d} \tag{14.99}$$

where

v_{Ed} is the design value of the punching shear stress;

V_{Ed} is the design value of the punching shear force;

u_1 is the basic control perimeter;

d is the effective slab depth;

β internal column: β = 1,15;

edge column: β = 1,4;

corner column: β = 1,5.

14.4.3 Structural measures to increase the punching shear resistance

From eq. (14.81) it appears that the punching resistance can be increased by increasing the depth of the slab or the dimensions of the column or by applying concrete having a higher strength class. These options have some drawbacks:

- increasing the depth of the slab leads to an increase of the self-weight of the structure;
- an increase of the column dimensions is contrary to architectural demands;
- a higher strength class of the concrete of results in higher costs for the entire slab whereas the extra strength is required only in a relatively small area around the column.

Another option to increase the punching shear resistance is to apply a column head and/or drop panel (Fig. 14. 90).

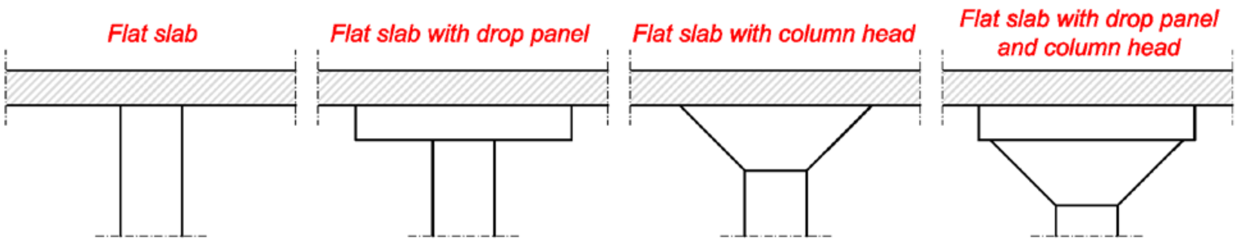


Fig. 14. 90: Flat slab with drop panel and/or column head

Column head

A column head is a local increase of column size just underneath the slab (Fig. 14. 91). If the column size increases gradually the floor is called a mushroom flat slab. This measure results in a considerable increase of the diameter of the punching cone.

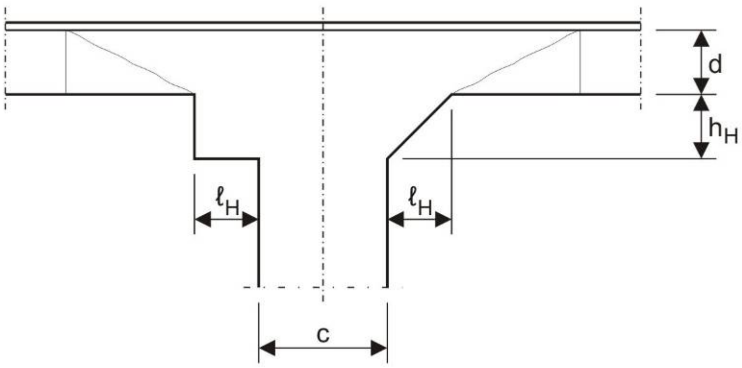


Fig. 14. 91 Increasing the punching shear resistance by applying a column head by a step-wise (left) or gradual (right) increase of slab depth; $l_H < 2{,}0\ h_H$ (EN 1992-1-1 fig. 6.17)

The design value of the shear stress for a circular column then results from:

$$v_{Ed} = \frac{V_{Ed}}{u_1\, d} \tag{14.100}$$

where: $u_1 = \pi \cdot (c + 2\ell_H + 4d)$ (14.34)

If the width of the column head l_H further increases and becomes larger then 2,0 h_H a so-called drop panel is created.

Drop panel

Now, two punching cones can arise, see Fig. 14. 92 (EN 1992-1-1 fig. 6.18). The shear stresses must be verified at both failure surfaces.
In case of a circular column and a circular drop panel, the verification formulae are:

Inner failure surface:

$$v_{\mathrm{Ed}} = \frac{V_{\mathrm{Ed}}}{u_1\, d_{\mathrm{H}}}$$

where:

$$d_{\mathrm{H}} = d + h_{\mathrm{H}}$$

$$u_1 = \pi \cdot (c + 4\, d_{\mathrm{H}}) \qquad (14.101a)$$

Outer failure surface:

$$v_{\mathrm{Ed}} = \frac{V_{\mathrm{Ed}}}{u_2\, d}$$

where:

$$u_2 = \pi \cdot (c + 2l_{\mathrm{H}} + 4d) \qquad (14.101b)$$

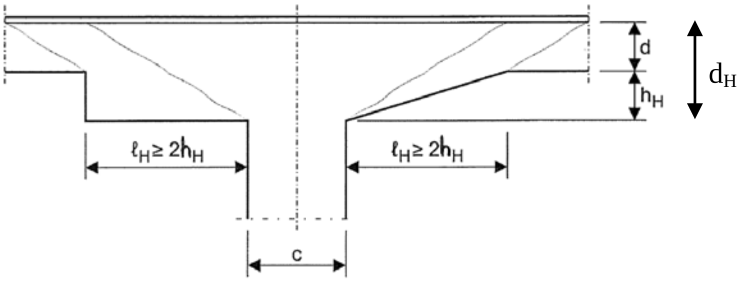


Fig. 14. 92: Example of a slab containing a drop panel (left) or a wide column head (right) in which two punching cones can arise (EN 1992-1-1 fig. 6.18)

Often in practice the drop panel is rectangular, not circular. The outer basic control perimeter is then calculated as for a rectangular loaded area.

Another option to increase the punching shear resistance is to provide shear reinforcement that anchors the punching cone to the surrounding concrete. Cracks that develop cross the shear reinforcement. The reinforcement then carries (part of) the punching shear load.

The shear reinforcement must be arranged close to the column and must extend to a cross-section where shear reinforcement is no longer required and the concrete can resist the design value of the shear stress: $v_{\mathrm{Rd,c}} > v_{\mathrm{Ed}}$. This cross-section is called the outer perimeter and is indicated in Fig. 14. 93 (u_{out} in NEN-EN 1992-1-1 fig. 6.22). The outer perimeter has the same shape as the basic control perimeter. Shear reinforcement should extend to a cross-section that is maximum 1,5d from u_{out} (EN 1992-1-1 cl. 6.4.5 (4)). This outer perimeter will be discussed later on.

According to the Eurocode at least two rows of link legs must be provided. The radial spacing, s_r, between these rows may not exceed 0,75d (EN 1992-1-1 cl. 9.4.3 & fig. 9.10). This is to ensure that no cracks can develop between the punching reinforcement that do not cross any shear reinforcement.

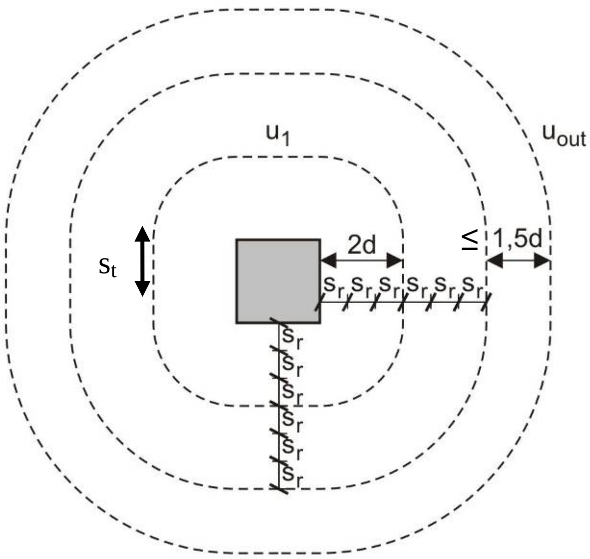


Fig. 14. 93: The area over which shear reinforcement is required (up to 1,5*d* from u_{out})

The first link leg may not be situated closer than 0,3*d* to the column, but also not more than 0,5*d* away from the column (EN 1992-1-1 cl. 9.4.3 & fig. 9.10). These requirements must ensure that a crack protruding from the column crosses the shear reinforcement. Fig. 14. 94 shows the arrangement of shear reinforcement relative to the first punching shear cracks that can develop.

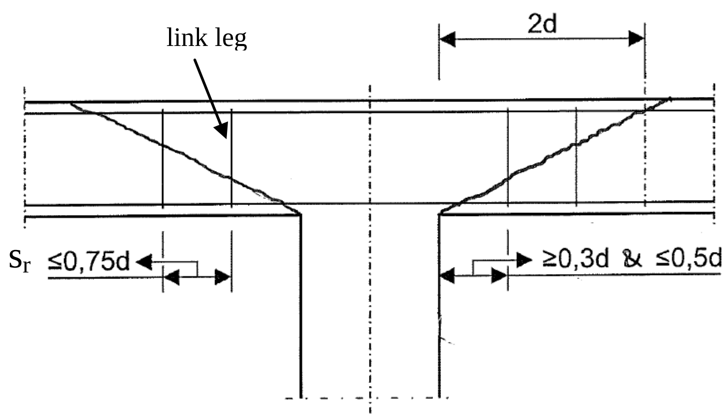


Fig. 14. 94: A punching shear crack extending to the basic punching perimeter and link legs to increase the punching shear resistance

When no shear reinforcement is provided, the punching shear resistance is:

$$V_{Rd,c} = v_{Rd,c}\, u_1\, d \qquad (14.102)$$

where $v_{Rd,c}$ follows from eq. (14.81) and u_1 from eq. (14.82). The question is how adding shear reinforcement contributes to the shear resistance. This is illustrated in Fig. 14. 95.

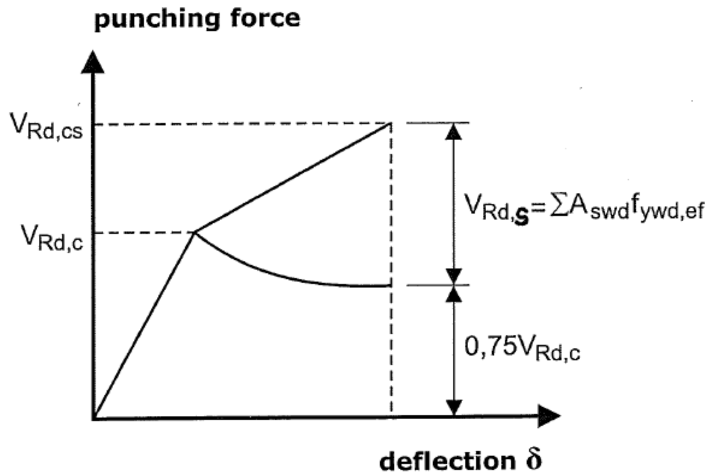


Fig. 14. 95: Schematic representation of the development of the contributions of the concrete and the shear reinforcement to the punching shear resistance

When the punching shear load is gradually increased, first the force $V_{Rd,c}$ is reached which is the resistance of the concrete. When no shear reinforcement is provided, punching shear failure now will occur. If shear reinforcement is present, the link legs are gradually stressed after cracking occurs and the load can be further increased. Experiments indicated that the contribution of the concrete does not drop to zero but stays at a level of about $0{,}75V_{Rd,c}$. This is also the result of the shear and longitudinal reinforcement action that reduces the width of the inclined cracks and, as a result, preserves the contribution of aggregate interlock over the crack faces.

The contribution of vertical punching shear reinforcement anchoring the cone to the surrounding slab is:

$$V_{Rd,s} = \Sigma A_{sw}\ f_{ywd,ef} \tag{14.103}$$

where ΣA_{sw} is the total cross-sectional area of the punching reinforcement that is crossed by the punching cone.

In case the shear reinforcement is inclined (angle α), the contribution is:

$$V_{Rd,s} = \Sigma A_{sw}\ f_{ywd,ef} \sin\alpha$$

The “effective” reinforcement design stress $f_{ywd,ef}$ is lower than the value f_{yd} = 435 N/mm^2 and is expressed as follows (EN 1992-1-1 cl. 6.4.5):

$$f_{ywd,ef} = 250 + 0{,}25d \leq f_{ywd} \tag{14.104}$$

where d is in mm.

The reasoning behind the steel design stress reduction is the importance of a proper functioning of the shear reinforcement in thin slabs. Above the columns these slabs are often heavily reinforced in two directions to resist the bending moments. This is top reinforcement. These slabs are often provided with a relatively high amount of shear reinforcement because of their small height. Underneath the longitudinal reinforcing bars it is not easy to have a good compaction of the concrete. This might cause poor bond of the links, which in turn reduces the stress developed in the links once cracking occurs. Moreover, reducing shear link steel stress results in crack width reduction, which in turn results in aggregate interlock, which makes that concrete still contributes to punching shear resistance.

The total punching shear resistance is (EN 1992-1-1 cl. 6.4.5 eq. (6.52)):

$$V_{\mathrm{Rd,cs}} = 0,75\,V_{\mathrm{Rd,c}} + V_{\mathrm{Rd,s}} \tag{14.105}$$

Equation (14.105) expressed as stresses results in (EN 1992-1-1 eq. (6.52)):

$$\frac{V_{\mathrm{Rd,cs}}}{u_1 d} = 0,75\frac{V_{\mathrm{Rd,c}}}{u_1 d} + \frac{V_{\mathrm{Rd,s}}}{u_1 d} \tag{14.106}$$

$$v_{\mathrm{Rd,cs}} = 0,75 v_{\mathrm{Rd,c}} + \frac{1,5\dfrac{d}{s_{\mathrm{r}}} A_{\mathrm{sw}}\, f_{\mathrm{ywd,ef}} \sin\alpha}{u_1 d} \tag{14.107}$$

where:

s_{r} is the distance in radial direction between individual perimeters of shear reinforcement;
A_{sw} is the total cross-sectional area of the shear reinforcement provided in each individual perimeter of shear reinforcement.

The following remarks must be made:
The previous equation has a factor 1,5 in the contribution of the shear reinforcement. This should theoretically be 2,0 (since the punching shear crack has an angle θ = 26,6° so the crack extends over 2,0d). The number of shear reinforcement perimeters, spaced at s_{r}, crossed by the crack is:

$$2,0\frac{d}{s_{\mathrm{r}}}$$

By also reducing this component of punching shear resistance by 25%, the NEN-EN 1992-1-1 expression is obtained. Experiments have shown that this approach gives good results. Shear links must be anchored at both sides of the punching shear cone. The smallest anchorage length of both is governing for the steel stress that can occur. Close to the top and bottom of the slab, the anchorgage length at one side of the crack is rather limited. This might result in a reduction of shear link effectiveness. The factor 0,75 takes this into account.

Note:
The amount of reinforcement A_{sw} has to be applied in each individual shear reinforcement perimeter; it is not the total amount of reinforcement that is crossed by a punching shear cone, since the latter is (including the 25% reduction discussed before):

$$0,75 \cdot 2,0\frac{d}{s_{\mathrm{r}}} A_{\mathrm{sw}}$$

The designer is, within certain limits (Fig. 14. 94), free to choose A_{sw} and s_{r}. For instance: a small number of shear reinforcement perimeters (large s_{r}), but each perimeter containing a large amount of reinforcement (large A_{sw}) or vice versa.

The amount of shear reinforcement that can be applied is restricted to a maximum. Two design rules must be applied.

Design rule 1

If the amount of shear reinforcement is too high the concrete just around the column might prematurely fail in compression before the stress in the shear reinforcement is fully developed (EN 1992-1-1 eq. (6.53)). This is similar to the concrete diagonal compression failure known from beams. From experiments it appears that the following holds (EN 1992-1-1 eq. (6.53)):

$$V_{\text{Rd,cs}} \leq V_{\text{Rd,max}} \tag{14.108}$$

where:

$$V_{\text{Rd,max}} = v_{\text{Rd,max}}\, u_0\, d \tag{14.109}$$

and:

$$v_{\text{Rd,max}} = 0,4\, \nu\, f_{\text{cd}}$$

where ν is a strength reduction parameter (EN 1992-1-1 eq. (6.6N)):

$$\nu = 0,6 \cdot \left(1 - \frac{f_{\text{ck}}}{250}\right)$$

u_0 = length of column perimeter (see Fig. 14. 80 for interior columns).

Design rule 2

The increase of punching shear resistance by applying shear reinforcement, is also limited by the following expression.

$$v_{\text{Rd,cs}} = 0,75 v_{\text{Rd,c}} + \frac{1,5 \dfrac{d}{s_{\text{r}}} A_{\text{sw}}\, f_{\text{ywd,ef}} \sin\alpha}{u_1 d} \leq k_{\text{max}} v_{\text{Rd,c}} \tag{14.110}$$

where according to the Dutch national annex, k_{max} = 1,6.

Often links (stirrups, double hairpins) are used as shear reinforcements which enclose the longitudinal reinforcement. Fig. 14. 96 presents the lay-out of such a type of reinforcement. The reinforcement is often labour-intensive to install. The shear reinforcement can be applied as two hairpins that have sufficient overlapping length at both their legs to transfer the force.

There are also other possibilities. A good solution is the application of studs. The studs are welded to a steel strip at one side and are provided with an enlarged stud end at the other side. If the surface area of the stud end is large enough and is close to and outside an intersection of the two-dimensional longitudinal bending reinforcement, yielding of the stud can be achieved. This is a very efficient way of reinforcing for shear. Applying this reinforcement is easier than providing links. Fig. 14. 97 shows an example.

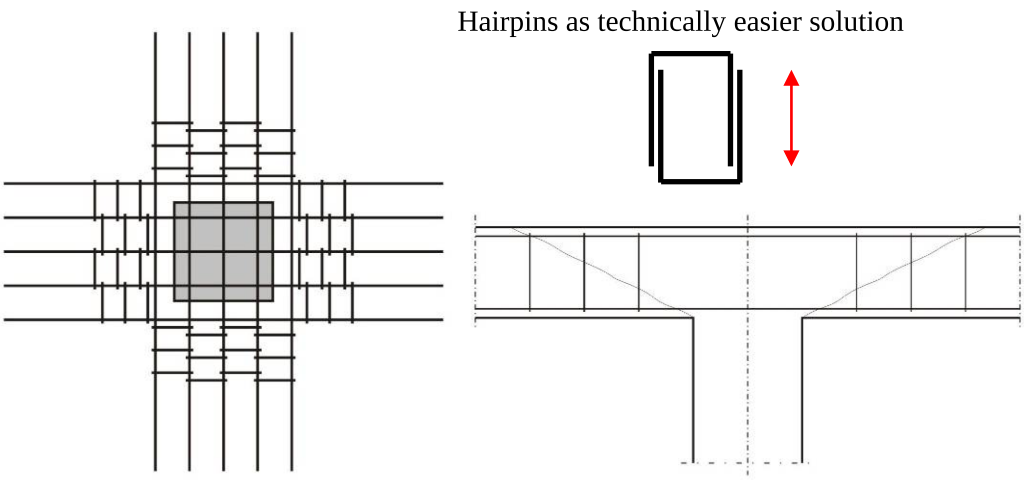


Fig. 14. 96: Application of links as shear reinforcement

Fig. 14. 97: Applying shear studs as punching shear reinforcement

As well as for bending and shear, also for punching shear a minimum amount of reinforcement is required, if $V_{Ed} > V_{Rdc}$. It is established from tests that a proper minimum cross-sectional area of an individual reinforcement link is (EN 1992-1-1 eq. (9.11)):

$$A_{sw,min} = \frac{0,08\ s_r\ s_t \sqrt{f_{ck}}}{1,5\ f_{yk}} \tag{14.111}$$

where s_r is the link spacing in radial direction (see Fig. 14. 93) and s_t is the link spacing in tangential direction (= the circumferential direction). The $A_{sw,min}$ makes that the designer does not use too small diameter bars. From the point of view of punching shear crack width control it is preferred to use closely spaced bars.

As stated before, the basic control perimeter u_1 is at a distance $2d$ from the loaded area. It should always be verified whether punching shear failure can also occur outside this area, see Fig. 14. 98. This figure shows other punching shear cones that might develop. If necessary, also these cones must be provided with shear reinforcement.

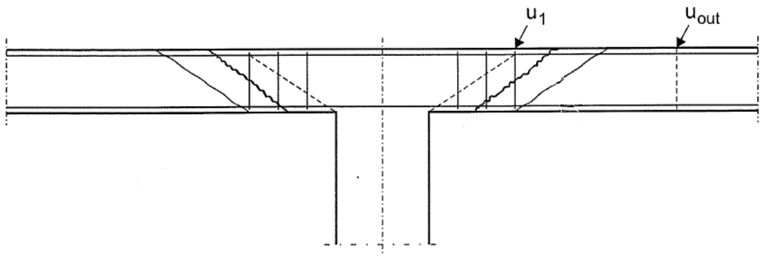


Fig. 14. 98: Formation of punching cones outside the basic control perimeter area

Outside the basic control perimeter checks should also be carried out. The most convenient way to do so is to calculate for which perimeter u_{out} no shear reinforcement is required. The length of this perimeter follows from the requirement that the concrete alone must be able to resist the full punching shear force (EN 1992-1-1 eq. (6.54)):

$$u_{out} = \frac{V_{Ed}}{v_{Rd,c}\, d} \tag{14.112}$$

If u_{out} is known, it is also known up to which position the shear reinforcement must continue since the perimeter u_{out} is constructed in the same way as u_1. The only difference is that u_1 is spaced $2d$ from the loaded area whereas this distance is unknown for u_{out} and follows from its length and shape.

A detailing rule states that outer shear reinforcement does not have to extend to u_{out}. It is allowed that the last perimeter of shear reinforcement is located maximum $1{,}5d$ from the outer control perimeter (EN 1992-1-1 cl. 6.4.5 (4)).

In radial direction the reinforcement spacing may not be larger than $s_r = 0{,}75d$. In circumferential direction (tangential direction) the maximum spacing is $s_t = 1{,}5d$, see Fig. 14. 93.

In theory all shear reinforcement perimeters require the same amount of reinforcement A_{sw}. In practice the tangential bar spacing s_t requirement (max. spacing is $1{,}5d$) makes that at increasing distance from a perimeter to the column, the number of links required is often defined by the requirement $s_t \leq 1{,}5d$ and not by A_{sw}. Reducing the link bar diameter is an option not to use too much link reinforcement, but the $A_{sw,min}$ requirement from eq. (14.111) makes that there is a limit to this. Therefore, the outer perimeters are often provided with (much) more punching shear reinforcement than follows from A_{sw}.

14.5 Special aspects

14.5.1 Effective span

In theory the span is the distance between centre lines of the supports. This is sometimes too conservative. The span to be used in the calculation depends on the type of bearings. The positions of the support reacting forces are governing. The distributions of these forces and, as a result, the positions of the resulting support reactions, influence the bending moment at mid-span. The effective span follows from the expression (EN 1992-1-1 cl. 5.3.2.2, see Fig. 14. 99):

$$l_{\text{eff}} = l_{\text{n}} + a_1 + a_2 \tag{14.113}$$

where l_{n} is the net distance between the faces of the supports and where a_1 and a_2 are at both ends 1 and 2 the distance between the face of the support and the resulting support reaction. The values a_1 and a_2 depend on the type of bearing and the dimensions of the structural components. They are, however, usually relatively small when compared with the total span. In the Eurocode, therefore, relatively simple practical values are given, see Fig. 14. 99 (EN 1992-1-1 fig. 5.4).

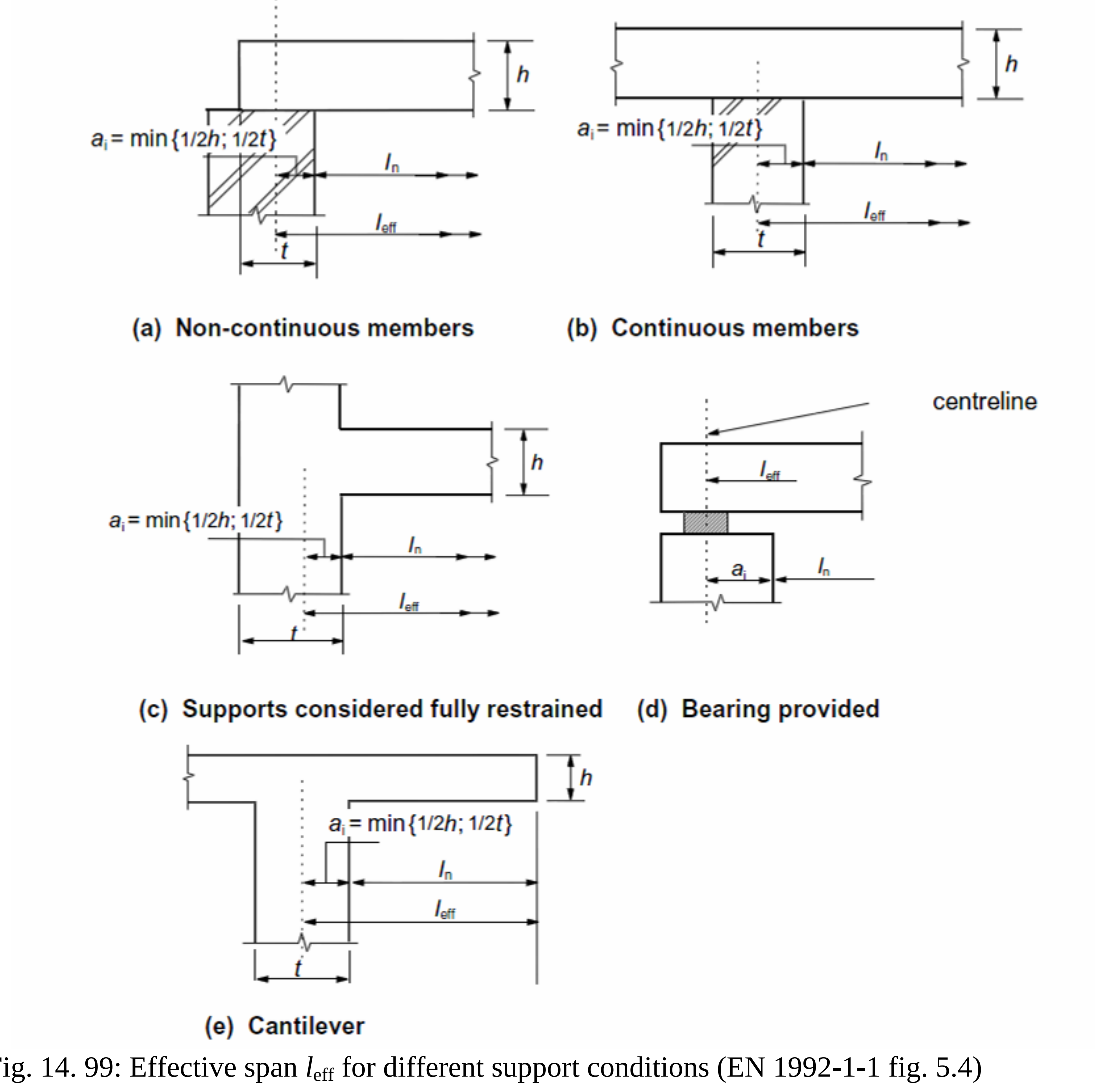


Fig. 14. 99: Effective span l_{eff} for different support conditions (EN 1992-1-1 fig. 5.4)

14.5.2 Transfer of concentrated (point and line) loads

Because of its stiffness in transverse direction, a one-way spanning monolithic slab will also transfer part of concentrated loads in this transverse direction. This implies that not only a directly loaded strip but also the slab area close to the concentrated load (with width b_{eff}) will be active in load transfer, see Fig. 14. 100.

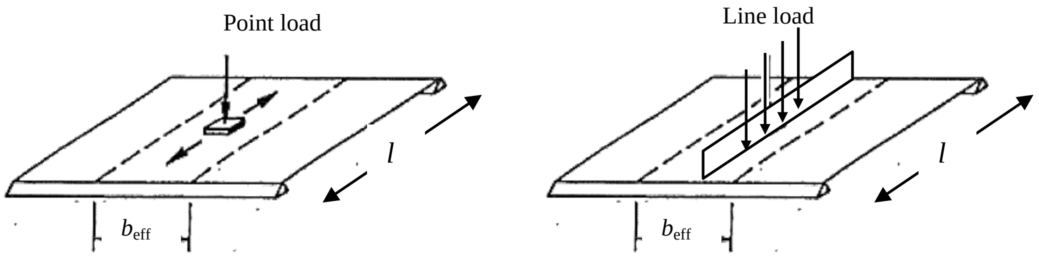


Fig. 14. 100: Effective width (b_{eff}) for a concentrated load on a one-way slab

Concentrated loads can be both line loads (e.g. masonry walls) or point loads (e.g. column load).

Line load
A line load can act over a certain length with different orientations. Assume that the slab is loaded over its full span length *l*, with the line load acting perpendicular to the supports (Fig. 14. 100 right). Grasser and Thielen [14.5] calculated the effective width for different support conditions. The width of the effective strip that resists the line load depends on slab span, the boundary conditions at both slab ends and the slab cross-section considered (bending moment at mid span (mv) or at a fixity (ms)), as given in Table 14. 12.

Point load
The maximum effective width of the slab area, b_{eff}, that may be taken into account when transferring a concentrated load also depends on the span, boundary conditions and distance of the load to the support.

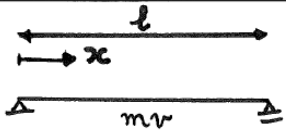
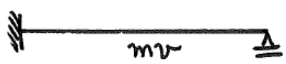
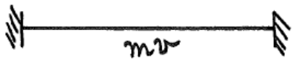
Based on calculations using the theory of elasticity, Grasser and Thielen [14.5] calculated the effective width for different support conditions. Their results are given in Table 14. 12.

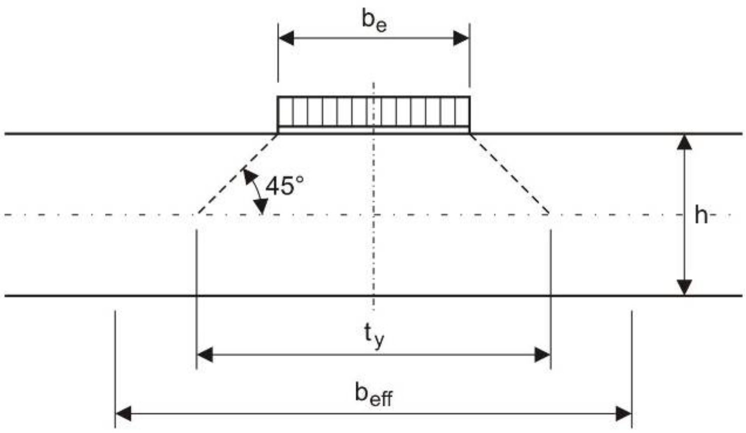
Point loads applied over a width b_e are first spread under 45º to the centre line of the plate (see the figure in Table 14. 12). The result is the basic strip width which is further increased depending on the distance from the concentrated load to the left support. A distinction is made between the effective width for a span moment ($M_{Ed,V}$), a support moment ($M_{Ed,S}$) and a shear force (V_{Ed}).

Obviously, the effective width may also be chosen smaller than the values presented in the table. For non-movable loads it is economic to choose a width of the strip that is as small as possible. In general, the reason to allow this is to enable more realistic stress distribution, which will be more in accordance to linear elastic analysis.

Of course the amount of transverse reinforcement should be adjusted to the required amount of principal reinforcement at the location of the concentrated load or line load.

Table 14. 12: Effective strip width (b_{eff}) for concentrated loads and evenly distributed (line) loads based on the theory of elasticity (mv = span moment; ms = support moment; vs = shear force; *x* = distance from left support to position of concentrated load) [14.5].

support condition	concentrated load	uniformly distributed load
	$b_{\text{eff}} = t_y + 2{,}5\,x\left(1-\frac{x}{l}\right)$	$b_{\text{eff}} = 1{,}35\,l$
	$b_{\text{eff}} = t_y + 0{,}5\,x$	
	$b_{\text{eff}} = t_y + 1{,}5\,x\left(1-\frac{x}{l}\right)$	$b_{\text{eff}} = 1{,}04\,l$
	$b_{\text{eff}} = t_y + 0{,}5\,x\left(2-\frac{x}{l}\right)$	$b_{\text{eff}} = 0{,}85\,l$
	$b_{\text{eff}} = t_y + 0{,}3\,x$	
	$b_{\text{eff}} = t_y + 0{,}4\,(l-x)$	
	$b_{\text{eff}} = t_y + x\left(1-\frac{x}{l}\right)$	$b_{\text{eff}} = 0{,}85\,l$
	$b_{\text{eff}} = t_y + 0{,}5\,x\left(2-\frac{x}{l}\right)$	$b_{\text{eff}} = 0{,}53\,l$
	$b_{\text{eff}} = t_y + 0{,}3\,x$	



14.6 References

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