15. Crack width control

15.1 Introduction

The principal idea of designing in reinforced concrete is to have the concrete resisting the compressive forces and the reinforcement the tensile forces.
Steel stresses actually start to develop after the concrete is cracked.
"Cracking" is therefore a phenomenon that is typical for the behaviour of concrete and is not a problem at all, unless crack widths are too large so that there is risk of corrosion of the reinforcing steel. The crack width that makes corrosion risky to occur, depends on the characteristics of the environment. The Eurocode contains requirements with respect to the crack width and concrete cover. The structure of course not only has to comply with the requirements of serviceability, but also with strength and stability during its design life (usually 50 years).
Crack width control is also important when particular requirements, for instance liquid tightness of the structure (EN 1992-3), should be respected (storage structures for aggressive liquids).
The allowable crack width depends on the environment a structure is exposed to. Exposure classes related to environmental conditions are given in table 4.1of EN 1992-1-1

The exposure classes are subdivided in 6 main classes:

- XO – no risk
- XC – carbonation
- XD – chlorides not originating from seawater
- XS – chloride from seawater
- XF – freeze-thaw
- XA – chemical attack

The exposure classes XF and XA concern the durability of the concrete itself. To prevent deterioration measures should be taken with regard to the mixture composition (minimum cement/binder content and maximum water/binder ratio) (EN 206-1 & NEN 8005). For the other exposure classes, except for XO of course, the requirements are related to preventing the reinforcing steel to corrode.

Class designation	Description of the environment	Informative examples where exposure classes may occur
1 No risk of corrosion or attack		
X0	For concrete without reinforcement or embedded metal: all exposures except where there is freeze/thaw, abrasion or chemical attack For concrete with reinforcement or embedded metal: very dry	Concrete inside buildings with very low air humidity
2 Corrosion induced by carbonation		
XC1	Dry or permanently wet	Concrete inside buildings with low air humidity Concrete permanently submerged in water
XC2	Wet, rarely dry	Concrete surfaces subject to long-term water contact Many foundations
XC3	Moderate humidity	Concrete inside buildings with moderate or high air humidity External concrete sheltered from rain
XC4	Cyclic wet and dry	Concrete surfaces subject to water contact, not within exposure class XC2
3 Corrosion induced by chlorides		
XD1	Moderate humidity	Concrete surfaces exposed to airborne chlorides
XD2	Wet, rarely dry	Swimming pools Concrete components exposed to industrial waters containing chlorides
XD3	Cyclic wet and dry	Parts of bridges exposed to spray containing chlorides Pavements Car park slabs
4 Corrosion induced by chlorides from sea water		
XS1	Exposed to airborne salt but not in direct contact with sea water	Structures near to or on the coast
XS2	Permanently submerged	Parts of marine structures
XS3	Tidal, splash and spray zones	Parts of marine structures
5. Freeze/Thaw Attack		
XF1	Moderate water saturation, without de-icing agent	Vertical concrete surfaces exposed to rain and freezing
XF2	Moderate water saturation, with de-icing agent	Vertical concrete surfaces of road structures exposed to freezing and airborne de-icing agents
XF3	High water saturation, without de-icing agents	Horizontal concrete surfaces exposed to rain and freezing
XF4	High water saturation with de-icing agents or sea water	Road and bridge decks exposed to de-icing agents Concrete surfaces exposed to direct spray containing de-icing agents and freezing Splash zone of marine structures exposed to freezing
6. Chemical attack		
XA1	Slightly aggressive chemical environment according to EN 206-1, Table 2	Natural soils and ground water
XA2	Moderately aggressive chemical environment according to EN 206-1, Table 2	Natural soils and ground water
XA3	Highly aggressive chemical environment according to EN 206-1, Table 2	Natural soils and ground water

(EN 1992-1-1 table 4.1: exposure classes related to environmental conditions)

Table 15-I gives an impression of the relative humidity of several environments.

Table 15-I Relative humidity of several environments

environment	relative humidity (RH)	explanation
‘very dry’ (X0)	Not accounted for in NEN 8005	very dry environment
‘dry’ (XC1)	RH < 60 %	dry environment
‘humid’ (XC3)	60 % ≤ RH < 85 %	outside unprotected
‘wet/dry’ (XC4)	85 % ≤ RH < 100 %	outside, cyclic wet/dry unprotected, tidal and spray zone
‘wet’ (XC2)	85 % ≤ RH < 100 %	outside, almost always wet, unprotected
‘permanently under water’ (XC1)	RH = 100 %	under lowest groundwater level

15.2 Crack width requirements

The requirements with regard to the allowable crack width w_{max} are based on the durability of the structure and esthetics. Requirements from EN 1992-1-1 table 7.1N are given in table 15-II.

Table 15-II Recommended values of w_{max} [mm] (EN 1992-1-1 table 7.1N)

<table>
<tr><th>Exposure Class</th><th>Reinforced members and prestressed members with unbonded tendons</th><th>Prestressed members with bonded tendons</th></tr>
<tr><td></td><td>Quasi-permanent load combination</td><td>Frequent load combination</td></tr>
<tr><td>X0, XC1</td><td>0,4[1]</td><td>0,2</td></tr>
<tr><td>XC2, XC3, XC4</td><td rowspan="2">0,3</td><td>0,2[2]</td></tr>
<tr><td>XD1, XD2, XS1, XS2, XS3</td><td>Decompression</td></tr>
<tr><td colspan="3">Note 1: For X0, XC1 exposure classes, crack width has no influence on durability and this limit is set to guarantee acceptable appearance. In the absence of appearance conditions this limit may be relaxed.
Note 2: For these exposure classes, in addition, decompression should be checked under the quasi-permanent combination of loads.</td></tr>
</table>

According to the Dutch National Annex to EN 1992-1-1 the values from table 15-II can be multiplied by a factor k_x:

$$k_x = \frac{c_{applied}}{c_{nom}} \leq 2,0$$

if the applied concrete cover on the reinforcement is larger than the nominal concrete cover required from durability requirements.

15.3 Crack formation

Crack formation has been studied since decades. Test series have been carried out on axially loaded reinforced concrete tensile bars, see fig. 15.1. By varying the concrete strength class, the reinforcement ratio, the bar diameter and/or the number of bars, a good impression was obtained of the basics of the cracking mechanisms.

In recent crack width models the behaviour of an actual structure is mostly modelled by defining an axially loaded, centrically reinforced concrete tensile bar as shown in fig. 15.1. Therefore this basic case will be dealt with in detail first.

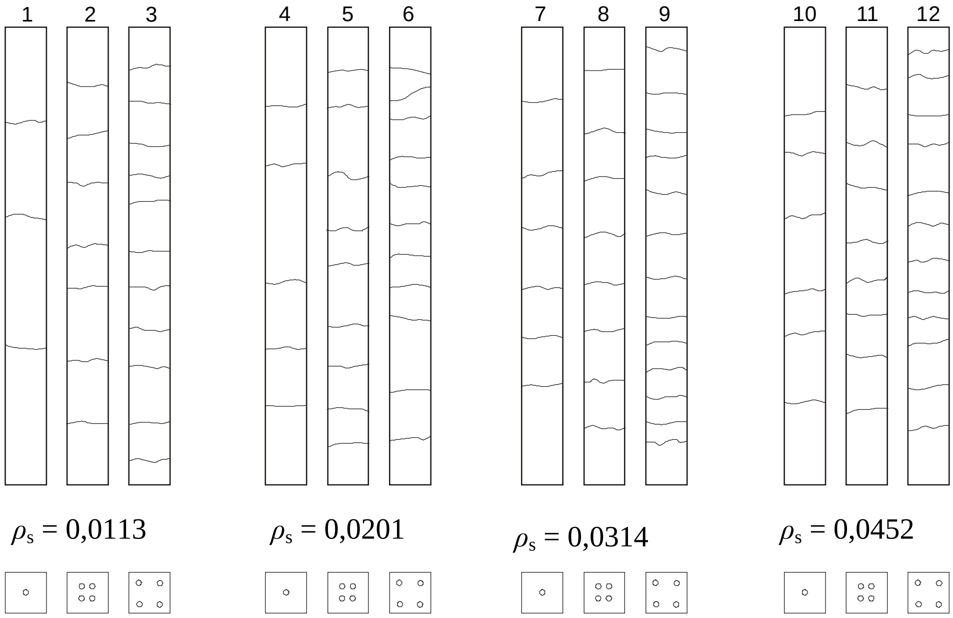

Fig. 15.1 Reinforced concrete tensile members subjected to axial tension (Stevin laboratory, TU Delft, 1976)

15.4 Bond between steel and concrete

When concrete cracks the reinforcement has to carry the tensile force. The reinforcing bars are stressed. As a result the reinforcing bars activate bond stresses to transfer force to the surrounding concrete. The bond stresses are caused by a slip of the bars relative to the concrete. By means of the bond between concrete and steel the steel force is gradually transferred to the concrete. For this load transfer mechanism the ribs on the bars are of importance.
On the basis of the analysis of numerous test results it turned out that crack widths can be calculated when assuming a constant bond stress between concrete and steel. This (mean) bond stress, which is independent of the magnitude of the slip between steel and concrete, is about two times the mean tensile strength f_{ctm} of the concrete: $\tau_{bm} = 2\ f_{ctm}$ (fig. 15.2).

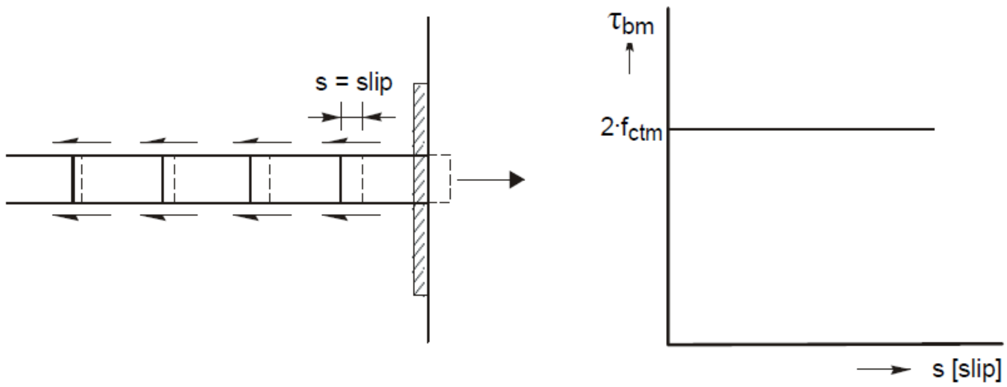

Fig. 15.2 Simplified bond stress distribution for short term static loading

15.5 Crack formation in a reinforced tensile bar (short-term loading)

As a basic tool for the calculation of crack width in a concrete structure, the concrete tensile bar subjected to axial tension is used, fig. 15.1. The behaviour of the bar will be studied in detail, assuming that the imposed strain gradually increases.
In the stage before the occurrence of the first crack, the strains of the steel and the concrete are equal. The contributions of the steel and the concrete to carrying the external force *N* are:

$$N_s = E_s A_s \, \varepsilon$$
$$N_c = E_c A_c \, \varepsilon$$

The total force is:

$$N_{tot} = N_s + N_c = E_s A_s \, \varepsilon + E_c A_c \, \varepsilon = E_c A_c \left(1 + \alpha_e \rho\right) \varepsilon$$

For $\varepsilon = \varepsilon_s = \varepsilon_c$:

$$N_{tot} = E_c A_c \left(1 + \alpha_e \rho\right) \varepsilon_c \qquad (15.1)$$

where:

$\alpha_e = \dfrac{E_s}{E_{cm}}$ is the ratio between the Young's moduli of steel and concrete;

$\rho = \dfrac{A_s}{A_c}$ is the reinforcement ratio.

When the strain (ε_c) increases at a certain moment the tensile stress in the concrete will reach the tensile strength of the concrete. Since the tensile strength of the concrete is subjected to scatter along the length of the specimen, the first crack will appear at a location where the tensile strength is the lowest, see figure 15.3a.

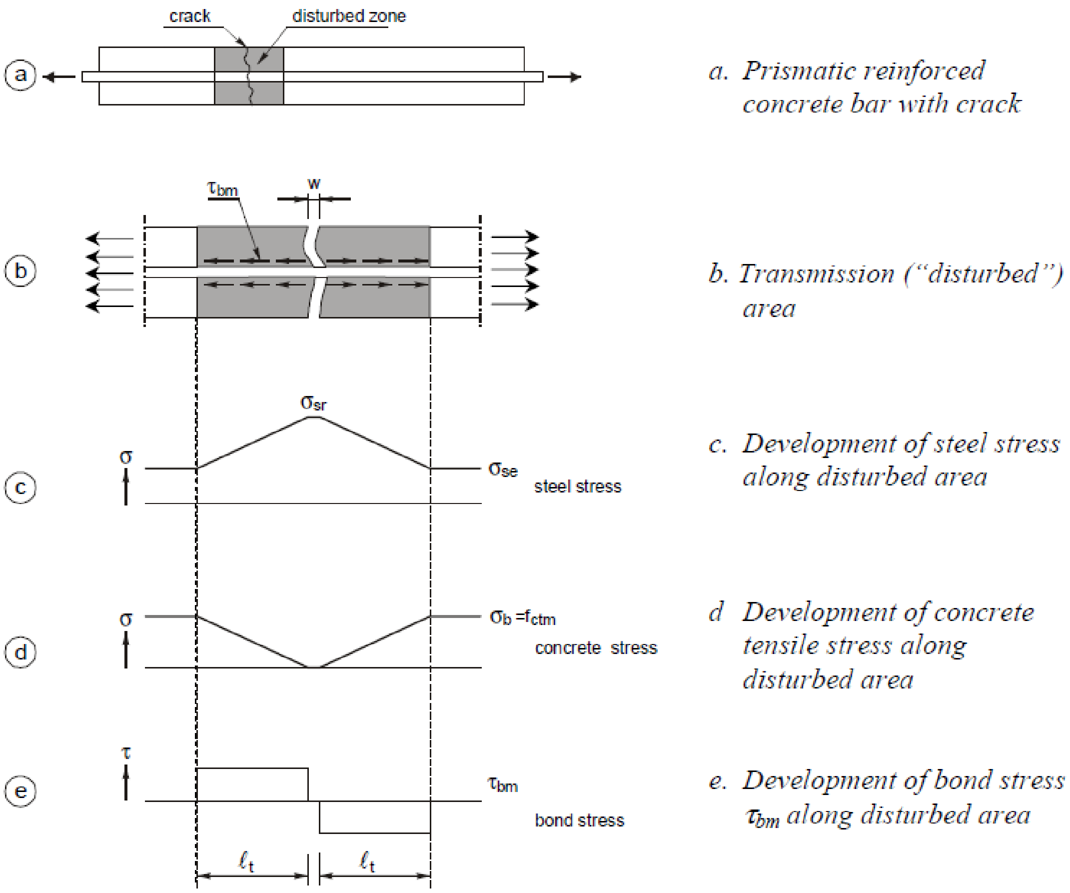

Fig. 15.3 A crack in a reinforced concrete tensile member and the transfer of forces from steel to concrete over a disturbed area (transfer length = l_t)

At the location where the concrete is cracked the concrete tensile stress $\sigma_{ct} = 0$. Here only the steel carries the tensile force. As a result of the bond stresses between the steel and the concrete, acting at both sides of the crack, the concrete is activated again in carrying the tensile force. At a certain distance l_t from the crack (the transfer length), the concrete carries the original amount of tensile force N. Outside the transfer lengths, the strains of concrete and steel are again equal, so that the undisturbed situation (as before cracking) is present.
The distance required to re-introduce part of the cracking force into the concrete depends on, among others, the bond strength τ_{bm}. For the calculation of this distance, the basics as shown in fig. 15.3 are used. Following the assumption of a constant bond stress, the course of the steel stress and the concrete stress along the transfer length l_t is linear, see figs. 15.3c and d.

When the bar is subjected to a strain (displacement-controlled test), the force N decreases as soon as a crack is formed. This behaviour can be explained from the formation of a crack, which causes a reduction of the stiffness of the tensile member. Since the total strain imposed on the bar is the same before and after cracking, the force drops from $N_{cr,1}$ to N_0 , see fig. 15.4.

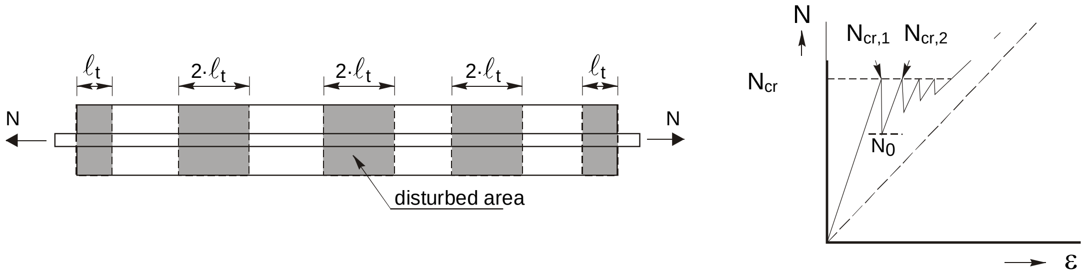

Fig. 15.4 Crack formation stage

The stage in which gradually new cracks are formed is denoted as the “crack formation stage”. In the crack formation stage the stress in the reinforcing steel σ_s in a crack reaches a maximum just before, at another location, a new crack arises. At that very moment the stress is $\sigma_s = \sigma_{sr}$, see fig. 15.3c.
If, as a simplification, the calculations are based on the mean concrete tensile strength f_{ctm}, the stress in the steel at the onset of a new crack is:

$$\sigma_s = \sigma_{sr} = \frac{N_{cr}}{A_s} = \frac{f_{ctm}}{\rho}\left(1+\alpha_e\rho\right) \qquad (15.2)$$

In the undisturbed areas, see fig. 15.4, the steel stress is directly proportional to the concrete stress:

$$\sigma_{se} = \alpha_e \; f_{ctm} \qquad (15.3)$$

At a further increase of the imposed strain the force increases again. However, the force cannot exceed $N_{cr,2}$, since then a new crack appears.
The concrete tensile stress σ_{ct} in a crack is zero, whereas at the end of the transfer length the concrete stress is f_{ctm} (fig. 15.3d). This implies that the force transmitted by bond over that length is:

$$N = A_c \; f_{ctm} \qquad (15.4)$$

The force transmitted by bond over l_t is:

$$N = \tau_{bm} \; l_t \; m \, \pi \, Ø \qquad (15.5)$$

Where m is the number of reinforcing bars and Ø is their diameter. Combining eqs. (15.4) and (15.5) gives an expression for the transfer length l_t:

$$l_t = \frac{1}{4}\frac{f_{ctm}}{\tau_{bm}}\frac{Ø}{\rho} \qquad (15.6)$$

The maximum crack width is equal to the difference between the elongation of the steel and the elongation of the concrete over the length $2l_t$, so:

$$w_{max} = 2\, l_t \left(\varepsilon_{sm} - \varepsilon_{cm}\right) \qquad (15.7)$$

where ε_{sm} and ε_{cm} are the mean steel strain and concrete strain, respectively, along the transfer length l_t. The course of the stresses at both sides of a crack are shown in fig. 15.5. The strains can be calculated from the stresses.

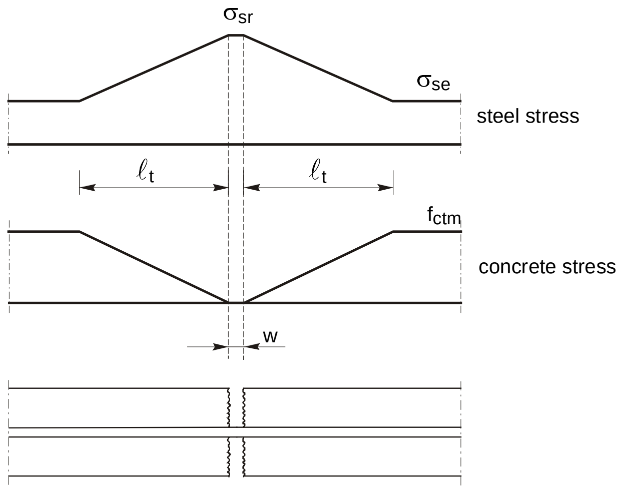

Fig. 15.5 Course of steel and concrete stresses at both sides of a crack

The mean steel strain is:

$$\varepsilon_{sm} = \frac{1}{2\,E_s}\left(\sigma_{sr} + \sigma_{se}\right) \tag{15.8}$$

Substitution of σ_{se} from eq. (15.3) into this equation results in:

$$\varepsilon_{sm} = \frac{1}{2\,E_s}\left(\sigma_{sr} + \alpha_e f_{ctm}\right) \tag{15.9}$$

The mean concrete strain over l_t is:

$$\varepsilon_{cm} = \frac{f_{ctm}/2}{E_c} = \frac{1}{2\,E_s}\,\alpha_e f_{ctm} \tag{15.10}$$

Substitution of ε_{sm} and ε_{cm} from eqs. (15.9) and (15.10) in eq. (15.7) yields:

$$w_{max} = \frac{1}{4}\frac{f_{ctm}}{\tau_{bm}}\frac{Ø}{\rho}\frac{1}{E_s}\sigma_{sr} \tag{15.11}$$

where σ_{sr} follows from eq. (15.2).

When the strain is further increased, more and more cracks will occur. The cracking process will continue until the tensile bar consists of “disturbed regions” only. When a certain number of cracks are formed, the disturbed regions start to overlap each other. The smallest spacing between two cracks is found, just at the end of a disturbed region (so at a distance l_t from an already existing crack) where a new crack has occurred. The largest spacing between two cracks is found, where a new crack has occurred at a distance just smaller than $2l_t$ from an already existing crack. The length in between the two cracks is then just too short for the concrete to reach its tensile strength again. The final crack spacing varies therefore between l_t and $2l_t$. When, finally, the reinforced member consists of disturbed regions only, the crack formation stage is finished. Although during further increase of the strain the external force increases, no new cracks are formed. The stage after the crack formation stage, in which no new cracks occur, but the existing cracks widen, is denoted as the “stabilised cracking stage”.

At a further increase of the strain, and as a result also an increase of the force N (fig. 15.4) the steel stress in the crack σ_s exceeds σ_{sr} (eq. 15.2). Because the force, transmitted from steel to the concrete does not increase (the bond stress is constant), the concrete strain between the cracks also does not increase. As a result, the increase of the crack width follows from the elongation of the steel only.

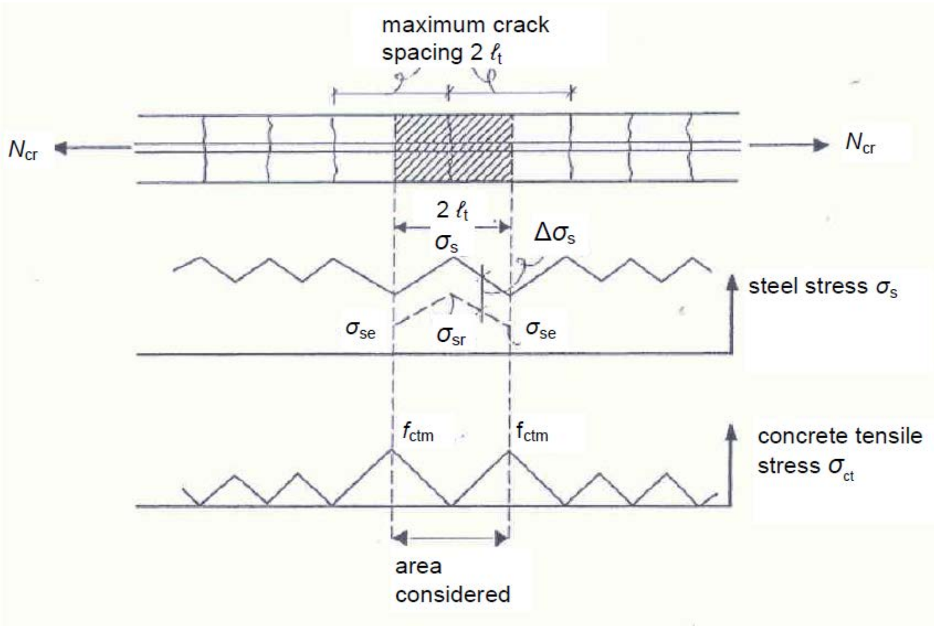

Fig. 15.6 Determination of the maximum crack width in the stabilized cracking stage

Figure 15.6 shows a crack where at both sides the maximum crack spacing is 2 ℓ_t. In the crack formation stage the maximum stress in the steel was σ_{sr} (eq. (15.2)). After the finalization of the crack formation stage the steel stress further increases, because of the increasing external tensile force N. The increase of the steel stress is $\Delta\sigma_s = \sigma_s - \sigma_{sr}$ The corresponding elongation of the steel over the distance 2 ℓ_t is totally converted to an increase of the crack width:

$$\Delta w = \frac{(\sigma_s - \sigma_{sr}) \cdot 2\ell_t}{E_s} \tag{15.12}$$

The total crack width in the stabilised cracking stage is obtained by adding Δw from eq. 15.12 to w_{max} from eq. (15.11). In combination with ℓ_t from eq. (15.6) the following expression is obtained:

$$w_{max} = \frac{1}{2}\frac{f_{ctm}}{\tau_{bm}}\frac{Ø}{\rho}\frac{1}{E_s}\left(\sigma_s - 0,5\,\sigma_{sr}\right) \tag{15.13}$$

The eqs. (15.11) and (15.13) are continuous since if in eq. (15.13) σ_s is replaced by σ_{sr} this expression reduces to eq. (15.11) (crack formation stage). Equation (15.13) is the general expression for the calculation of the maximum crack width in both the crack formation and the stabilised cracking stage. To calculate the crack width, it is only necessary to determine the transition point (strain) between the crack formation stage and the stabilised cracking stage. This is discussed in section 15.7.

Summarizing:

For the calculation of the crack width, two stages can be distinguished:

- *the crack formation stage where the crack width is maximum just before the moment a new crack occurs. In this stage, at increasing strain, the tensile force N does not exceed the cracking force* N_{cr} *. The maximum crack width follows from eq. (15.11);*
- *the stage where the crack pattern is fully developed. At increasing strain the force N increases. New cracks will not develop; the width of existing cracks increases. The maximum crack width depends on the steel stress in the crack and follows from eq. (15.13).*

15.6 Long term effects

During the service life of a concrete structure shrinkage of the concrete occurs. Furthermore, structures or parts of structures, can be subjected to long term constant loads, or dynamic loads. The influence of those effects is dealt with in the following.

The effect of shrinkage in the crack formation stage differs from that in the stabilised cracking stage. If in the crack formation stage shrinkage occurs while simultaneously the external imposed strain remains constant, the external force tends to increase. Since in the crack formation stage the external force cannot exceed the cracking load $N_{cr.}$ this implies that the existing crack widths will not increase. The result is that additional cracks will develop.
In the stabilised cracking stage the shrinkage does have an influence on the crack width. In this stage no new cracks are formed. The shortening of the concrete then can only result in widening of the already existing cracks. The influence of the shrinkage on the crack width is explained on the basis of the behaviour of a reinforced concrete element having a length 2 ℓ_t and restrained at both ends, see fig. 15.7(a). As a result of previous loading the element exhibits one crack in the centre and it is assumed that the stabilized cracking stage is reached. On behalf of symmetry only one half of the element is considered, see fig. 15.7(b)

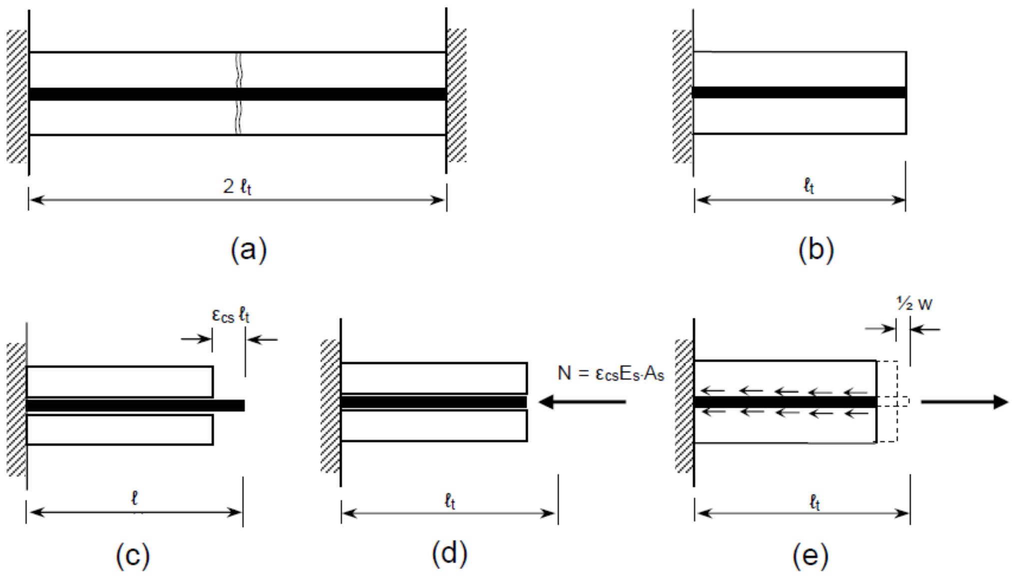

Fig. 15.7 Influence of shrinkage on the crack width (stabilised cracking stage)

The concrete tends to shrink, but this is counteracted by the steel. If the concrete would be able to shrink freely (assuming that there is no bond between the steel and concrete), the concrete strain would be equal to ε_{cs}, see fig. 15.7(c).
Note: Shrinkage implies a shortening of the concrete, so $\varepsilon_{cs} < 0$. In the following expressions it will be assumed that the absolute value of the shrinkage is used, so $\varepsilon_{cs} > 0$.

In order to restore the compatibility between steel and concrete, in a first step, a compressive force N is applied to the steel. The length of the steel is made equal to the length of the concrete, see fig. 15.7(d). The steel stress increases with $\Delta\sigma_s = \varepsilon_{cs}E_s$ whereas the stress in the concrete σ_c remains constant. This situation is regarded as the initial situation. In the next step the steel and the concrete are bonded and the same force N is applied to the steel, but now in the opposite direction. Now concrete and steel act together to carry the tensile force N. Between steel and concrete slip will occur. Because, with regard to the initial situation, there is an increase of the steel stress $\Delta\sigma_s = \varepsilon_{cs} E_s$, the slip between steel and concrete and the corresponding increase of the crack width follow directly from eq. (15.13).

The total crack width (load + shrinkage) in the stabilised cracking stage follows from:

$$w_{max} = \frac{1}{2}\frac{f_{ctm}}{\tau_{bm}}\frac{Ø}{\rho}\frac{1}{E_s}\left(\sigma_s - 0{,}5\,\sigma_{sr} + \varepsilon_{cs}E_s\right) \qquad (15.14)$$

Now the effect of a long term constant load and/or a varying load will be discussed.

It was stated before that for the bond stress between steel and concrete a value $\tau_{bm} = 2{,}0\, f_{ctm}$ gives good results for ribbed bars. Under a long term or dynamic load, the bond stress decreases. Tests have shown that a value $\tau_{bm,\infty} = 1{,}6\, f_{ctm}$ gives good results. In the crack formation stage the reduced bond strength results in an increase of the transfer length of 25%, and, as a result, a similar increase of the crack width. This follows directly from eq. (15.11), when instead of $\tau_{bm} = 2{,}0\, f_{ctm}$ the value $\tau_{bm} = 1{,}6\, f_{ctm}$ is used.

For the stabilised crack stage the situation is different. In most cases the load has been applied over a short period of time, so that the value $\tau_{bm} = 2{,}0\, f_{ctm}$ is realistic for the transfer length, and, as a result, also for the crack spacing. The influence of the concrete in between the cracks depends on the bond stress and, as a result, this influence decreases if the bond stress decreases. It can be assumed that this reduction is about 40%. This can be taken into account by replacing in eq. (15.14) the coefficient 0,5 by 0,3.
When taking these effects into account the following more general expression for the crack width is obtained:

$$w_{max} = \frac{1}{2}\frac{f_{ctm}}{\tau_{bm}}\frac{Ø}{\rho}\frac{1}{E_s}\left(\sigma_s - \alpha\,\sigma_{sr} + \beta\,\varepsilon_{cs} E_s\right) \tag{15.15}$$

where

σ_s is the steel stress in a crack under external tensile load;
σ_{sr} is the maximum steel stress in a crack in the crack formation stage $= \frac{f_{ctm}}{\rho}(1+\alpha_e \rho)$;
ε_{cs} is the shrinkage of the concrete;
ρ is the reinforcement ratio A_s / A_c;
f_{ctm} is the mean tensile strength of the concrete.

The values for τ_{bm}, α and β are given in Table 15.III.
Through recalculations these values for α (namely 0,5 and 0,3) have been slightly modified and have become 0,6 and 0,4 respectively in the Eurocode. In table15.III these values are enclosed by brackets.

Table 15,III Values for τ_{bm}, α and β from eq. (15.15) for various conditions. The values for α between brackets are the recalibrated values as applied in the Eurocode by means of the coefficient k_t (EN 1992-1-1 eq. (7.9))

	crack formation stage	stabilized cracking stage
Short term loading	α = 0,5 (0,6) β = 0 $\tau_{bm} = 2{,}0\, f_{ctm}$	α = 0,5(0,6) β = 0 $\tau_{bm} = 2{,}0\, f_{ctm}$
long term or dynamic loading	α = 0,5 (0,6) β = 0 $\tau_{bm} = 1{,}6\, f_{ctm}$	α = 0,3 (0,4) β = 1 $\tau_{bm} = 2{,}0\, f_{ctm}$

An expression for the maximum crack width w_{max} which is in agreement with the Eurocode expression can be obtained by substituting ℓ_t (eq. (15.6)) and σ_{sr} (eq. (15.2)) in eq. (15.13).

The result is:

$$w_{max} = 2\, l_t \frac{\sigma_s - 0{,}5 \dfrac{f_{ctm}}{\rho}(1+\alpha_e \rho)}{E_s}$$

The maximum crack spacing:

$$s_{r,max} = 2\, l_t$$

Influence of load duration:

k_t = 0,6 (short-term loading) or 0,4 (long-term loading)

The expression for w_{max} is (EN 1992-1-1 eq. (7.9)):

$$w_{max} = s_{r,max} \frac{\sigma_s - k_t \dfrac{f_{ct,eff}}{\rho_{p,eff}}(1+\alpha_e \rho_{p.eff})}{E_s}$$

where:

$f_{ct,eff}$ is the mean value of the concrete tensile strength of the concrete at the time when the crack may first be expected to occur.

$f_{ct,eff}$ = f_{ctm} or lower ($f_{ctm}(t)$ if cracking is expected earlier than at a concrete age of 28 days)

Taking into account the influence of shrinkage the equation becomes:

$$w_{max} = s_{r,max} \frac{\sigma_s - k_t \dfrac{f_{ct,eff}}{\rho_{p,eff}}(1+\alpha_e \rho_{p.eff}) + \varepsilon_{cs} E_s}{E_s}$$

The Eurocode uses the following expression for the maximum crack spacing (EN 1992-1-1 eq. (7.11)):

$$s_{r,max} = k_3\, c + k_1 k_2 k_4 \frac{Ø}{\rho_{p,eff}}$$

Since for k_3 and k_4 usually the recommended values 3,4 and 0,425 are used, $s_{r,max}$ reads:

$$s_{r,max} = 3{,}4\, c + 0{,}425\, k_1 k_2 \frac{Ø}{\rho_{p,eff}}$$

where:

c is the concrete cover to the longitudinal reinforcement;

k_1 is a coefficient which takes account of bond stress of the reinforcement:
= 0,8 for high bond (ribbed) bars;
= 1,6 for smooth bars;

k_2 is a coefficient which takes account of the distribution of the strain over the height of the concrete area considered (EN 1992-1-1 eq. (7.13)):
= 0,5 for bending;
= 1,0 for pure tension.

$\rho_{p,eff}$ is the reinforcement ratio of the tensile member.

15.7 The transition point between the crack formation stage and the stabilised cracking stage

Figure 15.8 shows the schematised behaviour of the tensile member. In the first linear branch (1) the concrete is uncracked. When the cracking load N_{cr} is reached, crack formation starts (2). At increasing deformation the load N can not exceed N_{cr} (the concrete strength variation from fig. 15.4 is not shown here). After the completion of crack formation the force N increases. The dotted line shows the N - $\Delta\ell / \ell$ relation of the steel reinforcement only. The line representing the behaviour of the reinforcement with the surrounding, cracked, concrete is assumed to be parallel to the line of the steel only (3). To calculate the position of this line it is assumed that the mean crack spacing is 1,5 ℓ_t. The resulting representative zone in between two cracks is shown in fig. 15.9.

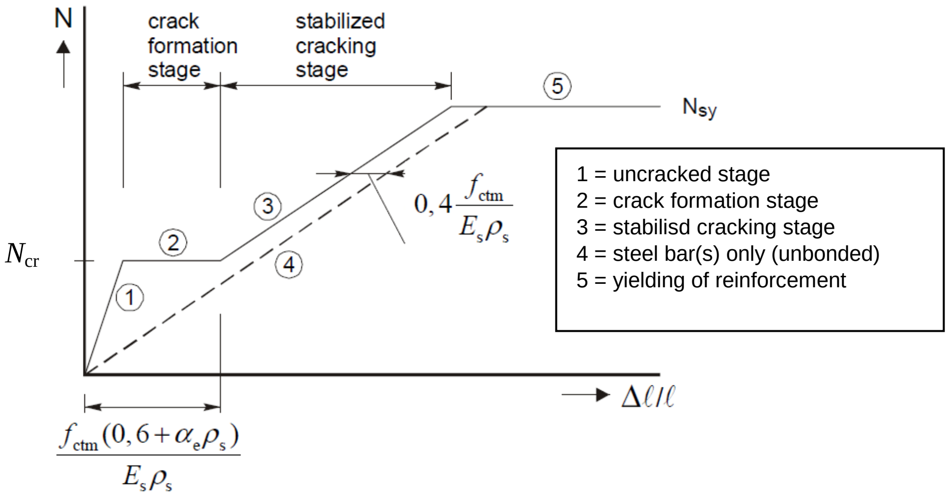

Fig. 15.8 Deformation of a reinforced concrete tensile member

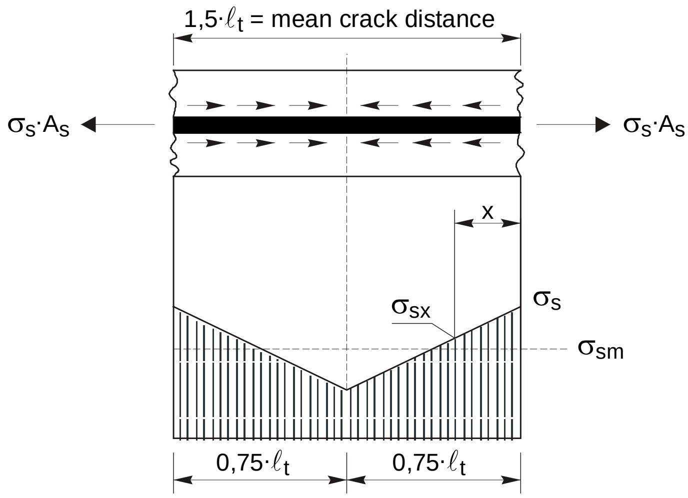

Fig. 15.9 Variation of stresses along a representative part of the member having a length equal to the mean crack spacing and located in between two cracks

The stress in the steel at a distance x = 0,75 ℓ_t is:

$$\sigma_{sx} = \sigma_s - \frac{0,75\, l_t\ \tau_{bm}\ \pi\, Ø}{\frac{1}{4}\pi Ø^2} \tag{15.16}$$

where: $\tau_{bm} = 2,0\ f_{ctm}$ and $l_t = \frac{1}{4} \cdot \frac{f_{ctm}\ Ø}{\tau_{bm}\ \rho}$ (eq. (15.6)).

Eq. (15.16) becomes:

$$\sigma_{sx} = \sigma_s - \frac{0,75\ f_{ctm}}{\rho} \tag{15.17}$$

The mean steel stress is:

$$\sigma_{sm} = \sigma_s - \frac{0,375\ f_{ctm}}{\rho} \tag{15.18}$$

and the mean steel strain is:

$$\varepsilon_{sm} = \frac{\sigma_s}{E_s} - \frac{0,375\, f_{ctm}}{E_s\ \rho} \approx \varepsilon_s - 0,4\frac{f_{ctm}}{E_s\ \rho} \tag{15.19}$$

With the aid of fig. 15.8 it is now possible to determine the strain for which the cracking pattern can be regarded to be complete (end of crack formation stage).
The horizontal branch (2) is defined by the cracking force:

$$N_{cr} = A_c\, f_{ctm}\, \left(1+\alpha_e \rho\right) \tag{15.20}$$

whereas the following branch (3) is described by:

$$N = E_s A_s \left(\varepsilon_s + 0,4\frac{f_{ctm}}{E_s\ \rho}\right) \tag{15.21}$$

By substituting $N = N_{cr}$ in eq. (15.21) and by using eq. (15.20) the intersection point of branches (2) and (3) is found:

$$\varepsilon_s = \frac{f_{ctm}\left(0,6+\alpha_e \rho\right)}{E_s\ \rho} \approx \frac{0,6\, f_{ctm}}{E_s\ \rho} \tag{15.22}$$

If the imposed strain of the reinforced concrete tensile member is smaller than the value resulting from eq. (15.22) the member is in the crack formation stage. For a higher value of the strain the stabilised cracking stage is reached. In most practical applications where imposed deformations apply, for instance by a temperature drop or concrete shrinkage at fixed boundary conditions, the imposed strain is mostly smaller that the value given by eq. (15.22). The crack pattern then is not completed and the structure is in the crack formation stage (2).
If, on the contrary, the member is subjected to a tensile load $N > N_{cr}$, fig. 15.8 shows that the member is in the stabilised cracking stage (3).

Note:
In order to simplify the calculations, fig. 15.8 is a restyled representation of the actual behaviour. The horizontal plateau (2) will in reality not occur since the cracking force

will gradually increase: the first crack will be formed at the weakest spot and each following crack will occur at a location where the tensile strength of the concrete is slightly higher. The most realistic description might therefore be to use the *lower bound 5% characteristic* concrete tensile strength for the first crack and to end with the *mean* tensile strength for the last crack. However, in the crack formation phase the maximum crack width is found at the highest cracking force. This implies that the designer should focus on the maximum cracking force. By using a constant cracking force based on the mean concrete tensile strength f_{ctm} this has been incorporated in the model.

15.8 The effective tensile area around the reinforcement

In the derivations it was, up to now, assumed that the reinforcing steel was uniformly distributed over the concrete cross-section. As a result, the forces, from the reinforcing steel transferred to the concrete by bond, do not have to spread over a larger concrete area. If this is the case, not only the bond properties play a role, but also the geometry of the element.

When the expression for the crack width was derived it was implicitly assumed that the transfer length l_t is small compared with the width of the cross-section. At the location where the tensile strength of the concrete is reached, the tensile stresses are then approximately uniformly distributed over the concrete cross-section. As a result the cross-section will fully crack through once the concrete tensile strength is reached.

If one dimension of the cross section is much larger than the other, the behaviour is different. At the position where the tensile strength of the concrete is reached cracking starts. The distribution of the tensile stresses that spread into the concrete is not uniform, see fig. 15.10a. The crack now does not proceed over the full width of the tensile member. Only if the force, introduced by the reinforcement, is more or less uniformly spread over the width of the element, the cross-section can completely crack (fig. 15.10b). Just around the reinforcing steel the behaviour is identical to that of the reinforced tensile member discussed in the previous sections. In the wide member only a few cracks reach the outer surface of the concrete, fig. 15.10b.
Because several internal cracks join, continuous cracks are formed. Such a crack has most often a disproportionally large width since at the outside of the concrete the deformation is concentrated in this small number of cracks. If the reinforcement would be concentrated at the outside of the cross-section, fig. 15.10c, the outer surface would demonstrate many cracks having small widths, whereas internally wide cracks occur. These findings demonstrate that there is a so-called "effective concrete area" around the reinforcement. The width of cracks that occur in this area is controlled by the reinforcement, whereas the crack width outside this area is uncontrolled. The relations derived in the previous sections apply to the effective concrete area only. An additional variable is used to distinguish between the standard reinforcement ratios ($\rho = A_s/A_c$ or $\rho_l = A_s/(bd)$) and the reinforcement ratio of the effective concrete area ($\rho_{eff} = A_s/A_{c,eff}$).

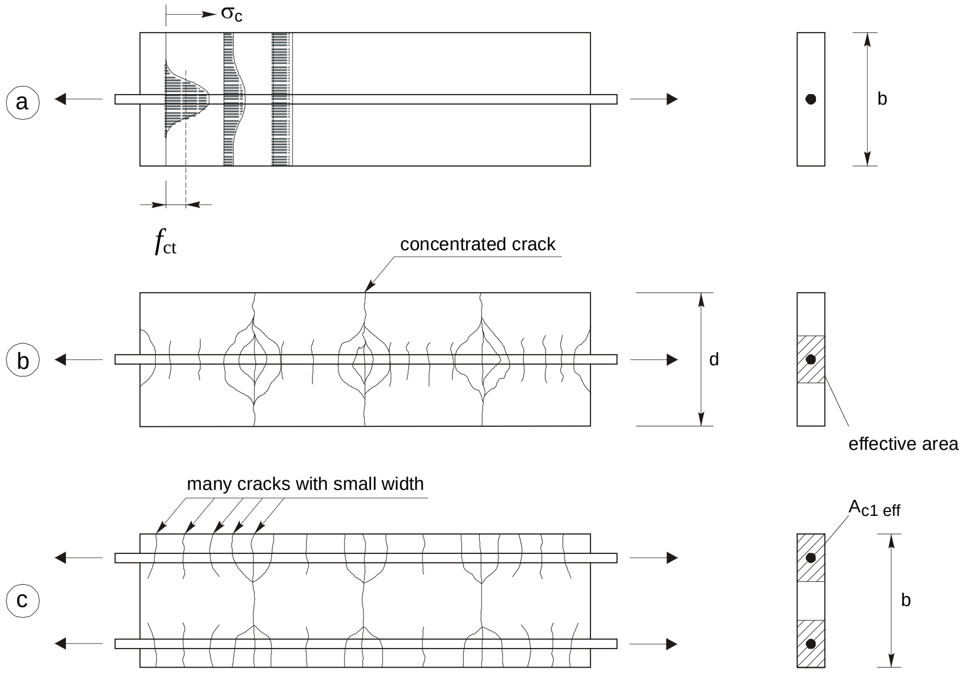

Fig. 15.10 Cracking behaviour of elements with concentrated reinforcement and a high ratio of element width to transfer length (b / l_t)

In elements loaded in bending similar phenomena occur. In the case of deep beams the main tension reinforcement limits the crack widths over an area closer to the reinforcement. If no substantial additional reinforcement over the height of the cross-section is provided (e.g. web reinforcement), the relatively small cracks at the bottom of the beam join and develop into wide cracks (fig. 15.11).

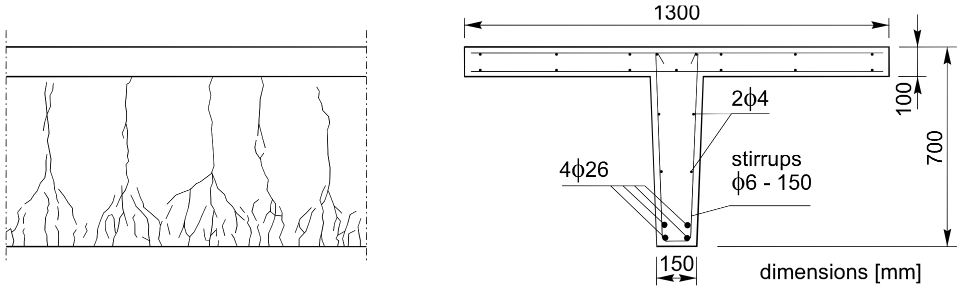

Fig. 15.11 Crack pattern in deep beam with only reinforcement at the bottom and hardly any web reinforcement loaded in pure bending

From experiments [15.4] it was found that the effective area around the reinforcement (the “hidden” reinforced concrete tensile member) for beams, walls and slabs can be defined as shown in fig. 15.12.

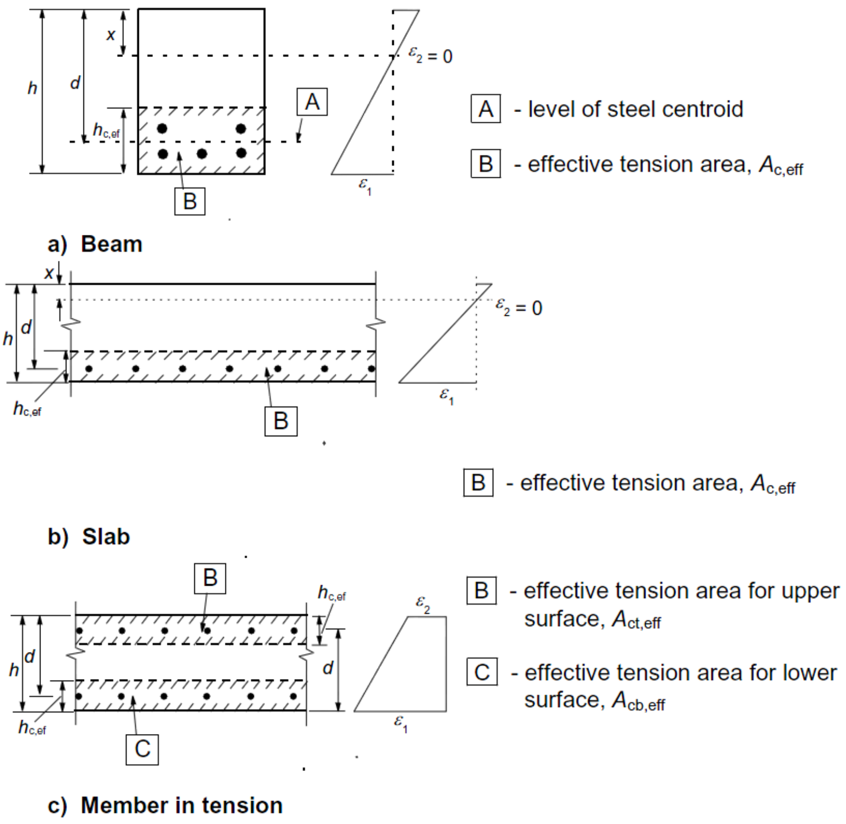

Fig. 15.12 Effective concrete area [15.4] (EN 1992-1-1 fig. 7.1)

15.9 Examples

Example 1: Deep beam

Fig. 15.13 shows an example where a hidden tensile tie may occur.

For the illustrated deep beam it has to be shown that the crack width w_k satisfies the requirement w_k < 0,20 mm, when the steel stress is 300 N/mm², caused by short-term loading.

Material properties

concrete strength class	: C30/37 (f_{ck} = 30 N/mm^2) (EN 1992-1-1 eq. (3.15))
mean axial tensile strength concrete	: f_{ctm} = 2,9 N/mm^2 (EN 1992-1-1 table 3.1)
modulus of elasticity concrete	: E_{cm} = 33000 N/mm^2 (EN 1992-1-1 table 3.1)
reinforcement class	: B500B (EN 1992-1-1 annex C)
design value tensile strength reinforcement	: f_{yd} = 435 N/mm^2 (EN 1992-1-1 fig. 3.8 & table 2.1N)
steel; Young's modulus of elasticity	: E_s = 200000 N/mm^2 (EN 1992-1-1 cl. 3.2.7 (4))
ratio E_s / E_{cm}	: α_e = 6,1
diameter main reinforcement (average)	: Ø = 22,8 mm (EN 1992-1-1 eq. (7.12))
total cross section main reinforcement	: A_s = 4025 mm^2

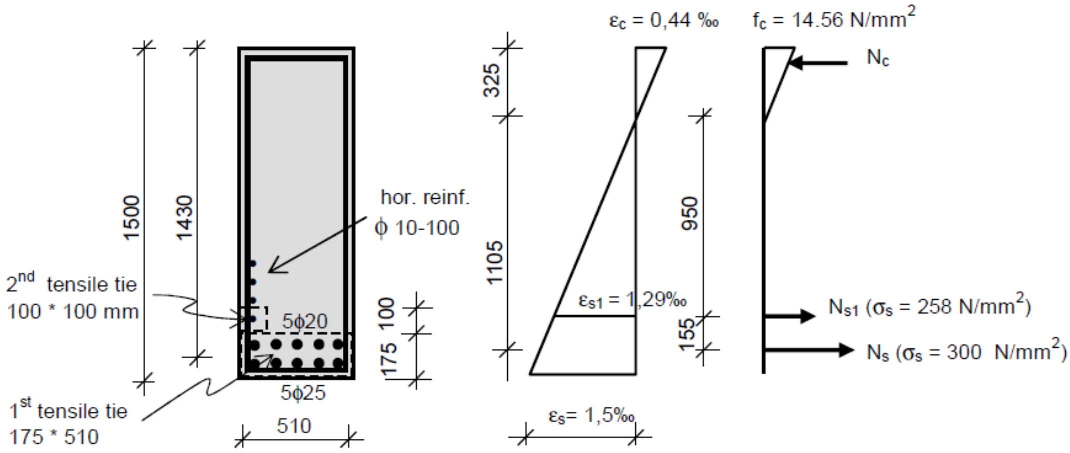

Fig. 15.13 Working example: cross-section and reinforcement of deep beam (left) and strain distribution over the height of the beam in SLS (right)

The effective height of the hidden tensile member (EN 1992-1-1 cl. 7.3.2 & fig. 7.1):

$$h_{eff} = 2{,}5\,(h - d) = 175 \text{ mm}$$

Effective cross-section:

$$A_{c,eff} = h_{eff}\, b = 175 \cdot 510 = 89250 \text{ mm}^2$$

Effective reinforcement ratio (EN 1992-1-1 eq. (7.10) without prestressing steel):

$$\rho_{p,eff} = A_s / A_{c,eff} = 4025 / 89250 = 0{,}045$$

Concrete strength class C30/37; mean direct tensile strength (EN 1992-1-1 table 3.1):

$$f_{ctm} = 2{,}9 \text{ N/mm}^2$$

It is now assumed that cracks occur when the concrete stress reaches the mean tensile strength. The cracking bending moment is (neglect the influence of the reinforcement):

$$M_{cr} = W_{bottom}\, f_{ctm} = \frac{1}{6} \cdot 510 \cdot 1500^2 \cdot 2{,}9 = 555 \cdot 10^6 \text{ Nmm}$$

The reinforcement ratio (ρ_l):

$$\rho_l = \frac{A_s}{b\, d} = \frac{4025}{510 \cdot 1430} = 0{,}0055$$

The ratio of the moduli of elasticity:

$$\alpha_e = \frac{E_s}{E_{cm(0)}} = \frac{200000}{33000} = 6{,}1$$

The height of the concrete compressive zone (x) in the cracked stage can now be calculated:

$$\frac{x}{d} = -\alpha_e\ \rho_l + \sqrt{(\alpha_e\ \rho_l)^2 + (2\ \alpha_e\ \rho_l)}$$

$$\frac{x}{d} = -6{,}1 \cdot 0{,}0055 + \sqrt{(6{,}1 \cdot 0{,}0055)^2 + (2 \cdot 6{,}1 \cdot 0{,}0055)}$$

$$\frac{x}{d} = 0{,}227 \qquad\qquad x = 0{,}227 \cdot 1430 = 325 \text{ mm}$$

The internal lever arm:

$$z = d - \tfrac{1}{3}x = 1430 - \tfrac{1}{3} \cdot 325 = 1322 \text{ mm}$$

The steel stress directly after cracking:

$$\sigma_{sr} = \frac{M_{cr}}{zA_s} = \frac{555 \cdot 10^6}{1322 \cdot 4025} = 104 \text{ N/mm}^2$$

Since σ_s = 300 N/mm² (see fig. 15.13) the actual bending moment applied to the beam is higher than M_{cr} which implies that the stabilised cracking stage is reached, also see fig. 15.8.

If the influence of shrinkage (assume that $\varepsilon_{cs} = -0{,}25 \cdot 10^{-3}$; EN 1992-1-1 cl. 3.1.4) is also accounted for, the expression for w_{max} following eq. (5.15) reads:

$$w_{max} = \frac{1}{2} \frac{f_{ctm}}{\tau_{bm}} \frac{Ø}{\rho_{eff}} \frac{1}{E_s} \left(\sigma_s - 0{,}5\ \sigma_{sr} + \varepsilon_{cs} E_s\right)$$

Using $\tau_{bm} = 2\ f_{ctm}$ and $Ø_m$ = 22,8 mm (mean bar diameter) the result is:

$$w_{max} = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{22{,}8}{0{,}045} \cdot \frac{1}{200000} \left\{300 - 0{,}5 \cdot 104 + 0{,}25 \cdot 10^{-3} \cdot 200000\right\} = 0{,}19 \text{ mm}$$

The requirement $w_{max} \leq 0{,}20$ mm is met.

Note:
EN 1992-1-1 uses $\rho_{p,eff}$ instead of ρ_{eff} as used in the previous sections since EN 1992-1-1 also includes prestressing steel. The influence of prestressing steel on crack width control is reduced because of its lower bond stress when compared with deformed (ribbed)

reinforcing steel (EN 1992-1-1 table 6.2).

It has to be avoided that a crack pattern occurs as shown in figure 15.11. To check whether this might occur, the strain distribution over the height of the beam must be known.

The steel strain is:

$$\varepsilon_s = \frac{\sigma_s}{E_s} = \frac{300}{200000} = 0,0015$$

The force in the steel is:

$$N_s = A_s\ \sigma_s = 4025 \cdot 300 = 1207,5 \cdot 10^3\ \text{N}$$

The compressive strain (ε_c) in the outer fibre:

$$\varepsilon_c = \frac{x}{d-x}\varepsilon_s = \frac{325 \cdot 1,5 \cdot 10^{-3}}{1105} = 0,441 \cdot 10^{-3}$$

The maximum compressive stress in the concrete:

$$\sigma_c = 0,441 \cdot 10^{-3} \cdot 33000 = 14,56\ \text{N/mm}^2$$

Check $\Sigma\, H = 0$:

$$N_c = \tfrac{1}{2} \cdot 325 \cdot 14,56 \cdot 510 = 1206,7 \cdot 10^3\ \text{N}$$

$$N_s = 1207,5 \cdot 10^3\ \text{N} \quad \text{(identical; OK)}$$

It is assumed that horizontal reinforcement Ø10-100 (A_s = 79 mm^2 per bar) is provided on both sides of the beam to avoid the occurrence of too wide cracks in the beam web above the hidden tensile tie.

By applying the tensile tie model, this reinforcement will be checked.
Along both sides of the beam tensile ties occur, see figure 15.12.

Effective concrete area around one bar:

$$b_{eff} = 2,5\ (c_{nom} + \frac{Ø}{2}) = 2,5 \cdot (35 + \frac{10}{2}) = 100\ \text{mm}$$

Reinforcement ratio:

$$\rho_{p,eff} = \frac{79}{100 \cdot 100} = 0,0079$$

For the first tensile tie (nr. 2 in fig. 15.13) directly above the main tensile tie it holds:

$$\varepsilon_{s1} = \frac{950 \cdot 1,5 \cdot 10^{-3}}{1105} = 1,29 \cdot 10^{-3}$$

This results in a steel stress:

$$\sigma_{s1} = 1,29 \cdot 10^{-3} \cdot 200000 = 258\ \text{N/mm}^2$$

The steel stress in the crack formation stage according to eq. (15.2):

$$\sigma_{sr} = \frac{2,9}{0,0079}(1 + 6,1 \cdot 0,0079) = 385 \text{ N/mm}^2$$

Apply $\tau_{bm} = 2\, f_{ctm}$ and Ø = 10 mm in eq. (15.14):

$$w_{max} = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{10}{0,0079} \cdot \frac{1}{200000} \left\{258 - 0,5 \cdot 385 + 0,25 \cdot 10^{-3} \cdot 200000\right\}$$

$$= 0,18 \text{ mm} \leq 0,20 \text{ mm}$$

As the strain decreases in upward direction, the crack width decreases and the provided reinforcement Ø10-100 is sufficient. It is not necessary to extend the reinforcement to the neutral axis. The unreinforced area has a very low strain; it is almost “stressless” (see fig. 15.14).

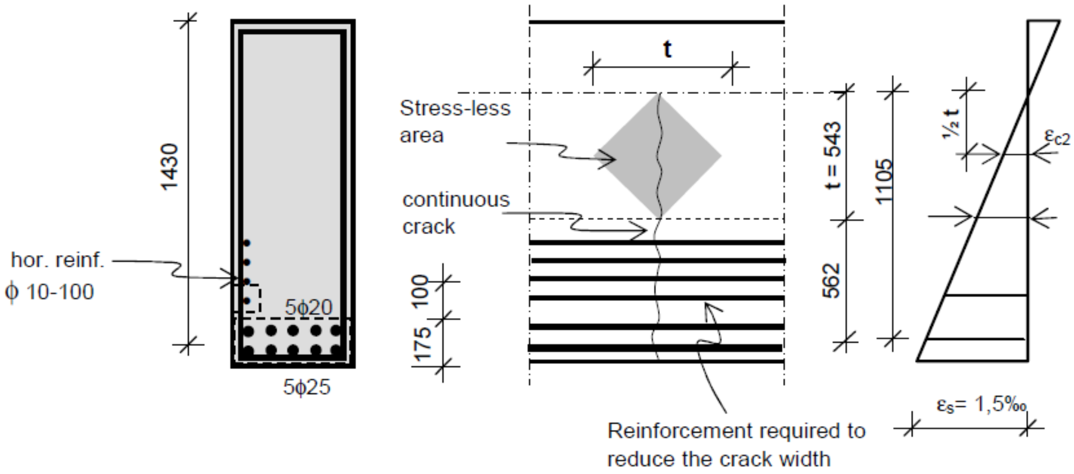

Fig. 15.14 "Stressless"-area above the reinforced area

The crack width follows from the crack spacing that occurs even without reinforcing steel present. It can be assumed that the crack spacing is about equal to the height of the unreinforced area (t in fig. 15.16). The maximum crack width occurs about halfway this area:

$$w_{max} = t\, \varepsilon_{c2}$$

where the fictitious concrete strain halfway down the unreinforced area is: :

$$\varepsilon_{c2} = \frac{\varepsilon_s \, \frac{1}{2} t}{(d - x)}$$

The expression for w_{max} :

$$w_{max} = \frac{\frac{1}{2} t^2}{(d - x)}\, \varepsilon_s$$

For $w_{max} \leq 0{,}20$ mm, $\varepsilon_s = 1{,}5‰$ and $d - x = 1105$ mm, the distance t should be less than 543 mm. As a result, additional reinforcement is required over a height $h_w = 1105 - 543 = 562$ mm.

In [15.4] a graph is given that can be used to determine the area where reinforcement should be applied, see figure 15.15).

The calculated value corresponds well with the one obtained from the graph. Additional crack distributing reinforcement is only required for relatively high beams (for $w = 0{,}2$ mm it implies that $h - x > 550$ mm). For smaller beams it is sufficient to check only the crack width in the main tensile tie.

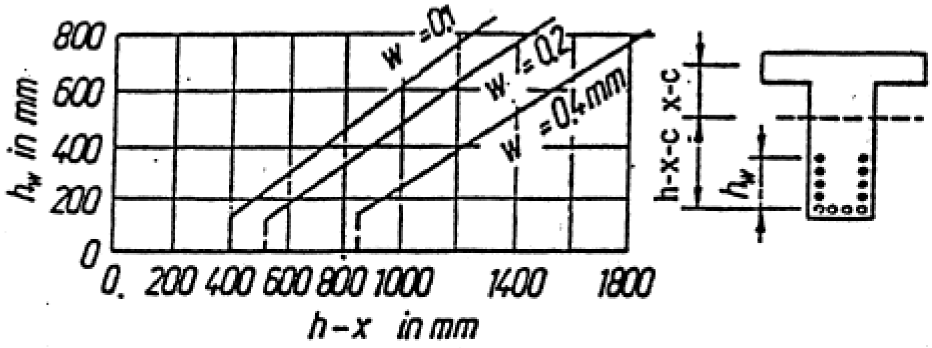

Fig. 15.17 Part of the web where crack width limiting reinforcement should be provided in order to avoid the occurrence of large cracks ("Sammelrisse") [15.4]

It appears that in high beams, crack width limiting reinforcement is required over a height of about 550 mm. This does not automatically imply that in the remaining parts no reinforcement is needed. In general practical minimum reinforcement should always be applied. Summarising one may conclude that the following reinforcement is to be provided:

- Horizontal reinforcement Ø10 – 100 mm as crack width limiting reinforcement (see calculation above).
- Other parts: horizontal reinforcement Ø10 – 300 mm.
- Vertical reinforcement, for instance also Ø10 – 300 mm.

Example 2 – Cantilever balcony slab

A cantilever balcony slab has to be designed (see fig. 15.16).

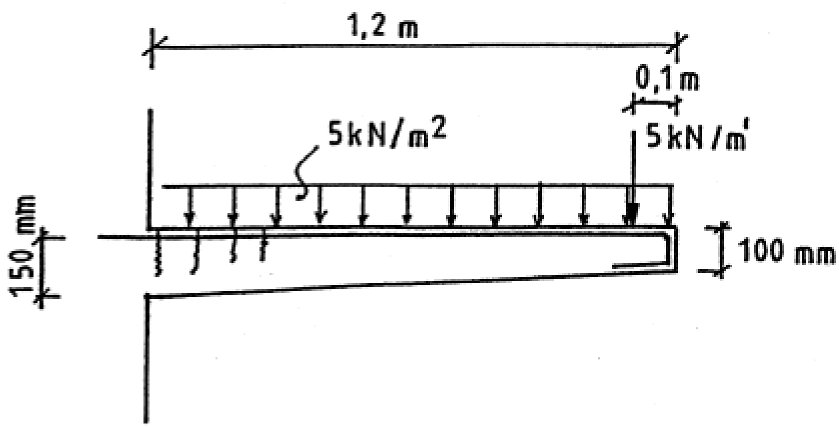

Fig. 15.16 Cross-section of a cantilever gallery slab

Material properties

concrete strength class	: C20/25
mean axial tensile strength concrete	: f_{ctm} = 2,21 N/mm^2
modulus of elasticity concrete	: E_{cm} = 30000 N/mm^2
reinforcement class	: B500B
modulus of elasticity steel	: E_s = 200000 N/mm^2
diameter main reinforcement	: Ø = 10 mm
ratio E_s / $E_{cm(0)}$	: α_e = 6,67 (short-term loading)
exposure class	: XC4
concrete cover	: $c_{nom} = c_{min,dur} + \Delta c_{dev}$ = 30 + 5 = 35 mm
effective height	: $d = h - c$ - Ø/2 = 150 – 35 - 10/2 = 110 mm

load factors:
γ_G = 1,2; γ_Q = 1,5.

Design value of the bending moment (M_{Ed}) is:

- variable load : $M_{Ed1} = 1,5 \cdot (\tfrac{1}{2} \cdot 5 \cdot (1,2)^2 + 5 \cdot 1,1) = 13,7$ kNm / m

Assume that selfweight can be calculated assuming a constant slab height h = 0,125 m:

- selfweight : $M_{Ed2} = 1,2 \cdot (0,125 \cdot 25 \cdot 1,2 \cdot 1,2 / 2) = 2,7$ kNm / m

- total load : $M_{Edtot} = 13,7 + 2,7 = 16,4$ kNm / m

The required reinforcement ratio ($\rho_{l,req}$) follows from standard tables:

$$\frac{M_{Ed,tot}}{b\ d^2} = \frac{16,4}{1,0 \cdot (0,11)^2} = 1355 \quad \text{result: } \rho_{l,req} = 0{,}31\%$$

$$A_s = 0,31 \cdot 0,11 \cdot 10^4 = 341 \text{mm}^2$$

Applied: Ø10 – 200 mm; $A_{s,prov}$ = 395 mm^2/m; $\rho_{l,prov}$ = 0,36%

The maximum bending moment in SLS is:

$$M = \frac{13,7}{1,5} + \frac{2,7}{1,2} = 11,4 \text{ kNm}$$

The height of the compression zone after cracking can be calculated with the following equation:

$$\frac{x}{d} = -\alpha_e \, \rho_l + \sqrt{(\alpha_e \, \rho_l)^2 + (2 \, \alpha_e \, \rho_l)}$$

$$\frac{x}{d} = -6,67 \cdot 0,0036 + \sqrt{(6,67 \cdot 0,0036)^2 + (2 \cdot 6,67 \cdot 0,0036)}$$

$$\frac{x}{d} = 0,197 \qquad x = 0,197 \cdot 110 = 21,7 \text{ mm}$$

The internal lever arm (z) is:

$$z = d - \tfrac{1}{3} x = 110 - \tfrac{1}{3} \cdot 21,7 = 103 \text{ mm}$$

The steel stress (σ_s) in SLS:

$$\sigma_s = \frac{M}{z \, A_s} = \frac{11,4 \cdot 10^6}{103 \cdot 395} = 280 \text{ N/mm}^2$$

The effective height of the hidden tensile tie (EN 1992-1-1 fig. 7.1):

$$h_{eff} = 2,5\,(h - d) = 2,5\left(c + \tfrac{1}{2}\text{Ø}\right) = 2,5\left(35 + \tfrac{1}{2} \cdot 10\right) = 100 \text{ mm}$$

Restriction (EN 1992-1-1 cl. 7.3.2 (3)):

$h_{eff} \leq \tfrac{1}{3}(h - x) = \tfrac{1}{3} \cdot (150 - 21,6) = 43 \text{ mm}$, which is governing.

The cross-sectional area ($A_{c,eff}$) of the tensile tie is:

$$A_{c,eff} = 43 \cdot 1000 = 43 \cdot 10^3 \text{ mm}^2$$

$$\rho_{eff} = \frac{395}{43 \cdot 10^3} = 0,0092$$

The cracking bending moment (M_{cr}):

$$M_{\mathrm{cr}} = W_{\mathrm{bottom}} f_{\mathrm{ctm}} = \frac{1}{6} \cdot 1000 \cdot 150^2 \cdot 2,21 = 8,29 \cdot 10^6 \text{ Nmm / m}$$

The steel stress directly after cracking (σ_{sr}):

$$\sigma_{\mathrm{sr}} = \frac{M_{\mathrm{cr}}}{zA_s} = \frac{8,29 \cdot 10^6}{103 \cdot 395} = 204 \text{ N / mm}^2$$

In the crack formation stage the steel stress is 204 N/mm². Under maximum SLS load the steel stress is 280 N/mm². The result is a stabilised cracking stage.

When taking into account an additional deformation due to shrinkage $\varepsilon_{cs} = 0,25 \cdot 10^{-3}$, the crack width is:

$$w_{\max} = \frac{10}{4 \cdot 0,0092 \cdot 200 \cdot 10^3} \cdot \left(280 - 0,5 \cdot 204 + 0,25 \cdot 10^{-3} \cdot 200 \cdot 10^3\right)$$
$$= 0,31 \text{ mm}$$

For environment class XC4 a maximum crack width of 0,30 mm is allowed.

In the past damage occurred in cantilever balconies because cracks perpendicular to the longitudinal axis of the elements occurred, see figure 15.17. This was caused by a phenomenon that was not accounted for. The cantilever slabs were monolithically connected to the floor slabs inside the building. Inside a building the relative humidity and temperature are about constant while at the outside they can vary strongly. A differential shrinkage of $\varepsilon_{cs} = 0,20 \cdot 10^{-3}$ and a temperature difference ΔT = 25° C at $\alpha_T = 1,0.10^{-5}$ result in an imposed deformation:

$$\varepsilon_{\mathrm{cs}} + \varepsilon_{\Delta\mathrm{T}} = 0,2 \cdot 10^{-3} + \alpha_T \, \Delta T = 0,2 \cdot 10^{-3} + 25 \cdot 10^{-5} = 0,45 \cdot 10^{-3}$$

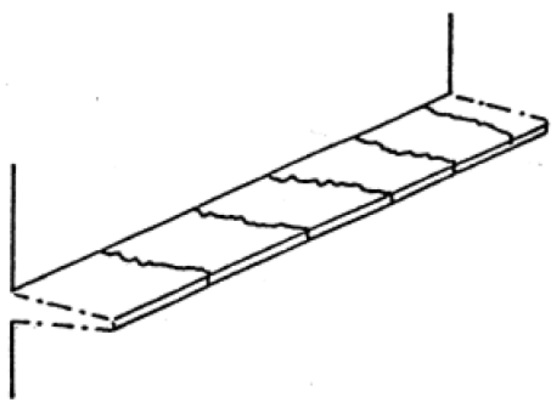

Fig. 15.17 Cracking in cantilever galleries

The cantilever slab then acts as a tensile member in its longitudinal direction. If the imposed deformation is (partly) restrained, cracks can occur. The cracking strain of concrete is about $0{,}12 \cdot 10^{-3}$, so a small degree of restraint causes cracking. In case of a restrained deformation the structure is mostly in the crack formation stage. This can be checked by eq. (15.22):

$$\varepsilon_s = \frac{f_{ctm}\,(0{,}6 + \alpha_e\,\rho_{eff})}{E_s\,\rho_{eff}}$$

Assume that the slab has a total longitudinal reinforcement ratio of 0,009:

$$\varepsilon_s = \frac{2{,}21 \cdot (0{,}6 + 6{,}67 \cdot 0{,}009)}{200 \cdot 10^3 \cdot 0{,}009} = 0{,}81 \cdot 10^{-3}$$

$$\varepsilon_{cs} + \varepsilon_{\Delta T} = 0{,}45 \cdot 10^{-3} < 0{,}81 \cdot 10^{-3}$$

This demonstrates that the slab is in the crack formation stage.

In case of long-term loading eq. (15.15) can be used to calculate the required amount of reinforcement to control cracking:

$$w_{max} = \frac{1}{2}\frac{f_{ctm}}{\tau_{bm}}\frac{Ø}{\rho}\frac{1}{E_s}\left(\sigma_s - \alpha\,\sigma_{sr} + \beta\,\varepsilon_{cs} E_s\right)$$

For Ø = 10 mm, w_{max} = 0,30 mm, f_{ctm} = 2,21 N/mm², α_e = 20, τ_{bm} = 1,6 f_{ctm}, α = 0,5 and β = 0 (long-term and varying load) the result is $\rho \geq$ 0,85%. The reinforcement might be Ø10 - 150 over the full length of the slab.

It appears that in the longitudinal direction more reinforcement is required than in the transverse direction. Usually another more cost-effective solution is chosen: simply supported slabs with expansion joints, supported by cantilever beams (fig. 15.20).

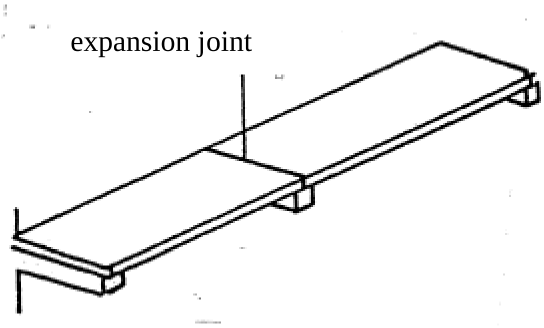

Fig. 15.18 Expansion joint between slabs to reduce the degree of restraint in longitudinal direction

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