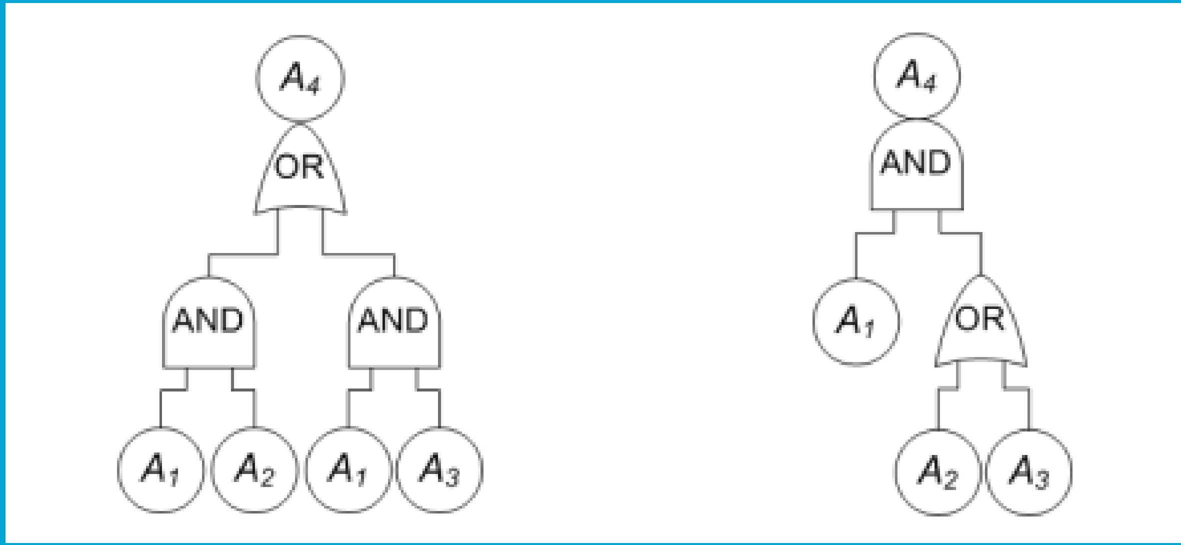
System's reliability: Fault Trees & Bayesian Networks



CIEM4210 / CIEM4220 / CIEM4230
CEG MSc CE Track Hydraulic and Offshore Structures (2024/2025)
HOS B-1 Probablistic Design

Oswaldo Morales Nápoles

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Outline

- Reminder of fundamental concepts of probability
- Discrete Variables
- Fault Trees
- Discrete Bayesian Networks

Probability

- P(A) = probability of event A   Mathematical definition → axioms

- Interpretations:

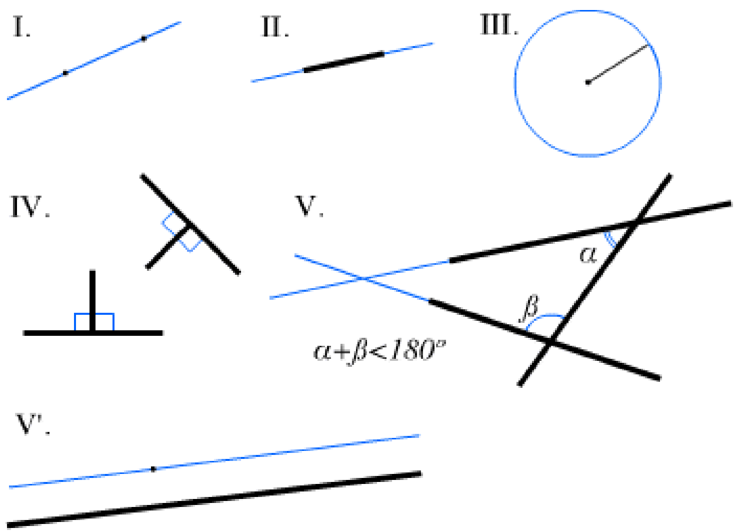
- **Classical:** Laplace (1819) *A philosophical essay on probabilities.* The number of favourable cases divided by the number of equi-possible cases

- **Frequentist:** Von Mises R. (1936) *Probability statistics and truth.* Limiting relative frequencies in a 'collective' or random 'sequence'

- **Subjective:** Ramsey (1931) or Savage (1956) *Foundations of statistics*. Degree of belief of a *rational* subject. Measured by observing choice behaviour. For example, if A>.B and B>.C then A>.C ">." Stands for preferable

Mathematical definition

- Kolmogorov axioms (1933)
- Probability can and should be developed from axioms in the same way as Geometry and Algebra
- Axiom: statements so evident that can be accepted without controversy
- Axioms:
  - 1. $P(A) \geq 0$ A element of Ω
  - 2. $P(\Omega) = 1$ Ω collection of elements
  - 3. P(A or B) =
- Few other technical axioms



- Euclid’s Postulates for geometry (comparison) :
  - To draw a line from any point to any point.
  - To produce a finite straight line continuously in a straight line.
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Sets

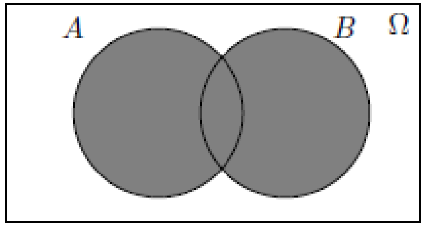


Figure 1.2: *Union of events A and B:* $A \cup B$; *A or B*

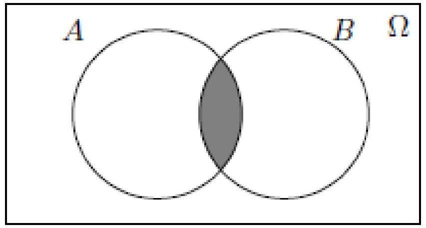


Figure 1.3: *Intersection of events A and B:* $A \cap B$; *A and B*

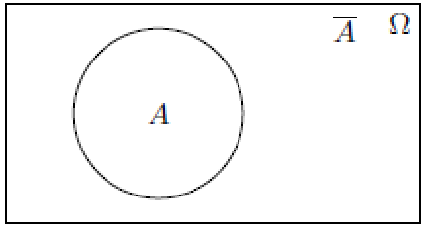


Figure 1.6: $A \cup \overline{A} = \Omega$ *and* $A \cap \overline{A} = \emptyset$; $\overline{A}$ *is called the complement of A*

$B - A = B/A$

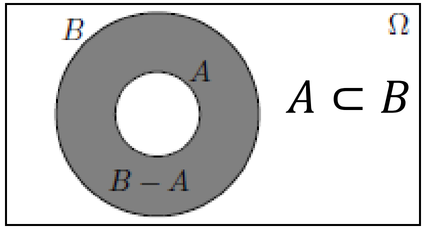


Figure 1.7: *A is a subset of B then:* $A \cup (B - A) = B$; $A \cap (B - A) = \emptyset$ *and* $A \cup B = B$

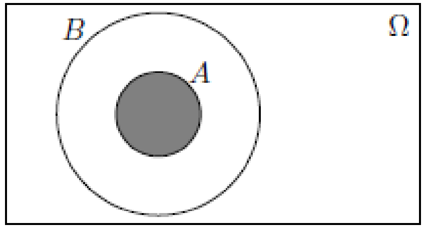


Figure 1.4: *A is a subset of B:* $A \subset B$; *A is a part of B or B contains A*

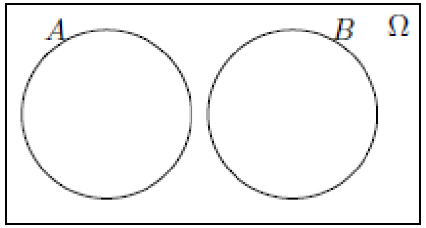


Figure 1.5: *A and B are mutually exclusive* $A \cap B = \emptyset$

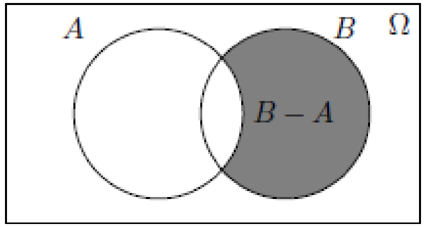


Figure 1.8: $A \cup (B - A) = A \cup B$, $A \cap (B - A) = \emptyset$

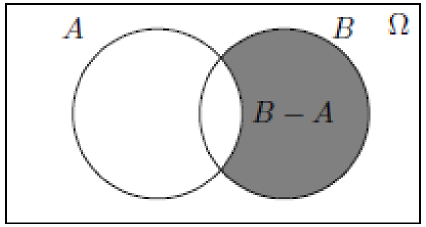
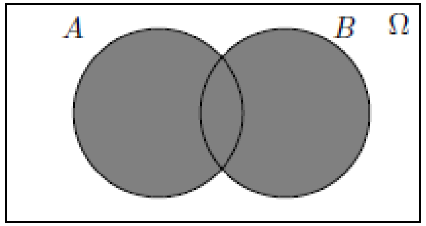


Figure 1.9: $(B - A) \cup (A \cap B) = B$, $(B - A) \cap (A \cap B) = \emptyset$

Basic Probability Calculus



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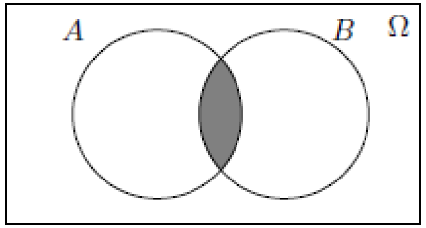

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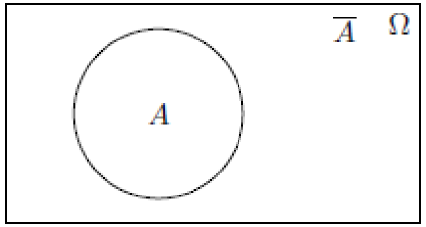

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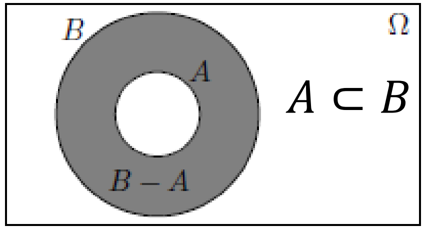

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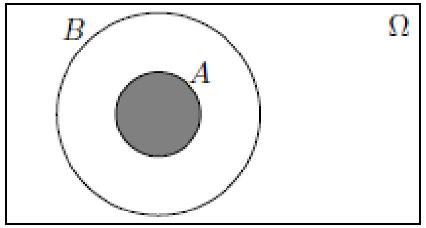

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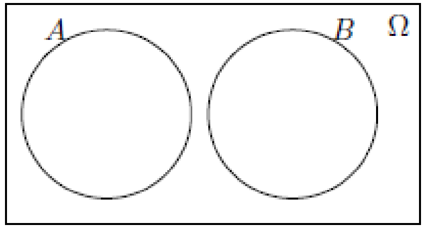

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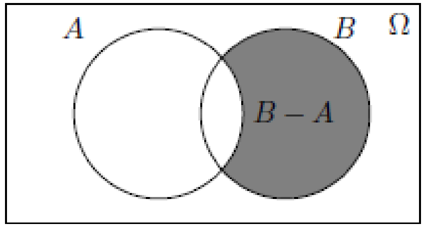

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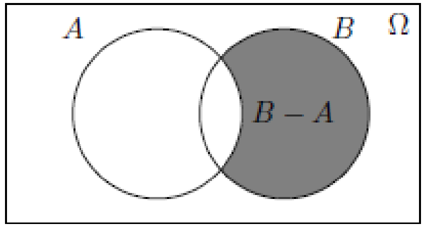

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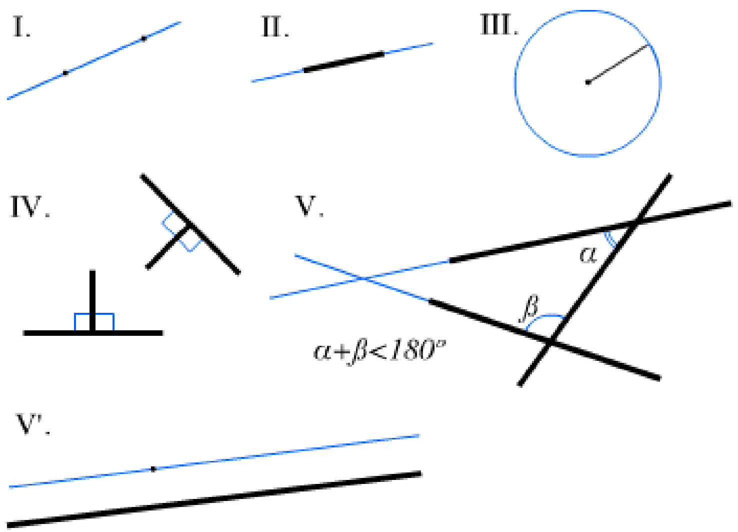
- Few other technical axioms



- Euclid's Postulates for geometry (comparison) :
  - To draw a line from any point to any point.
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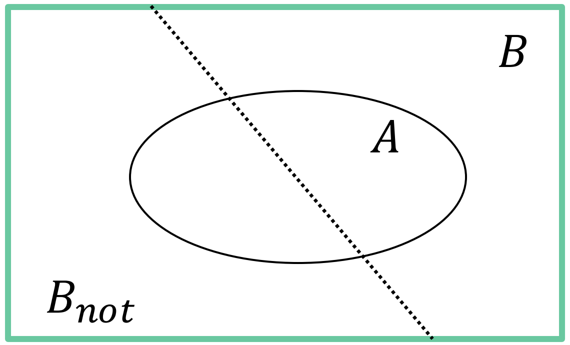
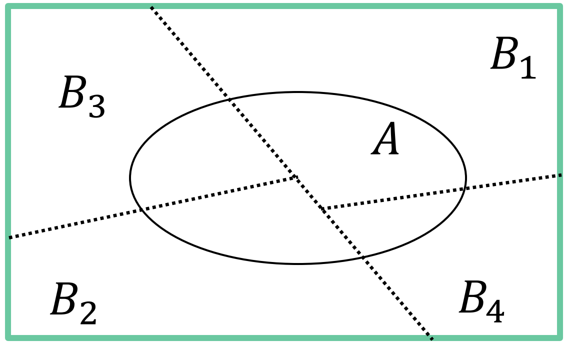
Total Probability

- $P(A) = P(A|B)P(B) + P(A|B_{not})P(B_{not})$ with $B \cap B_{not} = \emptyset$ and $B \cup B_{not} = \Omega$



- $P(A) = \sum_i P(A|B_i)P(B_i)$ with $B_j \cap B_k = \emptyset$ and $\bigcup_i B_i = \Omega$



- Generalization to continuous integral "in which the uncertainty is integrated out"

Discrete Random Variable

- n identical and independent “experiments”
  - Bernoulli trials
- each experiment may have a
  - “success” probability p or
  - “failure” with probability $(1 - p)$
- X : number of successes after n experiments

Discrete Random Variable

- Binomial:
- *n* identical and independent "experiments"
  - Bernoulli trials
- each experiment may have a
  - "success" probability p or
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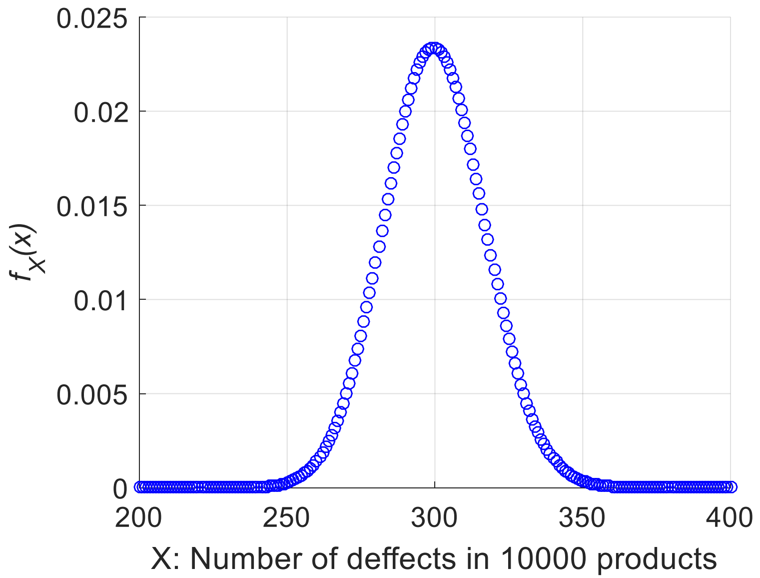
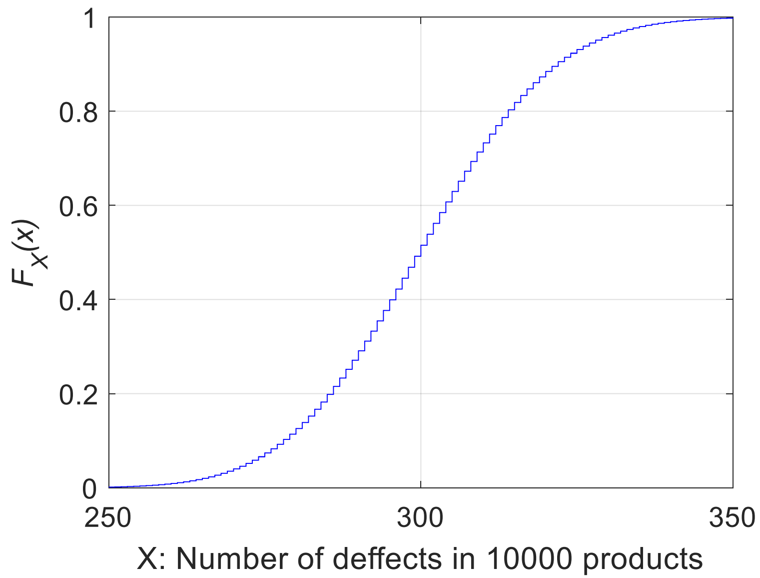
- X : number of successes after *n* experiments

- SSSSSSS (7 successes in 7 trials)
- FFFFFFF (7 failures in 7 trials)
- SSFFFFF, SFSFFFF, SFFSFFF, …, FFFFFSS (2 successes in 7 trials)
- $P(X=x)=\binom{n}{x}p^x(1-p)^{n-x}$
- $P(X=2)=\binom{7}{2}p^2(1-p)^{7-2}$
- Suppose we toss a fair coin 7 times →
  - $P(X=2)=0.1641$

Discrete Random Variable

- Binomial:
- Assume a producer knows P(defect) = 0,03 in a daily batch of $n = 10{,}000$ articles.
- $f_X(x) = P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$
  - mass function (pdf)
- $F_X(x) = P(X \leq x) = \sum_{\{X \leq x\}} P(X = x)$
  - cumulative distribution function (cdf)





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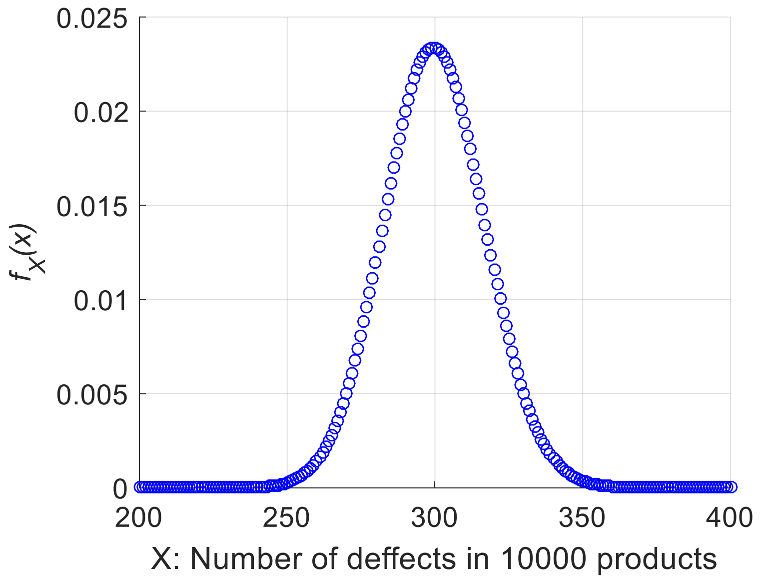
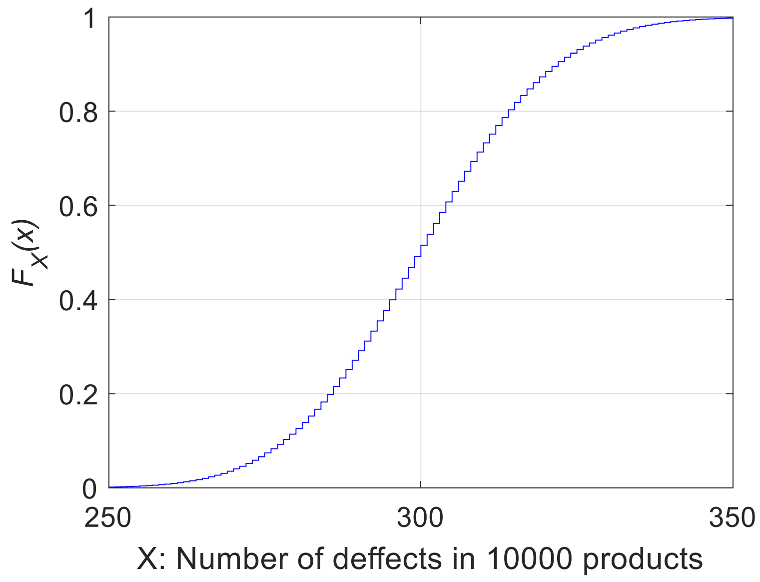
Discrete Random Variable

- Expectation: intuitively the "long-run" mean value: $E(X) = \sum_{x \in X} x\, P(X = x)$
- For binomial $E(X) = np$ — For our example $E(X) = 300$
- Median: x for which $P(X \leq x) = 0.5$ — For our example $50th\ percentile\ = 300$
- Mode: most probable value — For our example $= 300$





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Discrete Random Variable

Geometric:

- *n* identical and independent "experiments"
    - Bernoulli trials
- each experiment may have a
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- X : number of trials to first successes

- Example: number of wells to excavate before finding water

Discrete Random Variable

Geometric:

- *n* identical and independent "experiments"
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- each experiment may have a
  - "success" probability p or
  - "failure" with probability $(1-p)$
- X : number of trials to first successes
- Example:
  - S (successes in 1st trial)
  - FS (successes in 2nd trial)
  - FFS, FFFS, FFFFS, …, FFFFF…S
    (third, fourth, fifth, …)
- $P(X=x)=p(1-p)^{n-1}$

Remember binomial:
$P(X=x)=\binom{n}{x}p^x(1-p)^{n-x}$

- Example: number of wells to excavate before finding water

Discrete Random Variable

Geometric:
Finding water after drilling a well P(success) = 0,2

- $f_X(x) = P(X = x) = p(1-p)^{n-x}$
  - mass function (pdf)
- $F_X(x) = P(X \leq x) = \sum_{\{X \leq x\}} P(X = x)$
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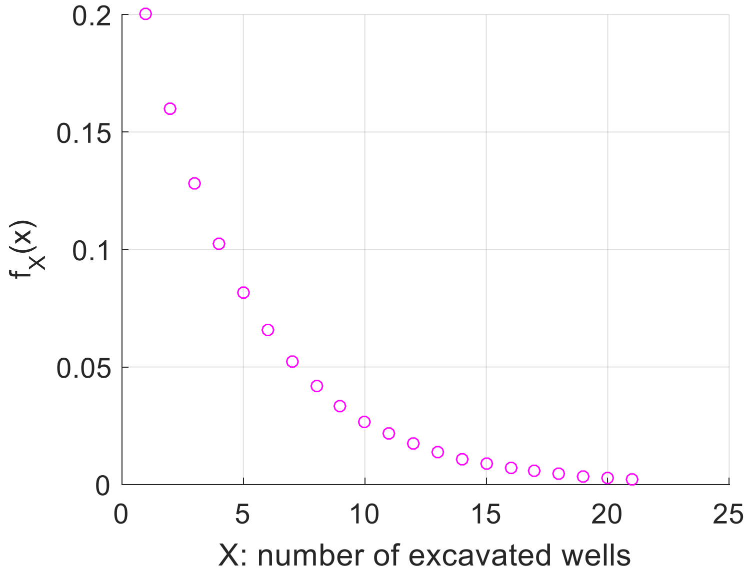
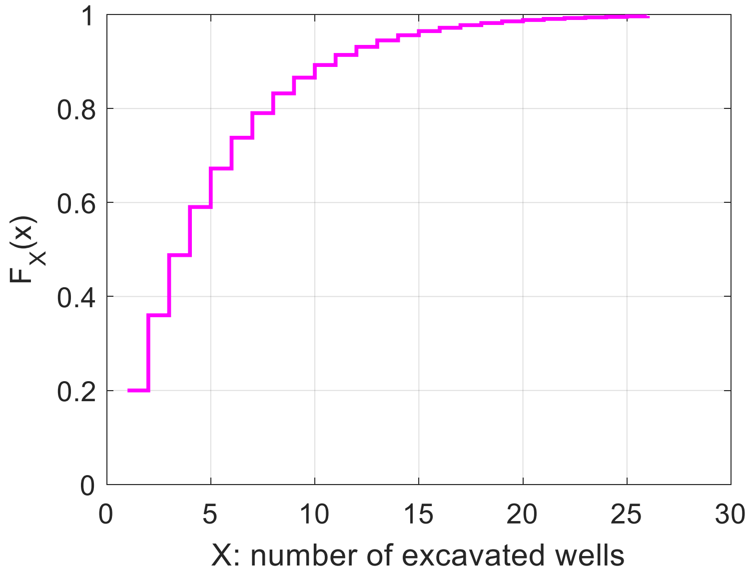




Discrete Random Variable

Binomial:

Expectation: intuitively the "long-run" mean value: $E(X) = \sum_{x \in X} x\, P(X = x)$

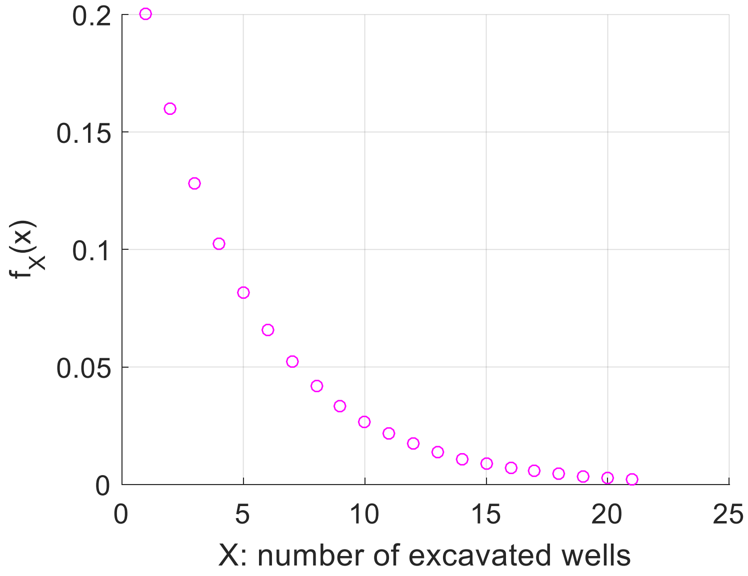
- For geometric $E(X) = 1/p$ example $E(X) = \frac{1}{0,2} = 5$

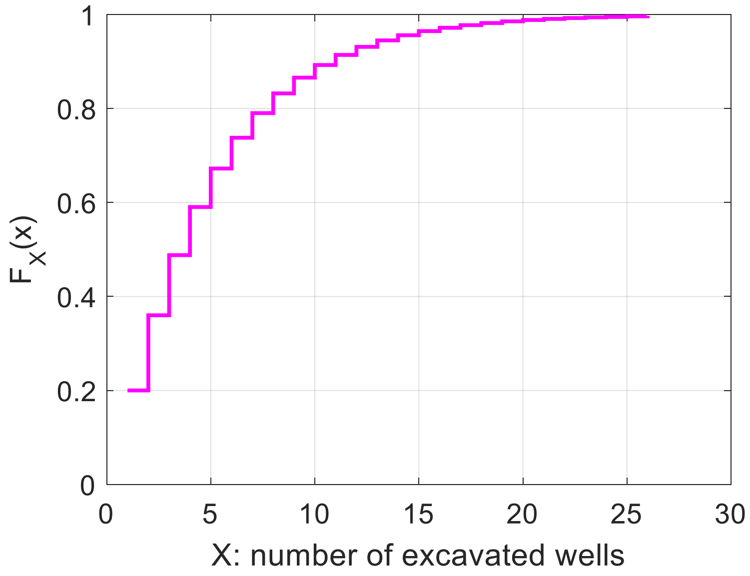
Median: x for which $P(X \leq x) = 0.5$

- $50th\ percentile = \lceil -1/\log_2(1-p) \rceil = 4$

Mode: most probable value

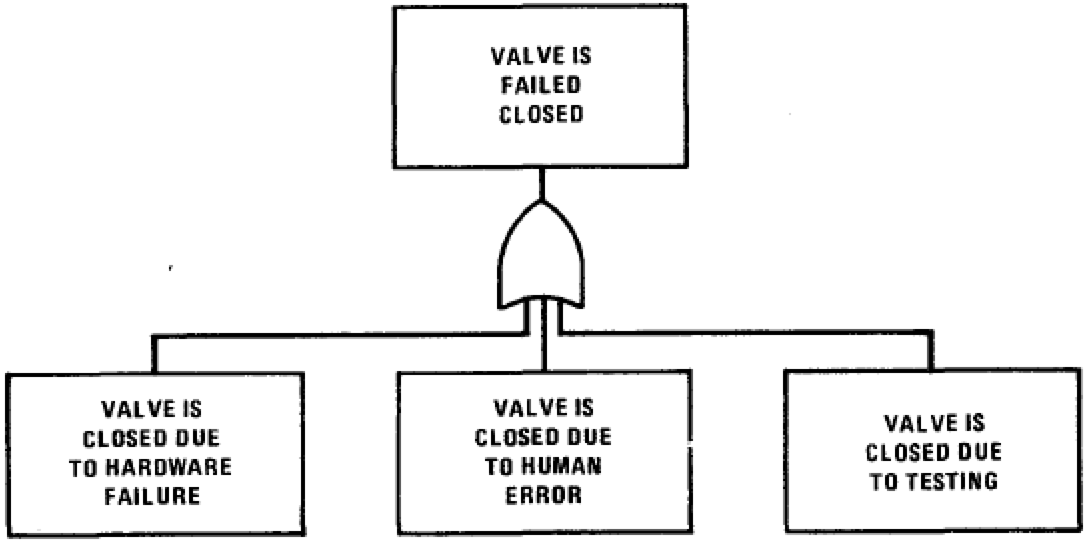
- For our example $= 1$





Fault Trees

- A graphic model
- parallel and serial combinations of faults
- result → occurrence of the predefined undesired event.
- (Fault tree handbook NUREG-0492)



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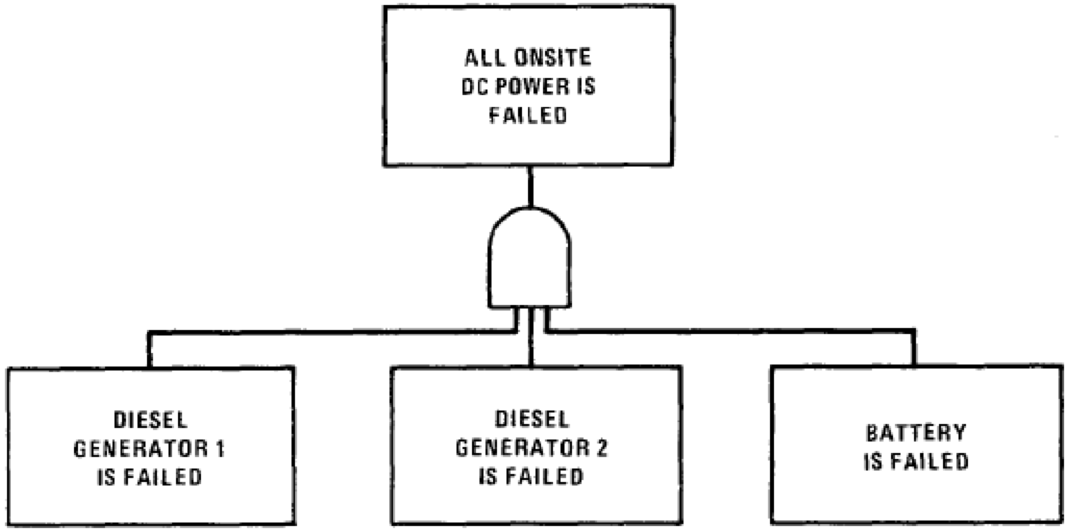

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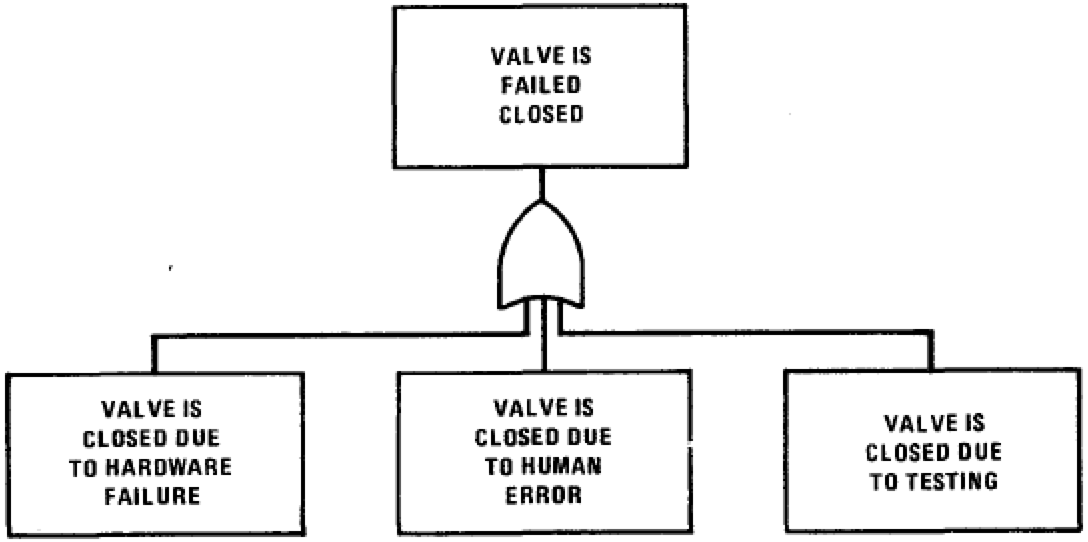
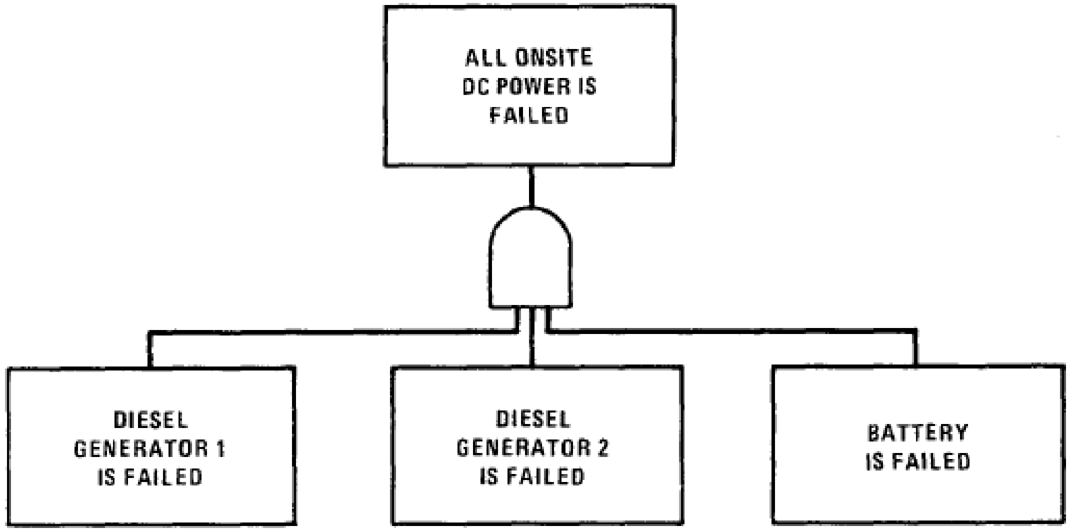
Fault Trees

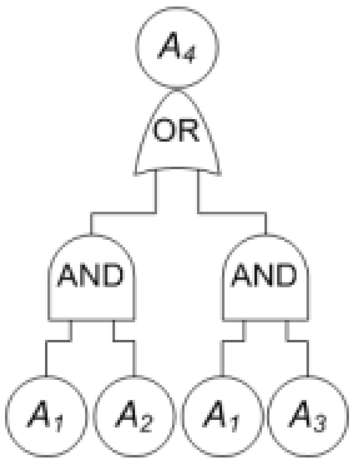
- Objective:
- Probability of system failure based on basic event probabilities
- Basic (intermediate) event probabilities knowing probability of system failure





Fault Trees

- Boolean algebra is used to operate with the Tree → Avoid double counting probabilities
- Probabilities are computed with the usual rules of probability



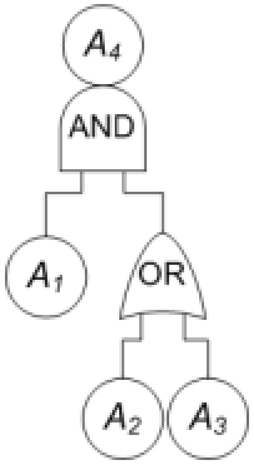


Table 6.1. *Laws of Boolean algebra*

Commutative laws	$X \cdot Y = Y \cdot X$
	$X + Y = Y + X$
Associative laws	$X \cdot (Y \cdot Z) = (X \cdot Y) \cdot Z$
	$X + (Y + Z) = (X + Y) + Z$
Distributive laws	$X \cdot (Y + Z) = X \cdot Y + X \cdot Z$
Idempotent laws	$X \cdot X = X$
	$X + X = X$
Absorption law	$X + X \cdot Y = X$
Complementation	$X + X' = \Omega$
	$(X')' = X$
De Morgan's laws	$(X \cdot Y)' = X' + Y'$
	$(X + Y)' = X' \cdot Y'$
Empty set/universal set	$\emptyset' = \Omega$

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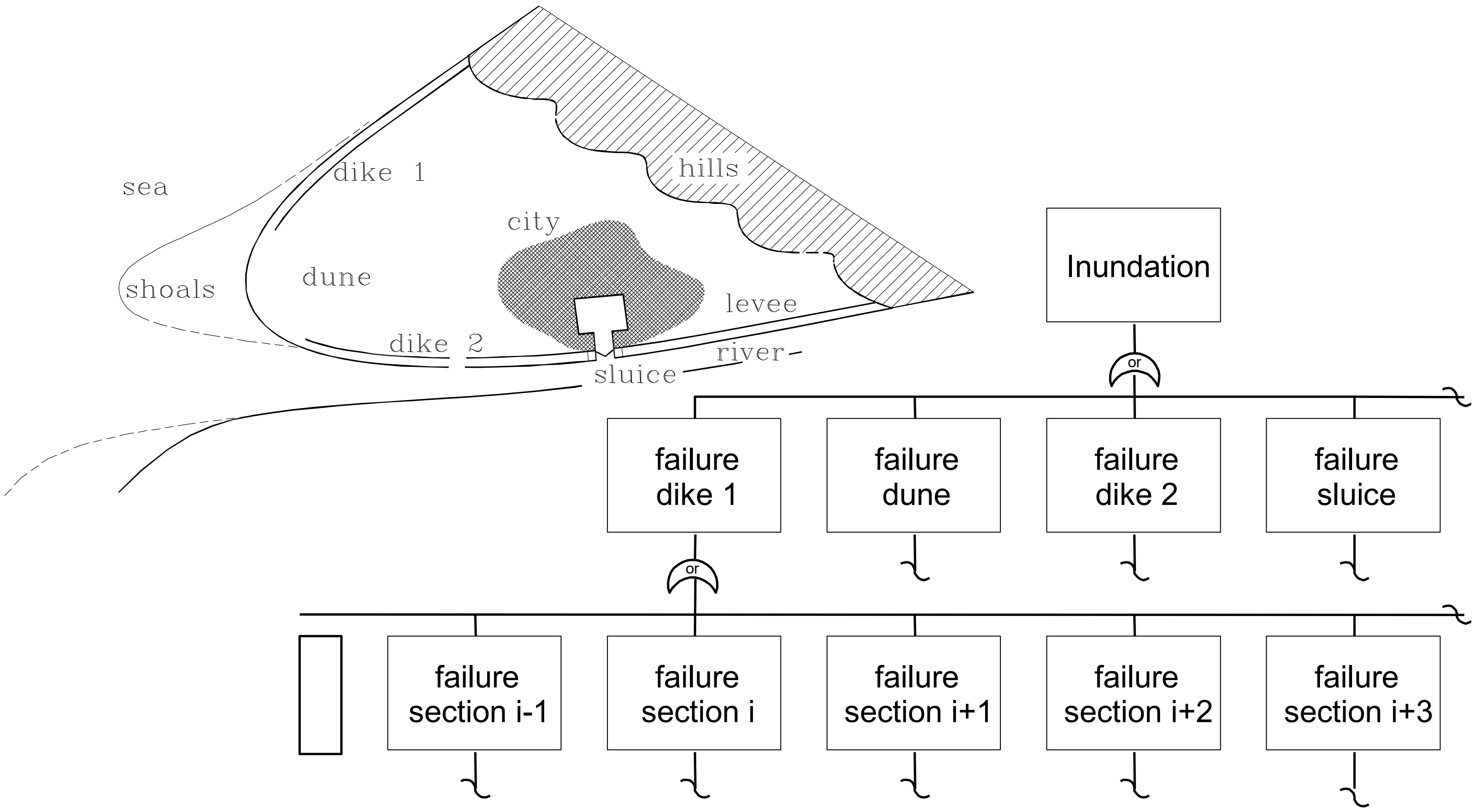
Example: Fault Tree Polder



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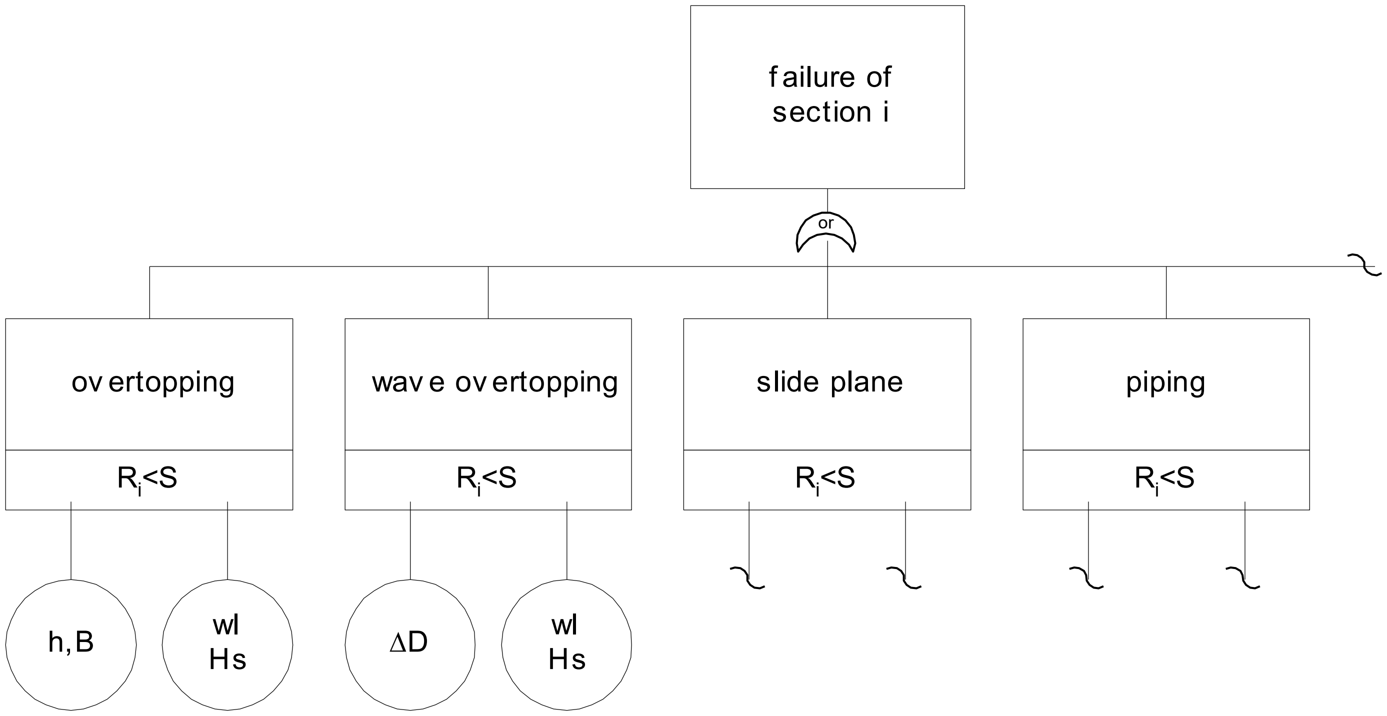

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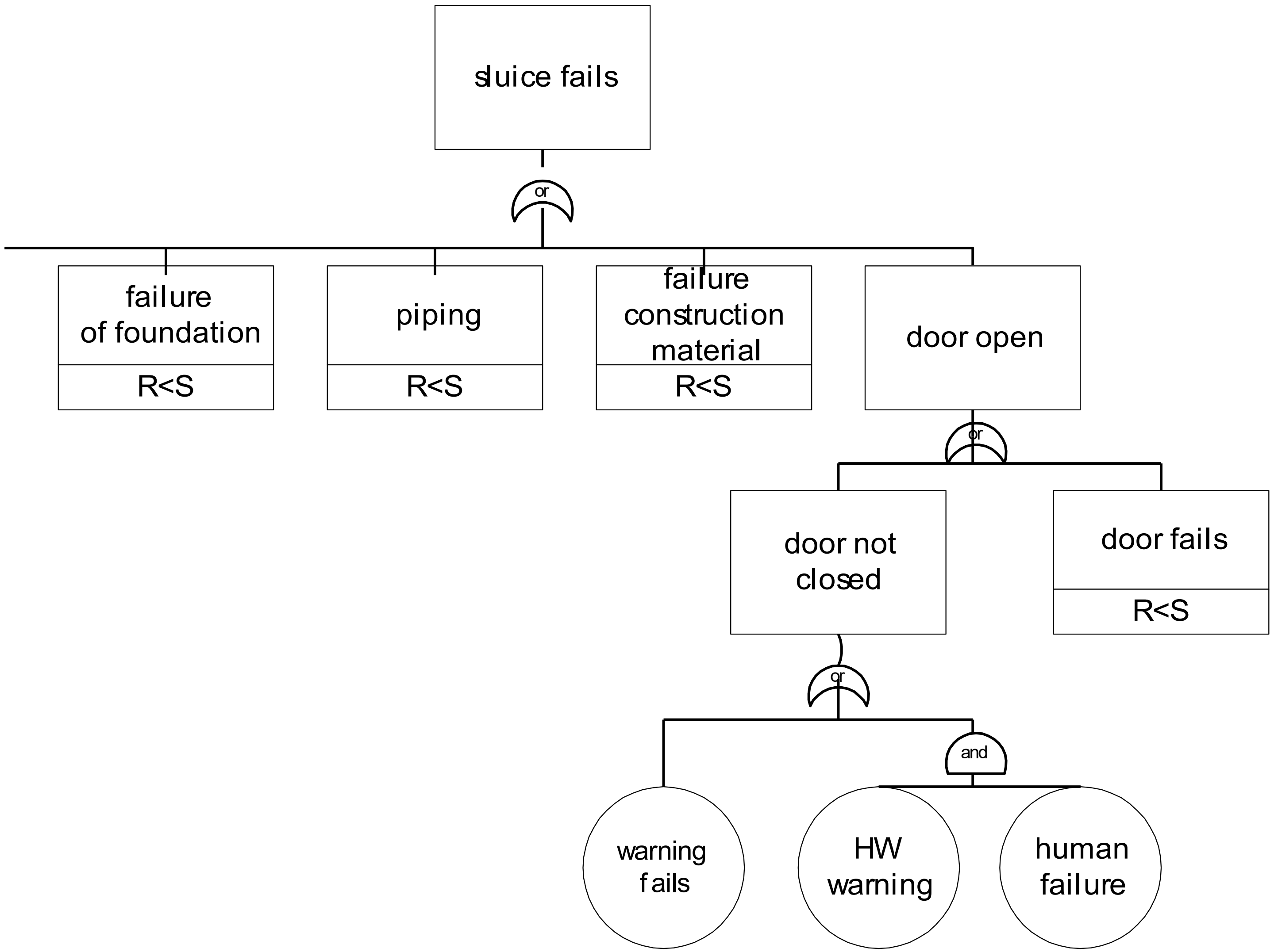

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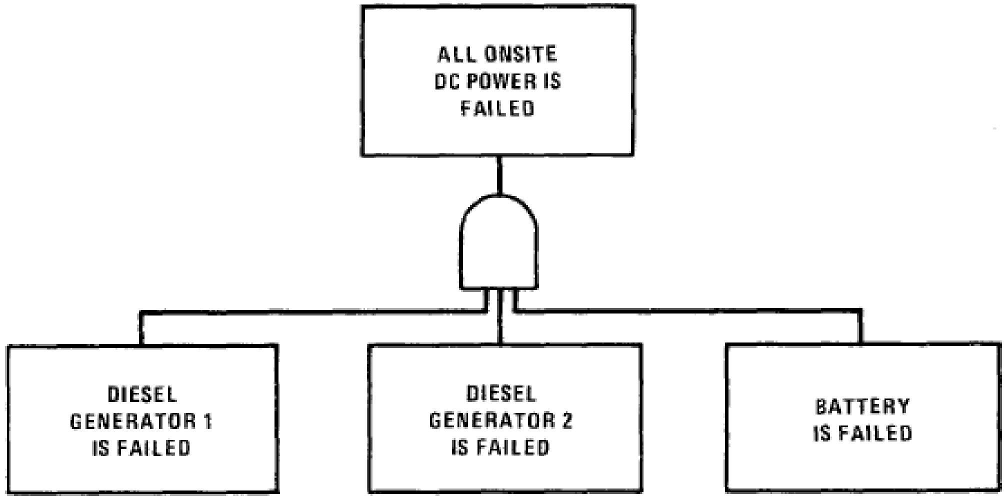
Fault Trees (AND)

- Assume
- P(Diesel Generator 1 Fails) = 0,02 on demand
- P(Diesel Generator 2 Fails) = 0,02 on demand
- P(Battery Fails) = 0,05 on demand

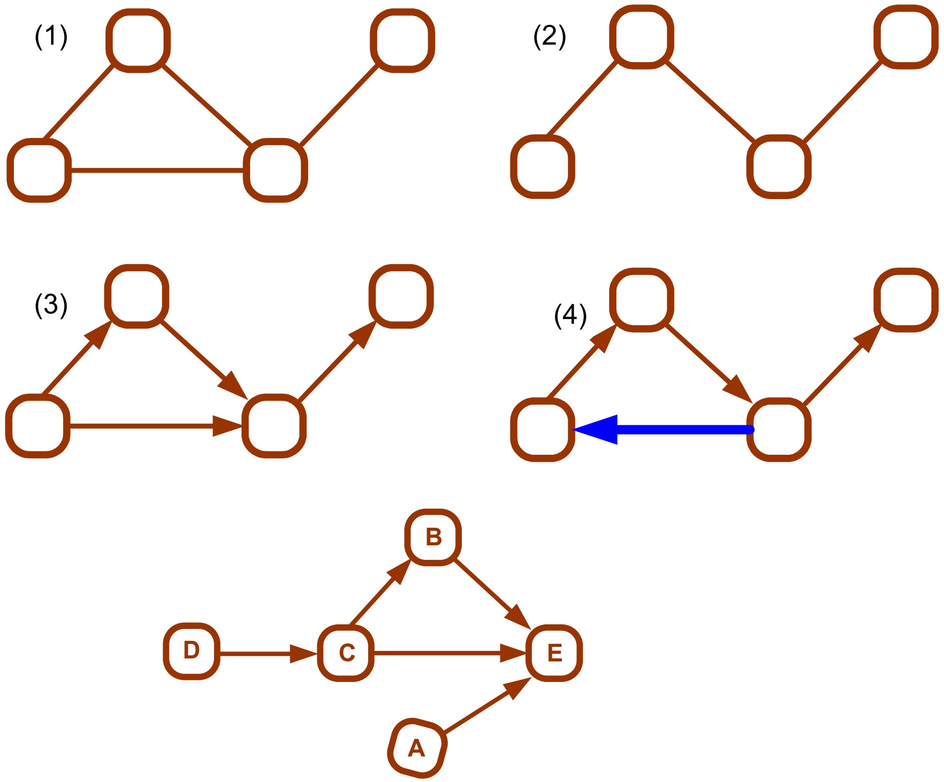
- P(All onsite DC power is failed)?



What is a Bayesian Network

$$f_{X_1,\ldots,X_n} = \prod_{i=1}^{n} f_{X_i|Pa(X_i)}$$

- BNs Directed Acyclic Graph (3)
- Nodes represent random variables
- A,B,C (parents) of E (child);
- D NOT a direct influence for E (ancestor)
- $A \perp B$; $A \perp C$; $A \perp D$; $D \perp E \mid C$
- Information (influence) flow / sampling order:
- {A, {D→C →B}} → E
- {{D, A} →C → B} → E



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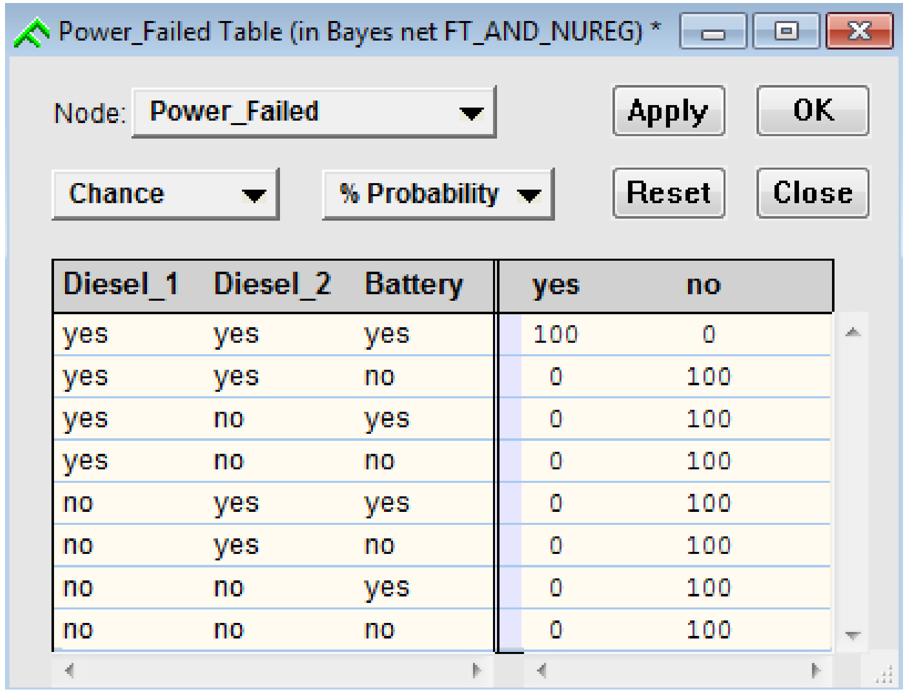
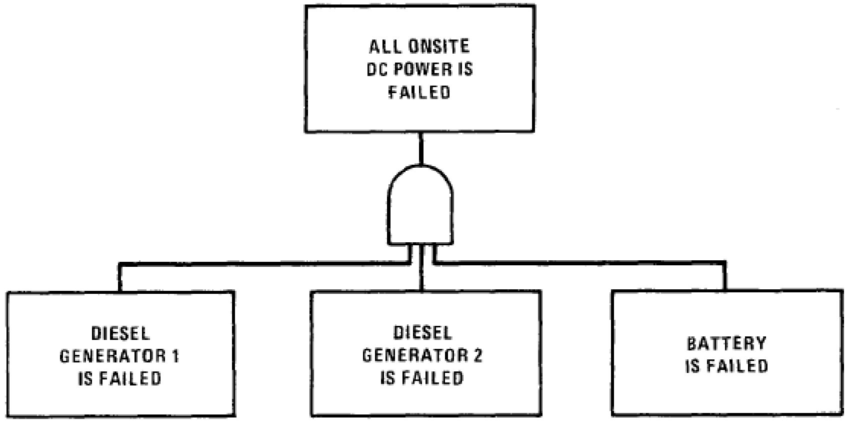
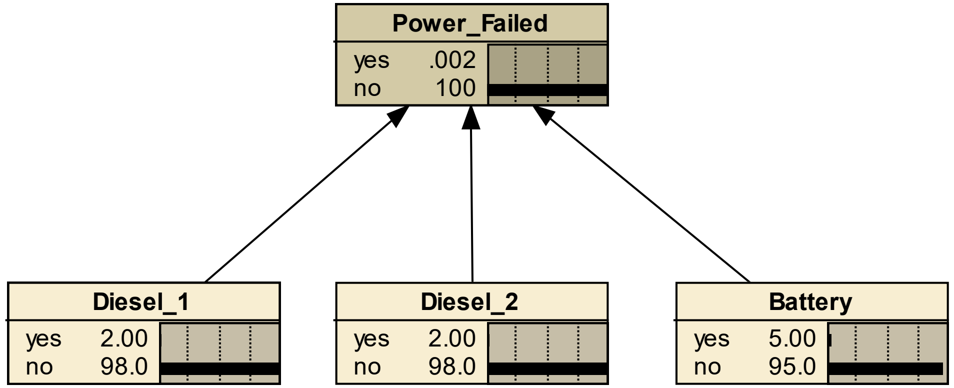

Power_Failed Table (in Bayes net FT_AND_NUREG) *

Node: Power_Failed | Apply | OK

Chance | % Probability | Reset | Close

Diesel_1	Diesel_2	Battery	yes	no
yes	yes	yes	100	0
yes	yes	no	0	100
yes	no	yes	0	100
yes	no	no	0	100
no	yes	yes	0	100
no	yes	no	0	100
no	no	yes	0	100
no	no	no	0	100



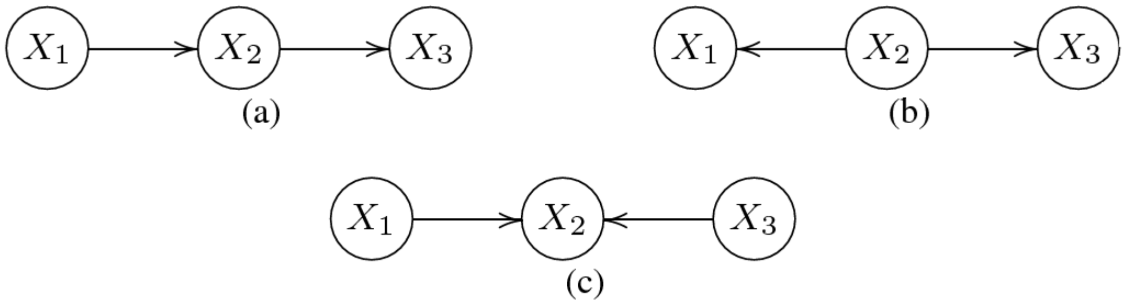


Information encoded in the graph (Semantics)

- a) X_1 is conditionally independent of X_3

$$(X_1 \not\perp X_3)$$
$$(X_1 \perp X_3 | X_2)$$

- b) X_1 is conditionally independent of X_3



- c) X_1 is independent of X_3 but not conditionally independent

$$X_1 \perp X_3$$
$$X_1 \not\perp X_3 | X_2$$

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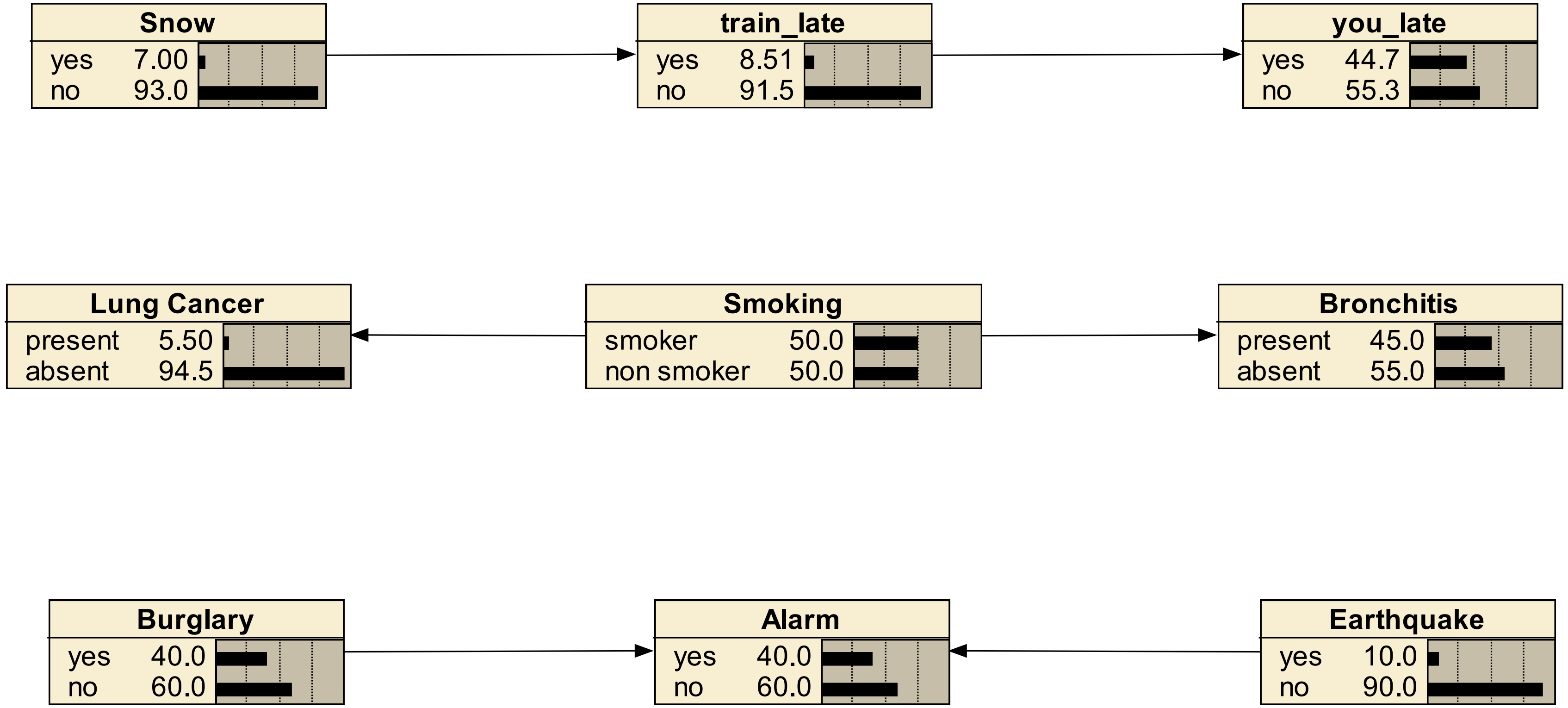
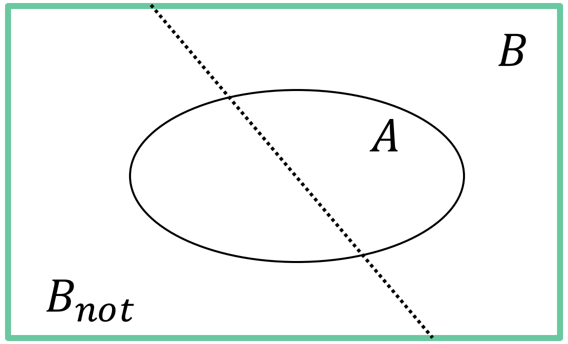
Information encoded in the graph (Semantics)



Fault Trees (AND with dependence)

$$P(A) = P(A|B)P(B) + P(A|B_{not})P(B_{not})$$

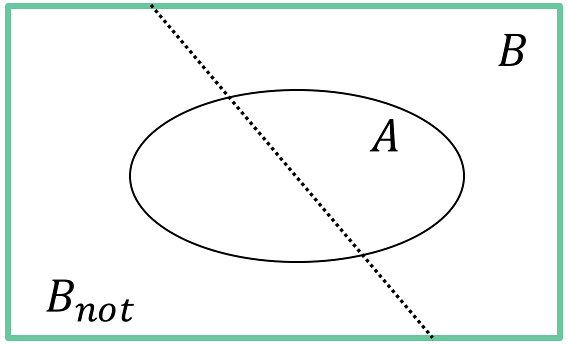
with $B \cap B_{not} = \emptyset$ and $B \cup B_{not} = \Omega$



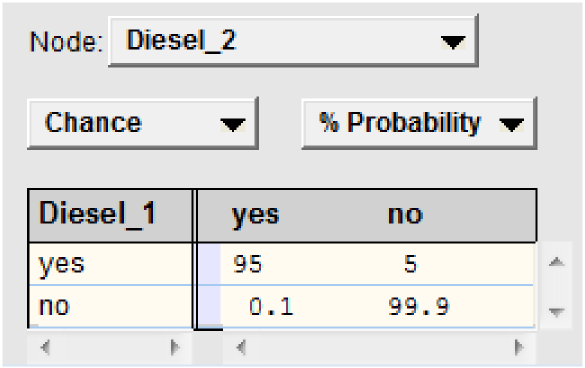
- $P(Diesel_2 = Yes) = 0.02$
- $P(Diesel_2 = Yes|Diesel_1 = No) = 0.001$ (from experts, data, or some other source)
- From total probability, we can evaluate $P(Diesel_2 = Yes|Diesel_1 = Yes)$:
  - $P(Diesel_2 = Yes) = P(Diesel_2 = Yes|Diesel_1 = Yes)$

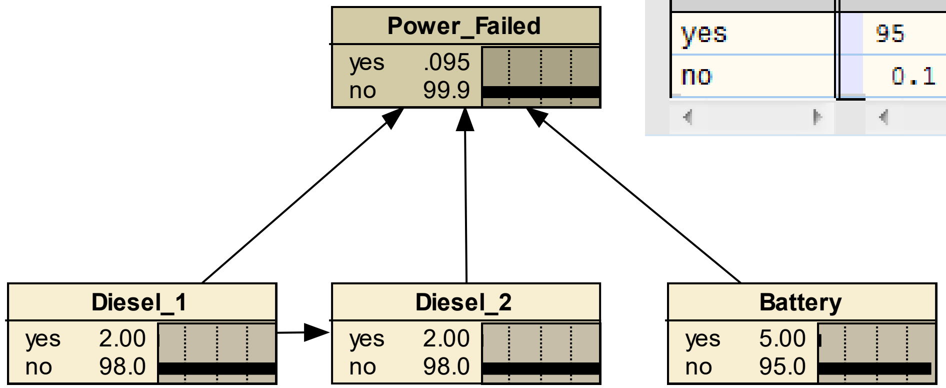
Fault Trees (AND with dependence)

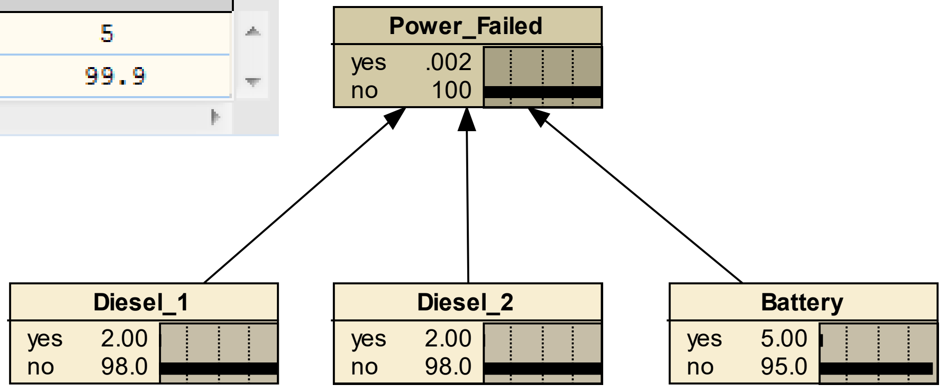
$P(A) = P(A|B)P(B) + P(A|B_{not})P(B_{not})$
with $B \cap B_{not} = \emptyset$ and $B \cup B_{not} = \Omega$



- $P(Diesel_2 = Yes) = 0.02$
- $P(Diesel_2 = Yes|Diesel_1 = No) = 0.001$ (from experts, data, or some other source)
- From total probability, we can evaluate $P(Diesel_2 = Yes|Diesel_1 = Yes)$:
  - $P(Diesel_2 = Yes) = P(Diesel_2 = Yes|Diesel_1 = Yes) * P(Diesel_1 = Yes) +$
  - $P(Diesel_2 = Yes|Diesel_1 = No) * P(Diesel_1 = No)$
  - $\rightarrow 0.02 = P(Diesel_2 = Yes|Diesel_1 = Yes) * 0.02 + 0.001 * 0.98$
- Then, we can compute $P(Power\_Failed)$







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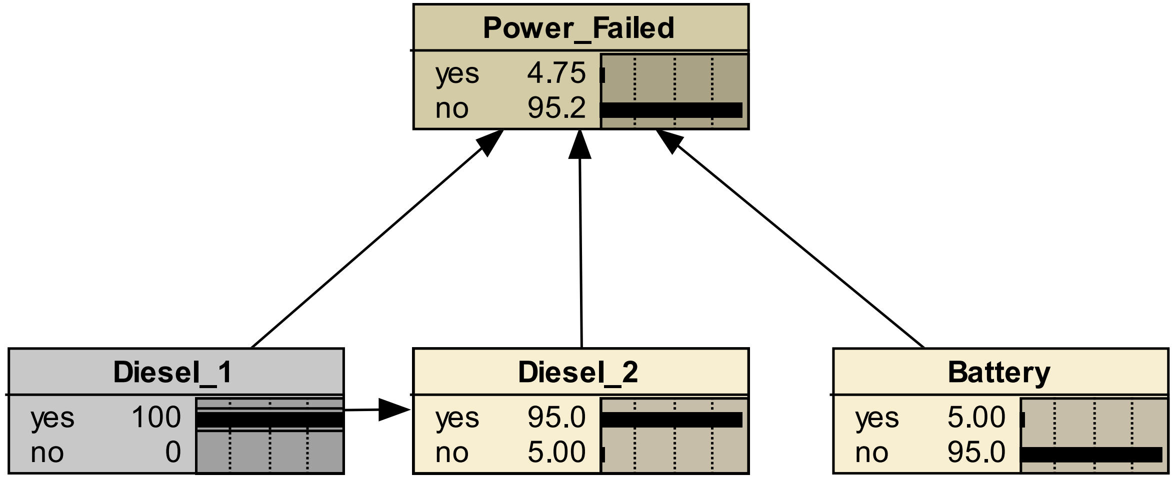
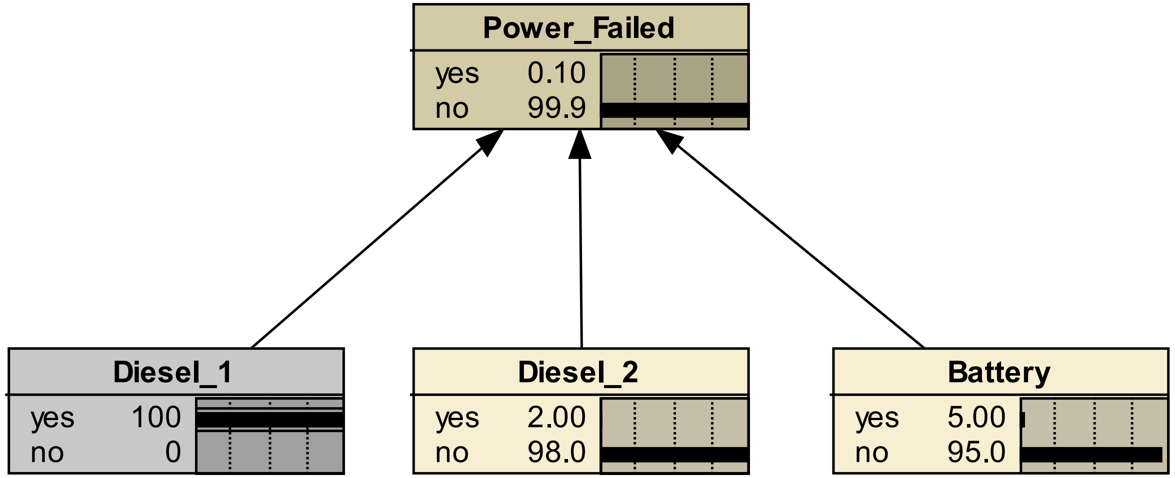
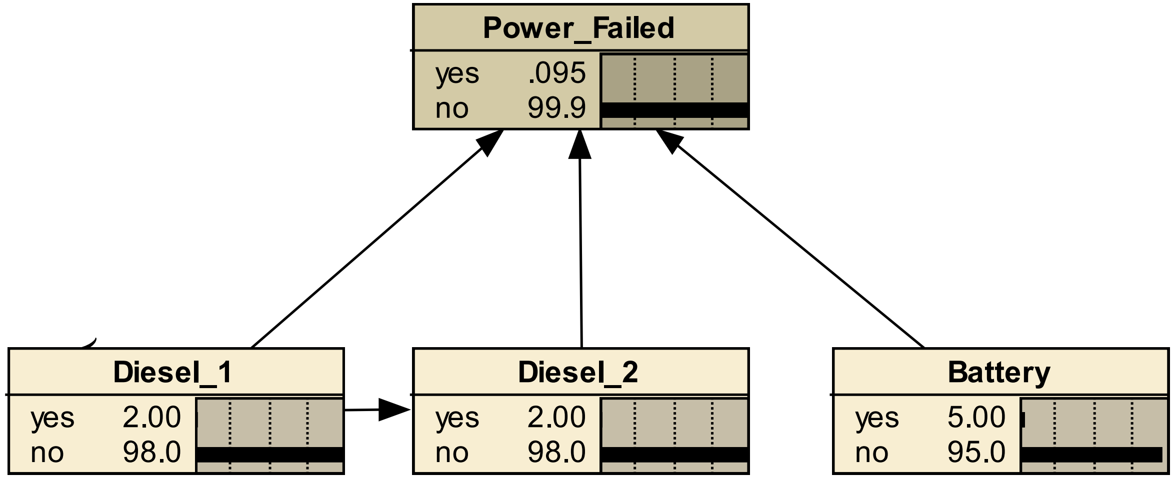
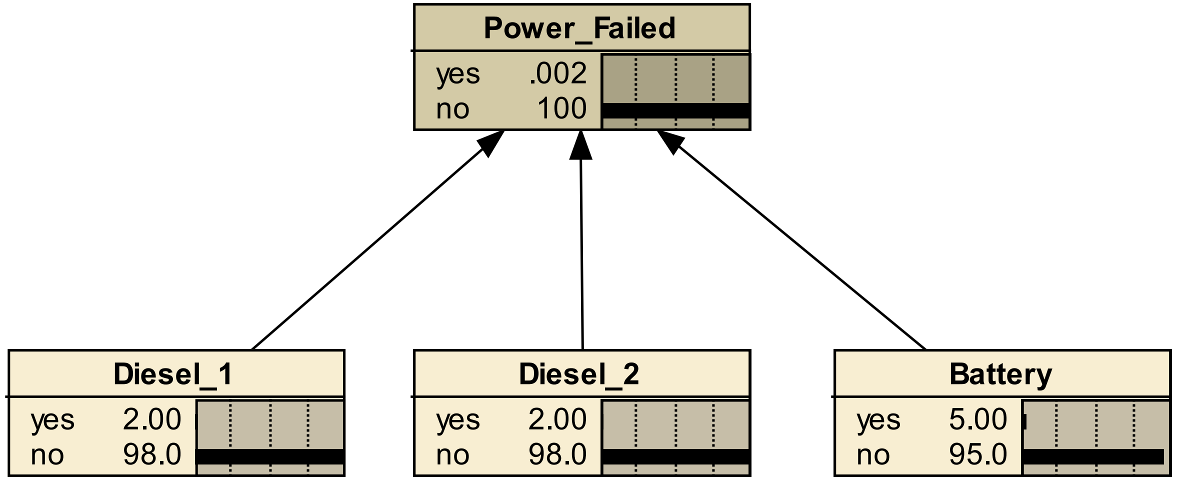
Conditional distributions





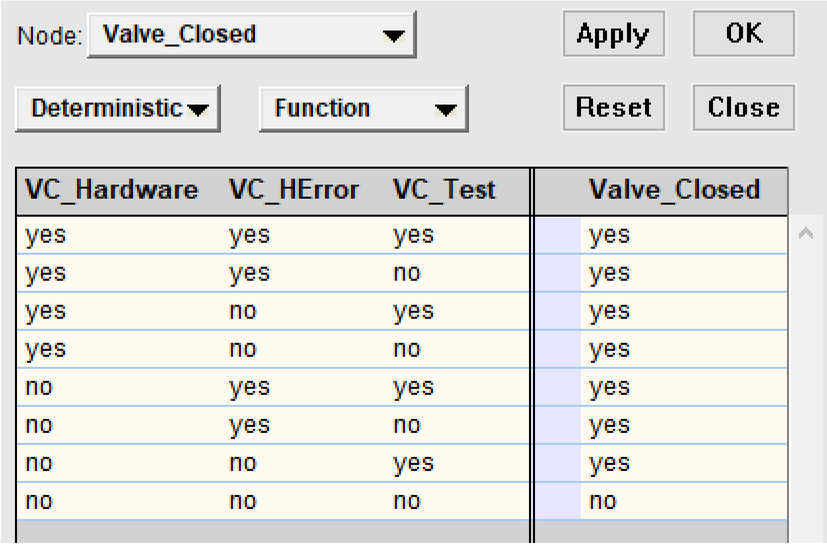




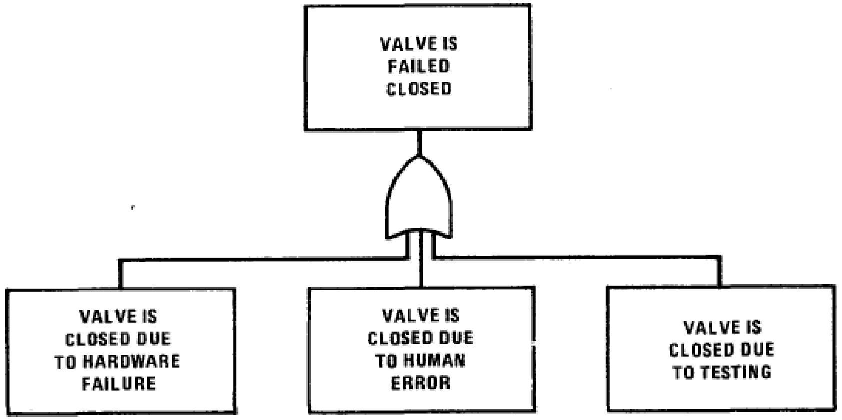
Fault Trees (OR)

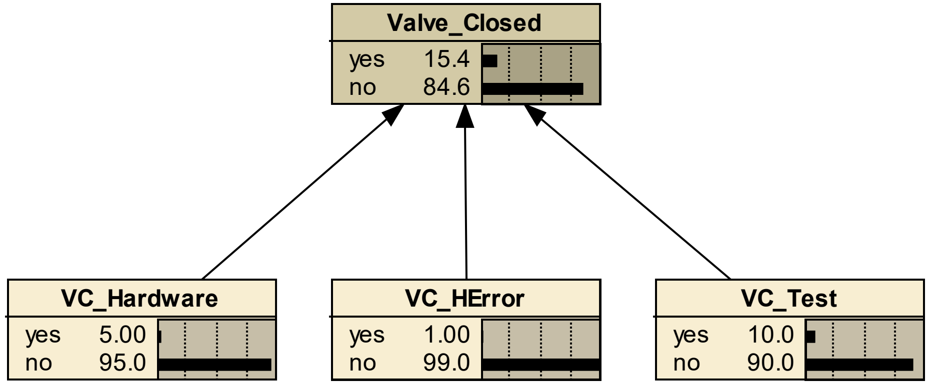
- $P(VC\_Hardware) = 0.05$, on demand
- $P(VC\_HError) = 0.01$, on demand
- $P(VC\_Test) = 0.1$, on demand

- What is $P(Valve\_Closed)$?



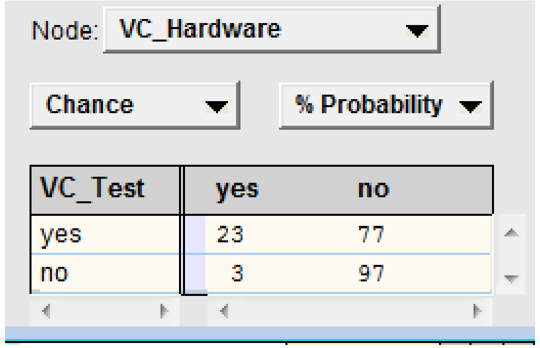
VC_Hardware	VC_HError	VC_Test	Valve_Closed
yes	yes	yes	yes
yes	yes	no	yes
yes	no	yes	yes
yes	no	no	yes
no	yes	yes	yes
no	yes	no	yes
no	no	yes	yes
no	no	no	no

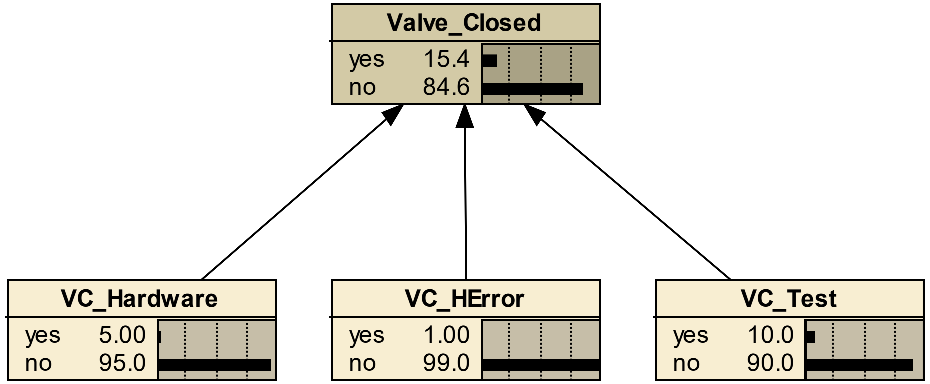


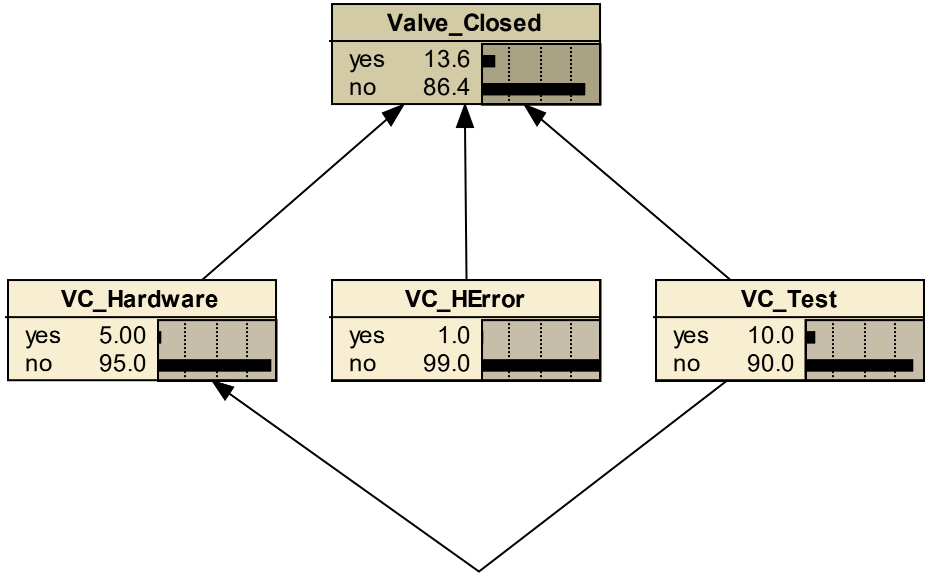


Fault Trees (OR with dependence)

- $P(VC\_Hardware = No) = 0.95$
- $P(VC\_Hardware = No|VC\_Test = Yes) = 0.77$ (from experts, data, or some other source)
- From total probability, we can evaluate $P(VC\_Hardware = No|VC\_Test = No)$:
  - $P(VC\_Hardware = No) = P(VC\_Hardware = No|VC\_Test = No) * P(VC\_Test = No) +$
  - $P(VC\_Hardware = No|VC\_Test = Yes) * P(VC\_Test = Yes)$
  - $\rightarrow 0.95 = P(VC\_Hardware = No|VC\_Test = No) * 0.90 + 0.77 * 0.10$
- Then, we can compute $P(Valve\_Closed)$







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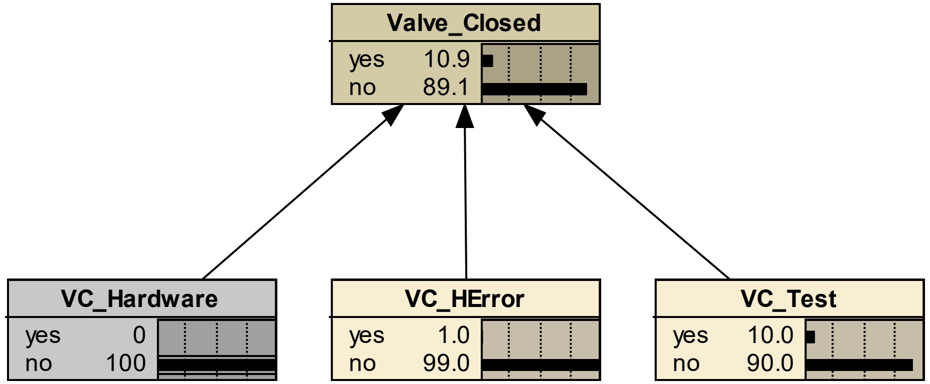
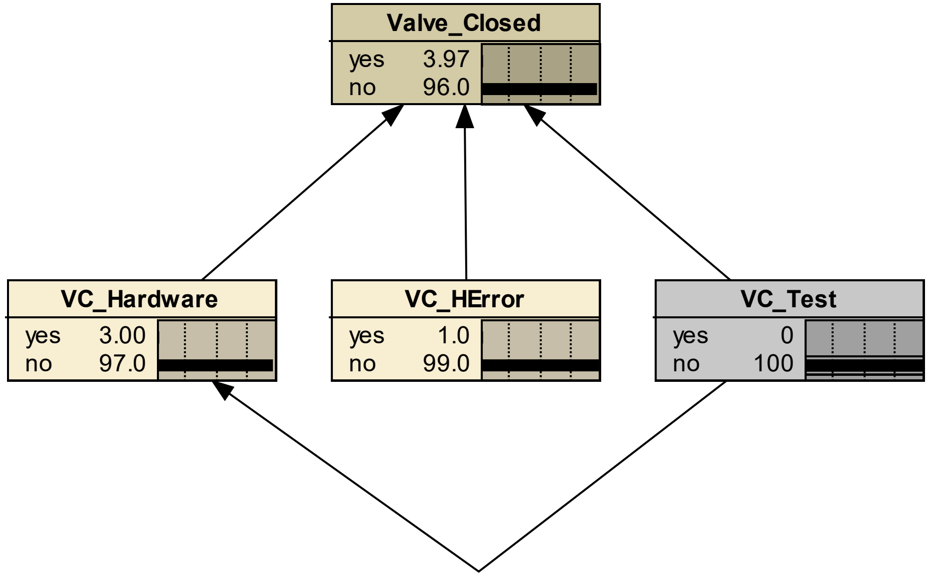
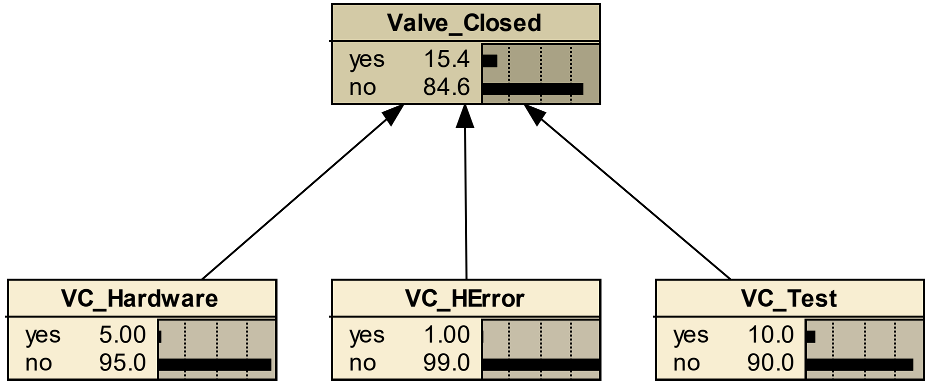
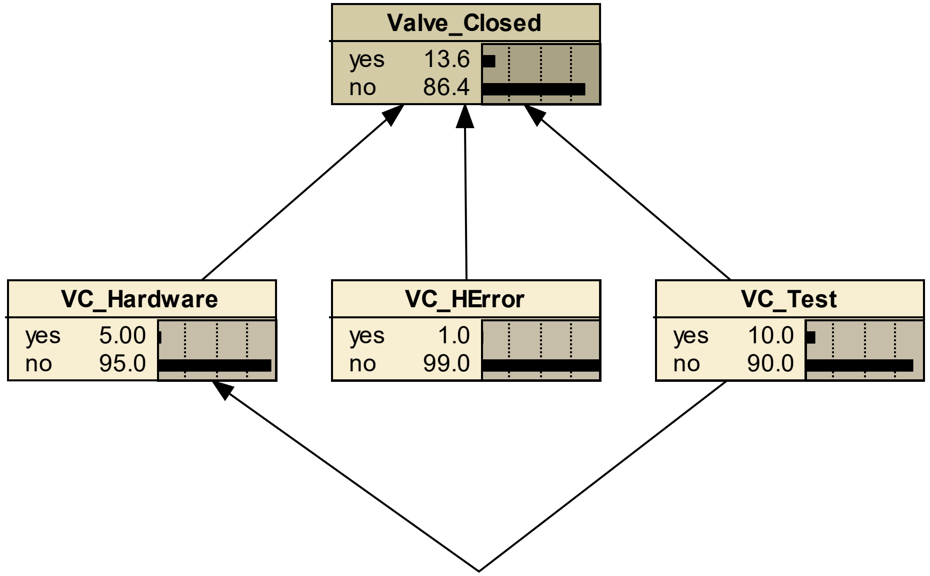
Fault Trees (OR with dependence)









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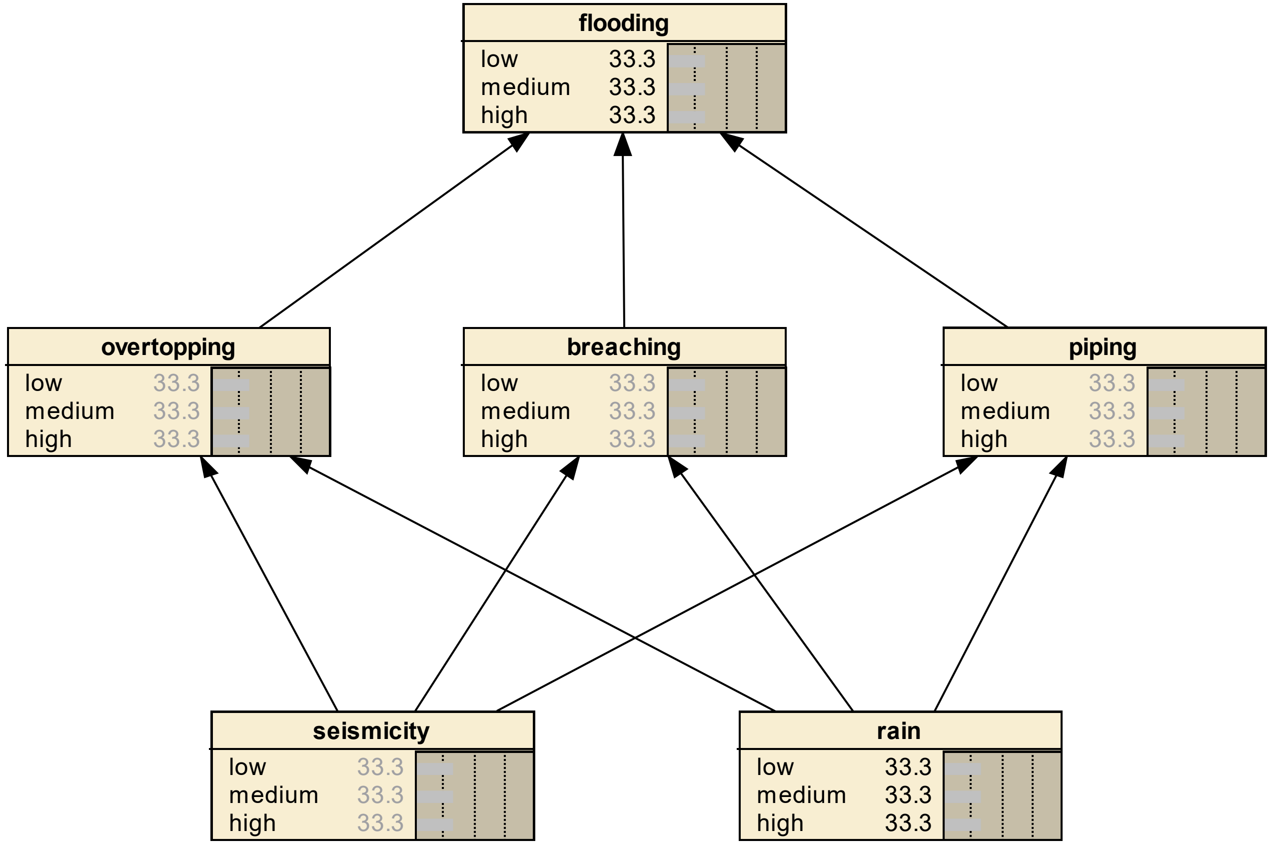
Simplified Flooding (during lecture)



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Probabilistic **M**odelling of real-world phenomena through **O**bse**R**vations and **E**licitation (MORE)

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What is **MORE?**

Real-world phenomena (e.g., rainfall, earthquakes, cars crossing bridges, ocean waves) are random and **unpredictable!**

How can we take this into account in our engineering research and design?

What is **MORE?**

Real-world phenomena (e.g., rainfall, earthquakes, cars crossing bridges, ocean waves) are random and **unpredictable!**

How can we take this into account in our engineering research and design?

In this module, you will use advanced probabilistic methods that **incorporate observations and expert opinion to support decisions** that make our **lives safer and more manageable**.

Who is **MORE**?

- **You!**
- Elisa Ragno (HE)
- Femke Vossepoel (Geo Eng)
- Patricia Mares Nasarre (HE)
- Anna Storiko (WM)
- Max Ramgraber (Geo Eng)
- Oswaldo Morales Napoles (HE)
- Robert Lanzafame (HE)
- Juan Pablo Aguilar-López (HE)

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What will we do in MORE?

Week	Day	time	Topic	Assignment	Group Project	Lecturer
1			**BAYES THEOREM AND MCMC**	A1	Form Group and Find Topic	Elisa Anna Anna Anna
2			**DEPENDENCE MODELLING:** Nonstatioanry distributions Multivariate distributions	A2 A3	Form Group and Find Topic	Juan Elisa Elisa Elisa
3				A4	Deadline Project Proposal	Elisa Patricia Elisa Oswaldo
4	6-12-2024	13:45	Groups Working on Data for their project (no class)	A5	Refine Proposal and Data collection	Oswaldo Elisa Oswaldo -
5			**DATA ASSIMILATION**	A6	Data Collection and Processing	Max Max Max Oswaldo
6	20-12-2024	13:45	**EXPERT JUDGMENT** No Class	A7	Finalize Project Proposal including detailed porcessing of data	Oswaldo Oswaldo - -

Anduryl, PyBanshee, Matlatzinca, Chimera and the modelling of risk and reliability

Oswaldo Morales Nápoles

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